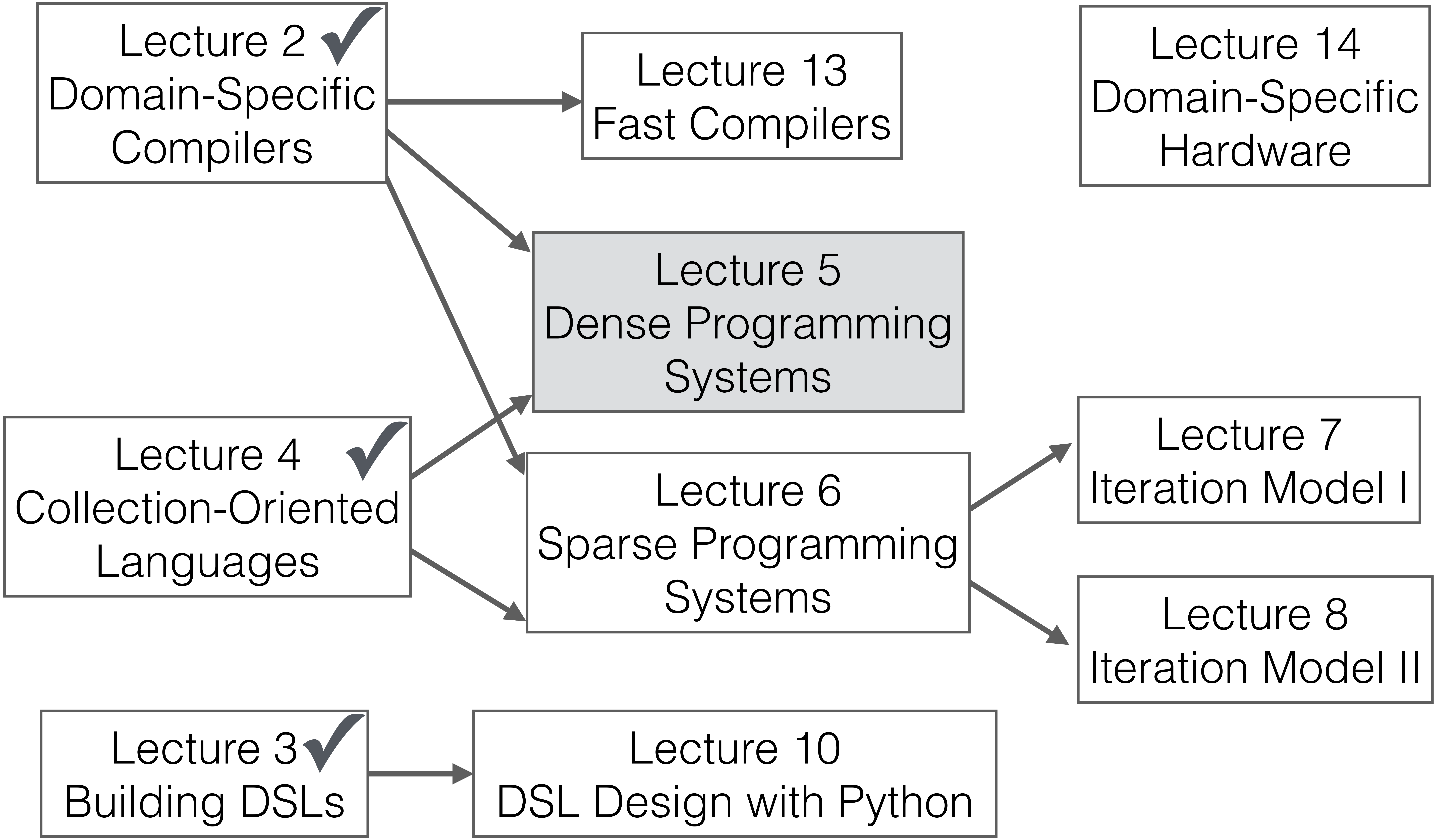
Lecture 5 - Dense Programming Systems

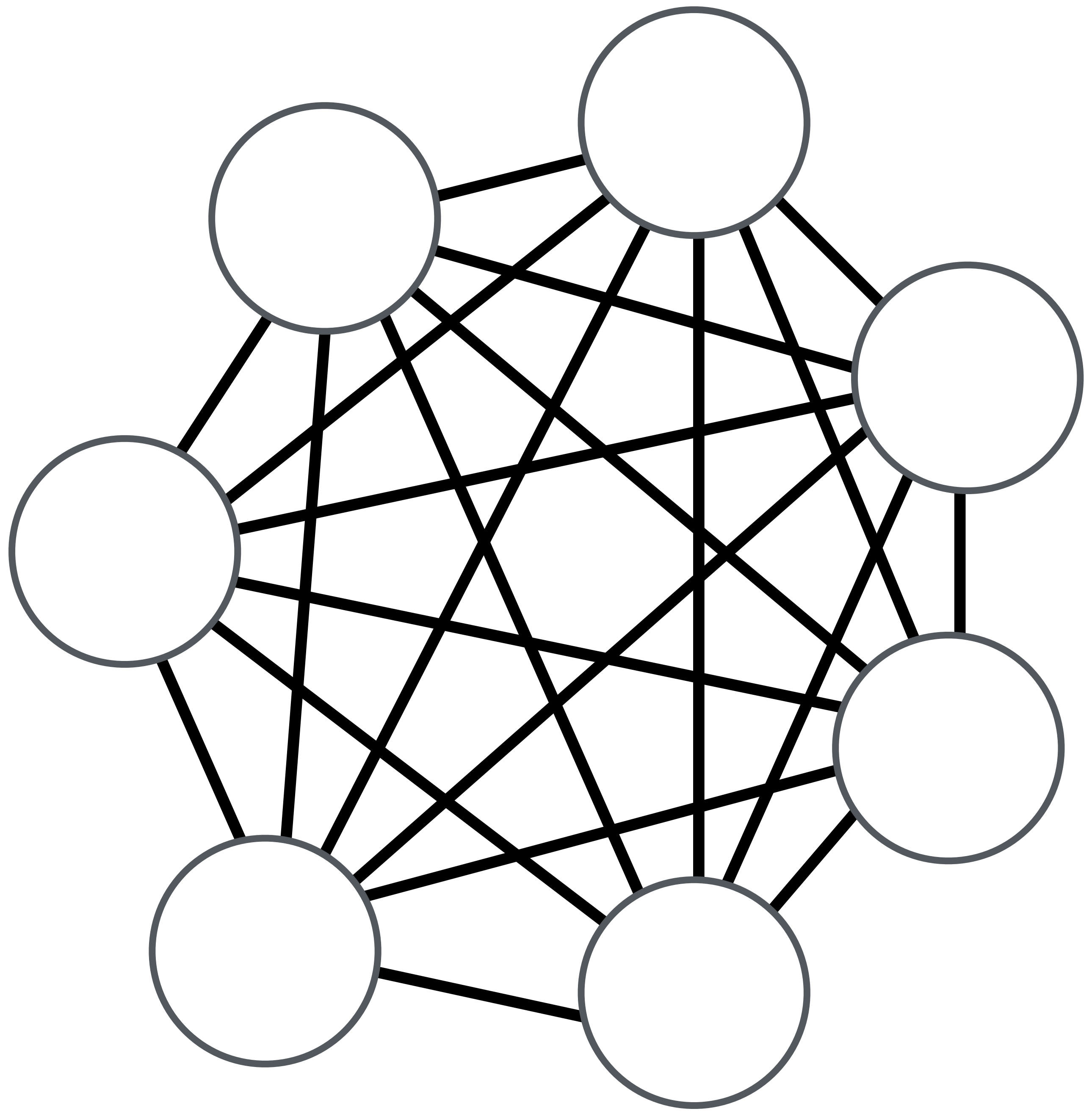
Stanford CS343D (Fall 2021)
Fred Kjolstad

Lecture Overview

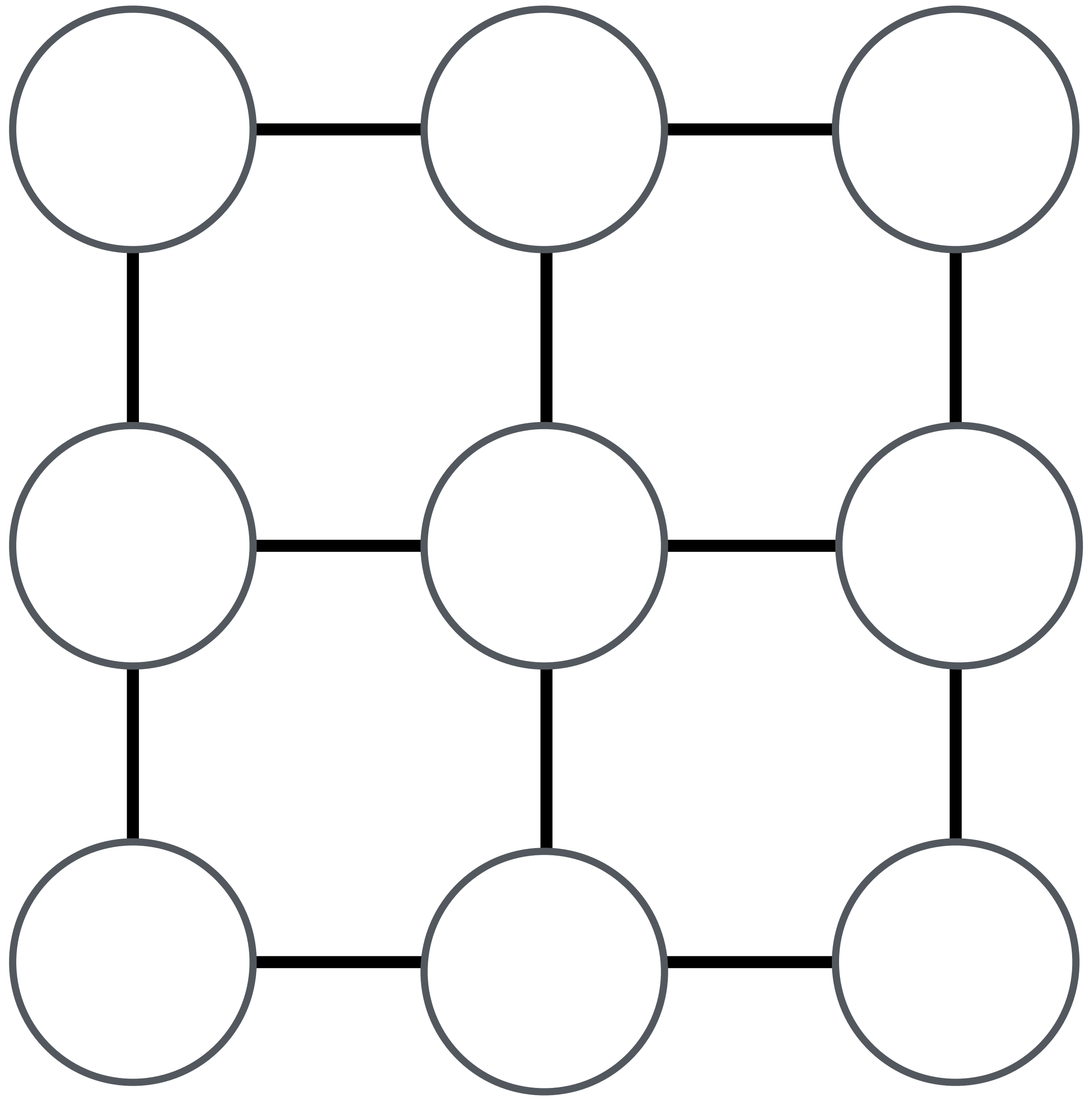


Terminology: Regular and Irregular

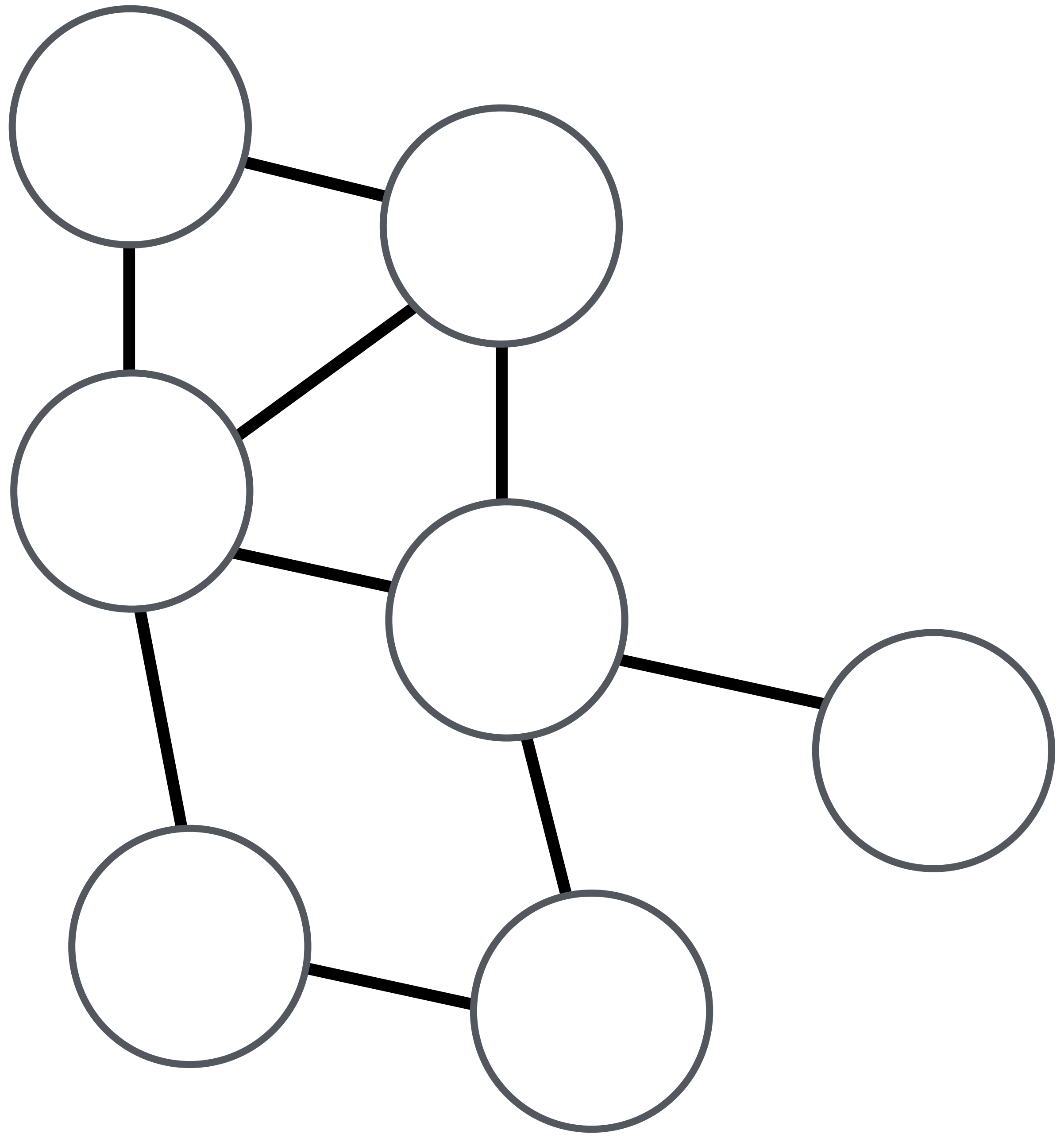
Fully Connected Regular System



Regular System

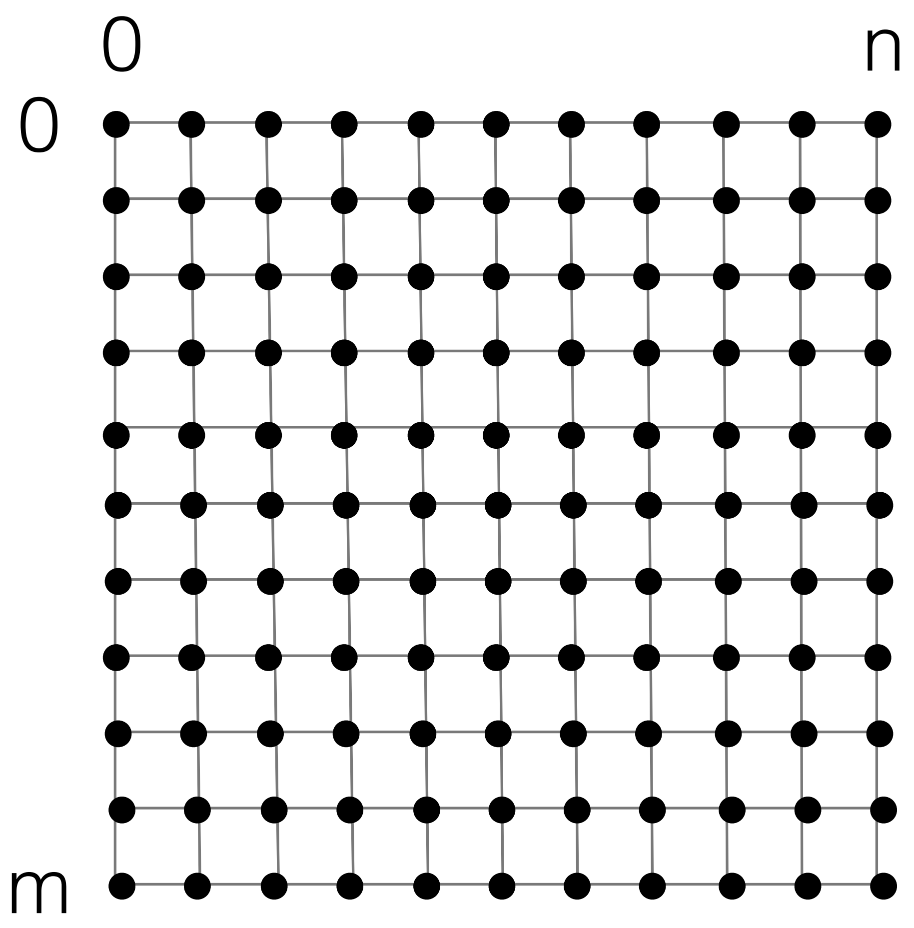


Irregular System



Terminology: Dense and Sparse

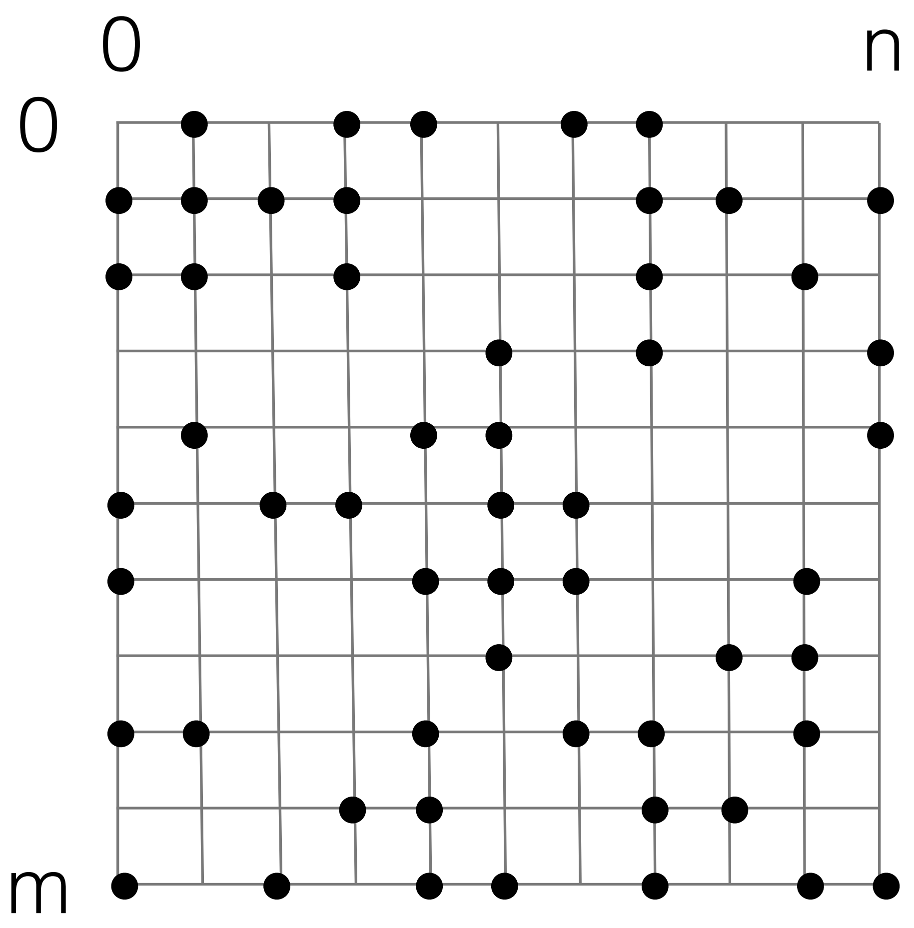
Dense loop iteration space




```
for (int i = 0; i < m; i++) {
  for (int j = 0; j < n; j++) {
    y[i] += A[i*n+j] * x[j];
  }
}
```

$$y = Ax$$

Sparse loop iteration space

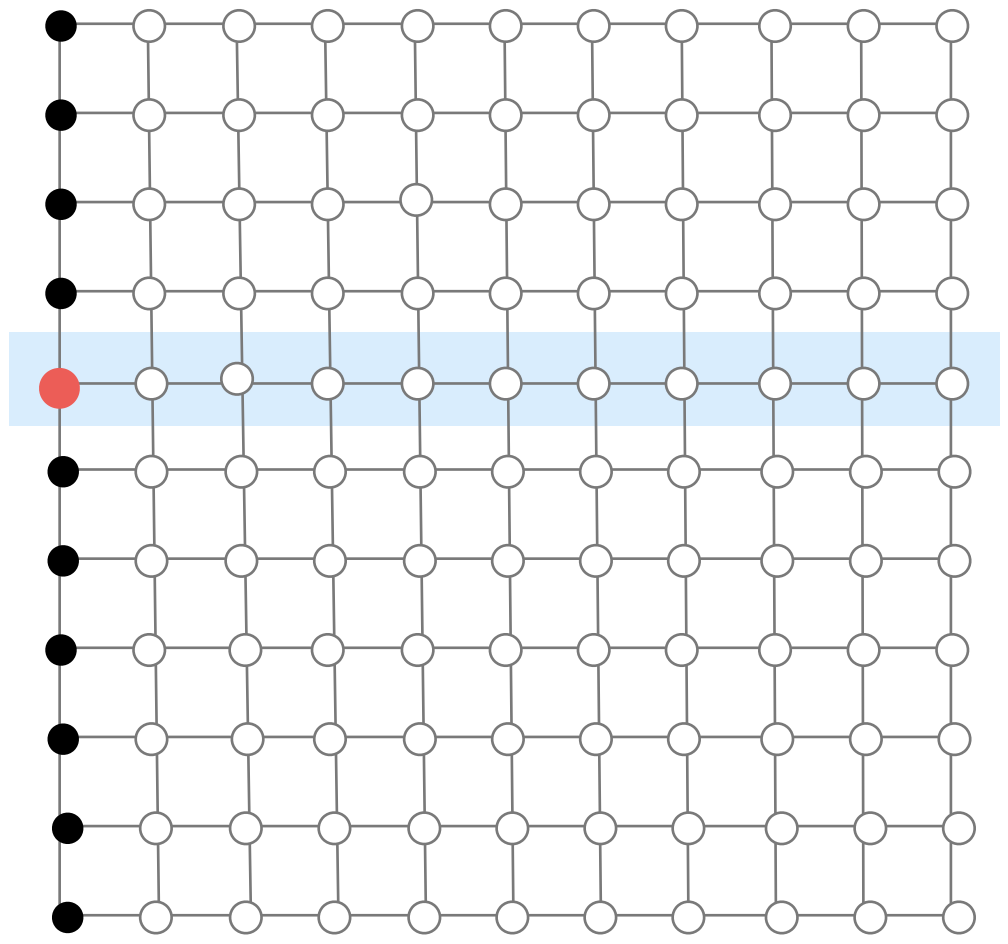



```
for (int i = 0; i < m; i++) {
 for (int pA = A2_pos[i]; pA < A_pos[i+1]; pA++) {
    int j = A_crd[pA];
    y[i] += A[pA] * x[j];
  }
}
```

$$y = Ax$$

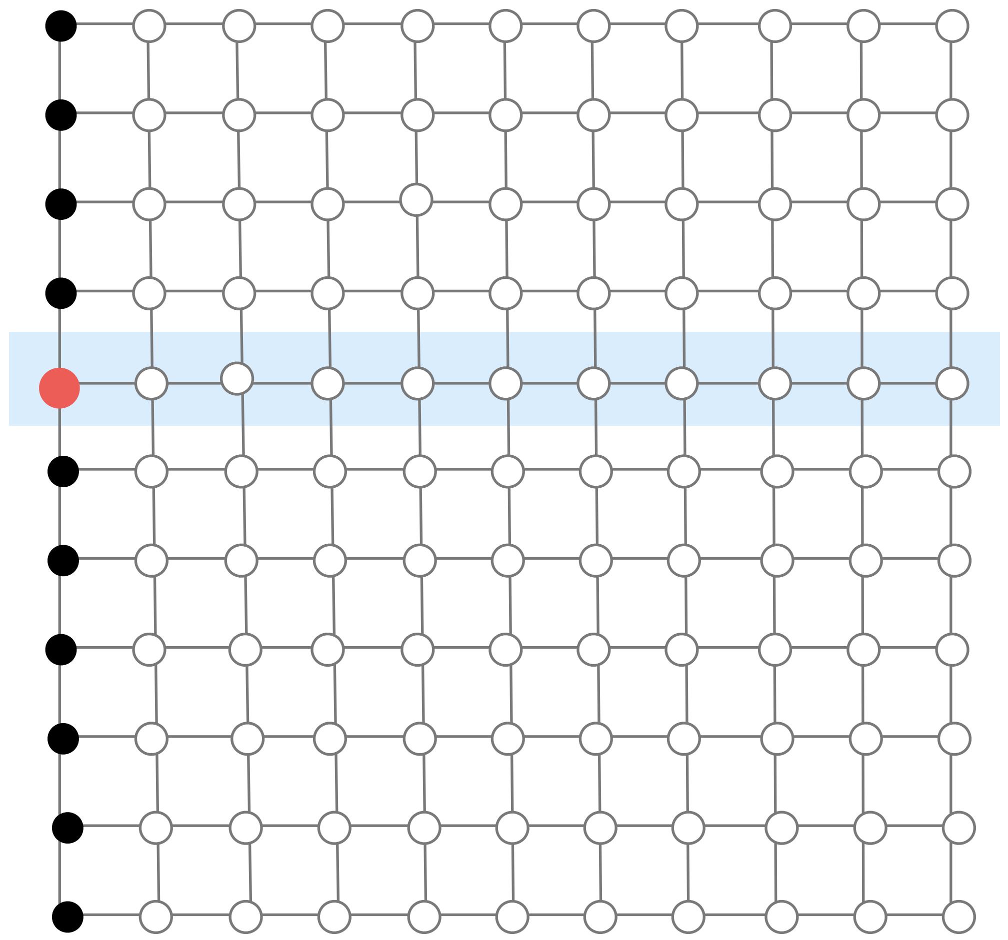
Dense applications

Dense Matrix-Vector Multiplication

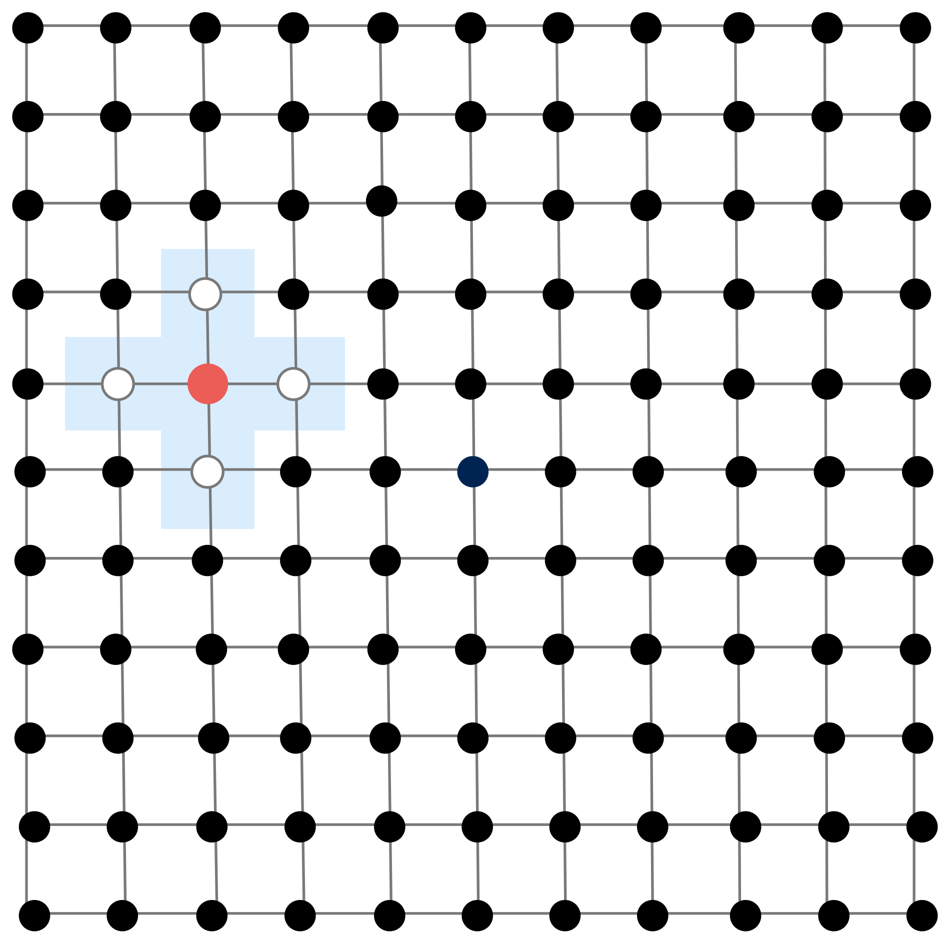


Dense applications

Dense Matrix-Vector Multiplication

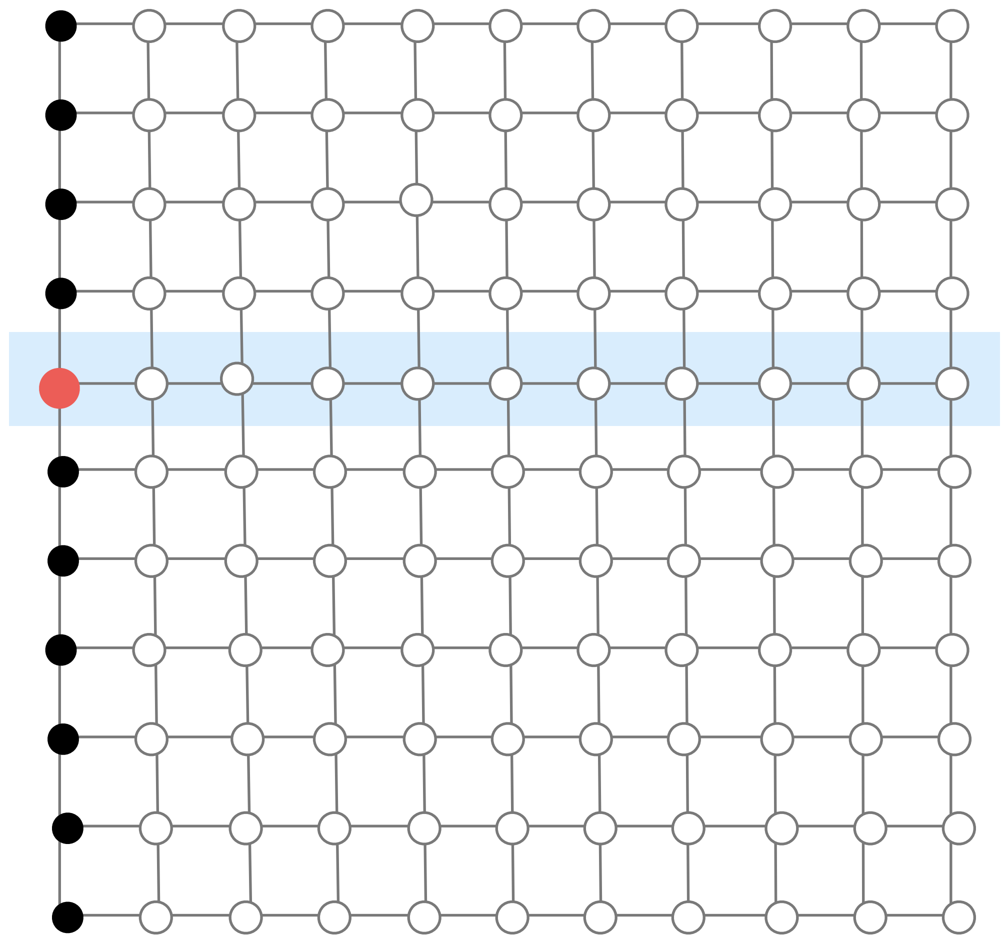


Stencils

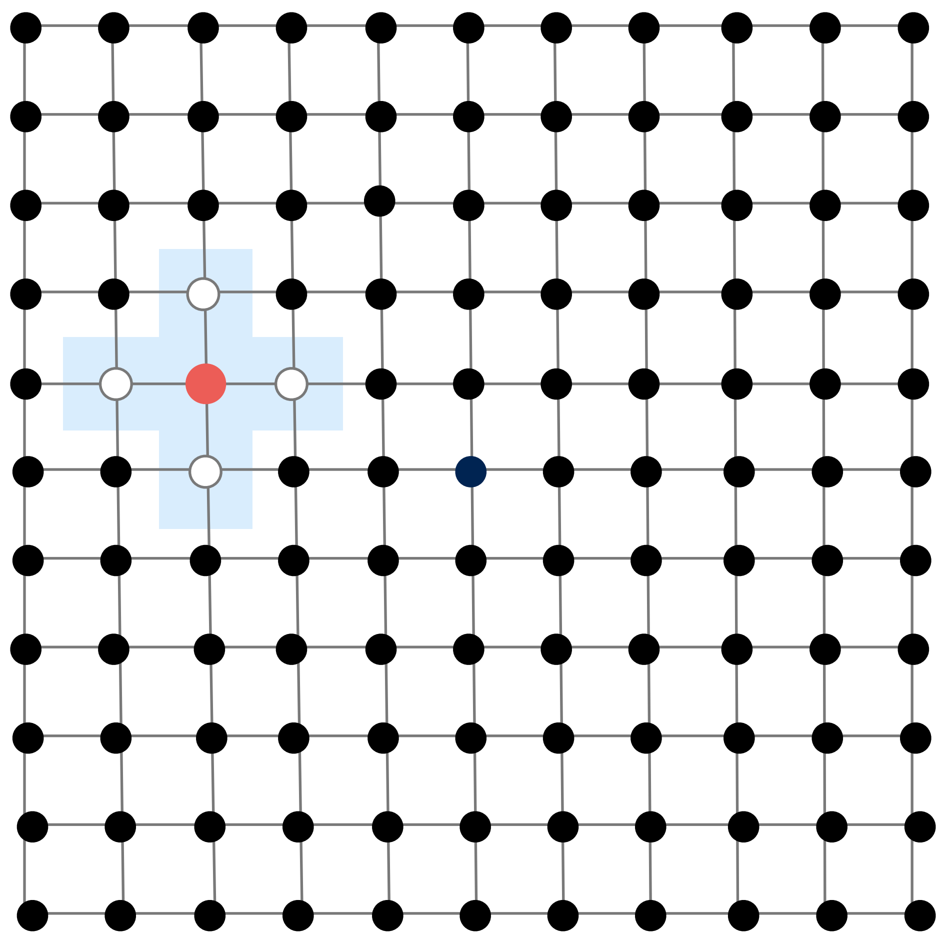


Dense applications

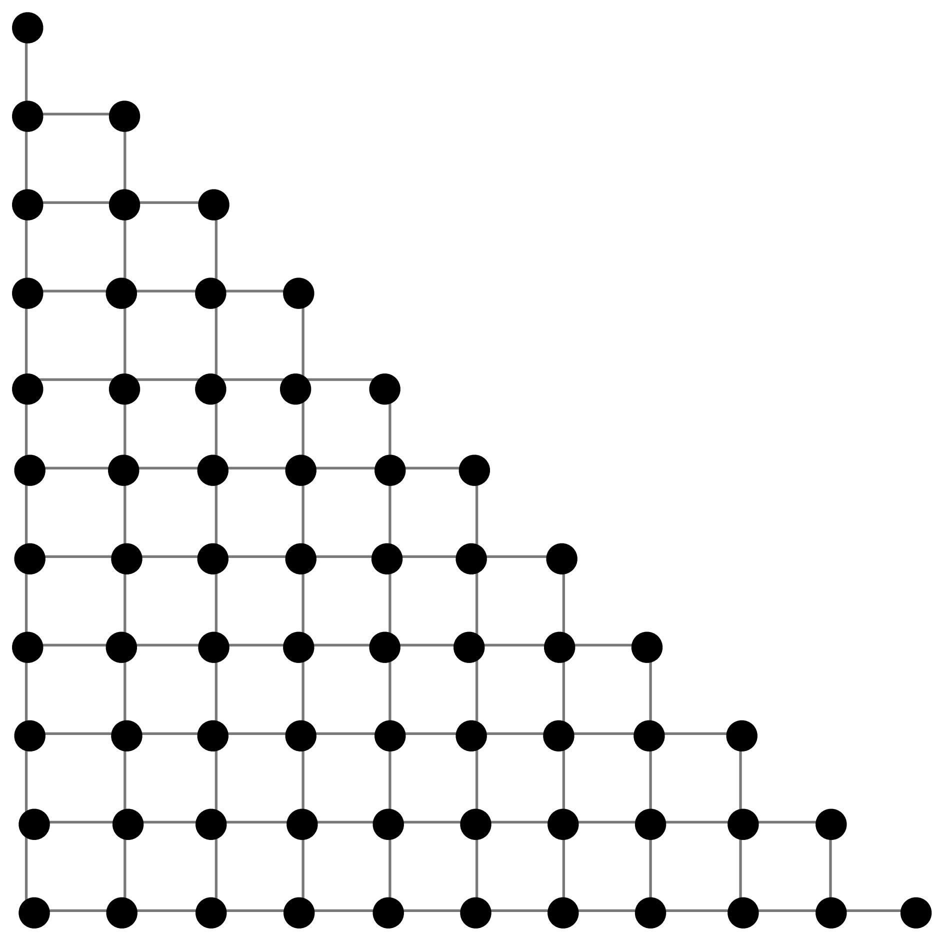
Dense Matrix-Vector Multiplication



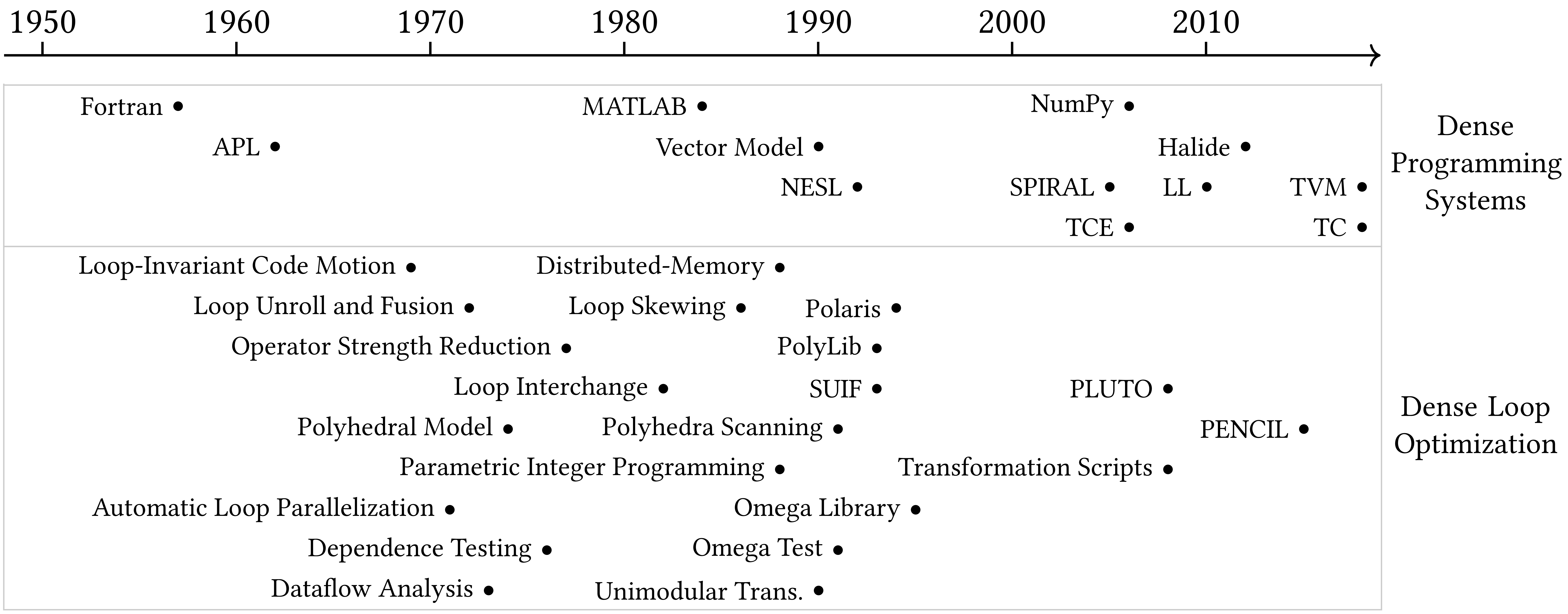
Stencils



Triagonal Solve



Timeline of some important developments in compilers and programming languages for dense compilers



Traditional compiler loop transformations

Reorder (interchange)

→

```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    A[i][j] = B[i][j] + C[i][j];
```

```
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    A[i][j] = B[i][j] + C[i][j];
```

Traditional compiler loop transformations

Split (Stripmine)

→

```
for (int i=0; i<m; i++)
  a[i] = b[i] + c[i];
```

```
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    a[i] = b[i] + c[i];
```

Traditional compiler loop transformations

Vectorize →

```
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    a[i] = b[i] + c[i];
```

```
for (int k=0; k<m; k+=4)
  a[k:k+4] = b[k:k+4] + c[k:k+4];
```

Traditional compiler loop transformations

```
for (int i=0; i<m; i++)
  a[i] = b[i] + c[i];

for (int i=0; i<m; i++)
  d[i] = -b[i];
```

Fusion →

```
for (int i=0; i<m; i++)
  a[i] = b[i] + c[i];
  d[i] = -b[i];
```

Traditional compiler loop transformations

Collapse (flatten)

→

```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    A[i*m+j] = -B[i*m+j];
```

```
for (int ij=0; ij<m*n; ij++)
  A[ij] = -B[ij];
```

Two models of loop optimization: source code rewrite and mathematical frameworks

Source Code Rewrite

```
for (int i=0; i<m; i++) {
  a[i] = b[i] + c[i];
}
```

split(4) →

```
for (int k=0; k<m; k+=4) {
  for (int i=k; i<k+4; i++) {
    a[i] = b[i] + c[i];
}
```

Mathematical loop optimization frameworks include the polyhedral model

Two models of loop optimization: source code rewrite and mathematical frameworks

Source Code Rewrite

```
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  a[i] = b[i] + c[i];
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split(4) →

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for (int k=0; k<m; k+=4) {
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}
```

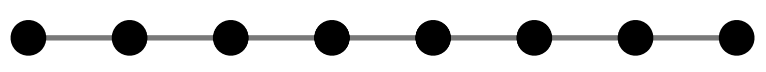
Mathematical Frameworks

```
for (int i=0; i<m; i++) {
  a[i] = b[i] + c[i];
}
```

```
for (int k=0; k<m; k+=4) {
  for (int i=k; i<k+4; i++) {
    a[i] = b[i] + c[i];
}
```

convert to integer domain ↓

code generation ↑



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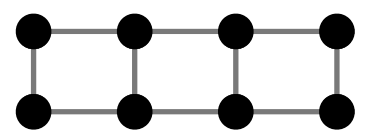

Mathematical loop optimization frameworks include the polyhedral model

Optimizing dense codes require complex tradeoffs between parallelism, locality, and work efficiency

Clean C++: 9.94 ms per megapixel

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

Fast x86 C++: 0.9 ms per megapixel

```
void fast_blur(const Image &in, Image &blurred) {
 __m128i one_third = _mm_set1_epi16(21846);
 #pragma omp parallel for
 for (int yTile = 0; yTile < in.height(); yTile += 32) {
  __m128i a, b, c, sum, avg;
  __m128i tmp[(256/8)*(32+2)];
  for (int xTile = 0; xTile < in.width(); xTile += 256) {
   __m128i *tmpPtr = tmp;
   for (int y = -1; y < 32+1; y++) {
    const uint16_t *inPtr = &(in(xTile, yTile+y));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_loadu_si128((__m128i*)(inPtr-1));
     b = _mm_loadu_si128((__m128i*)(inPtr+1));
     c = _mm_load_si128((__m128i*)(inPtr));
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(tmpPtr++, avg);
     inPtr += 8;
   }}
   tmpPtr = tmp;
   for (int y = 0; y < 32; y++) {
    __m128i *outPtr = (__m128i *)(&(blurred(xTile, yTile+y)));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_load_si128(tmpPtr+(2*256)/8);
     b = _mm_load_si128(tmpPtr+256/8);
     c = _mm_load_si128(tmpPtr++);
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(outPtr++, avg);
}}}}}
```

Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines. *Ragan-Kelley et al.* (2012)

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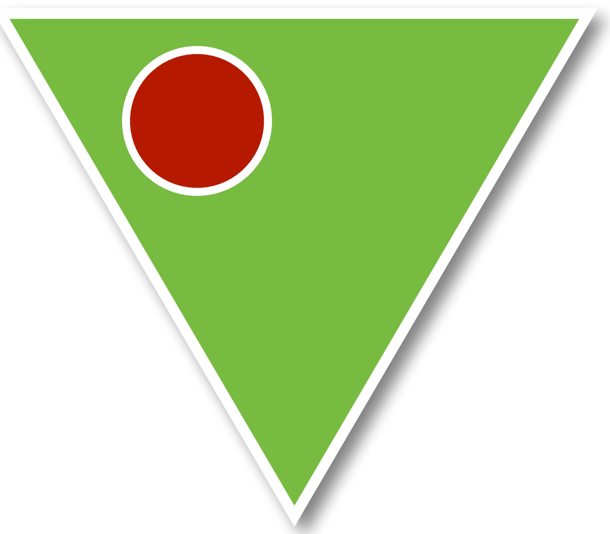
}
```



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   _m128i *tmpPtr = tmp;
   for (int y = -1; y < 32+1; y++) {
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   }}
   tmpPtr = tmp;
   for (int y = 0; y < 32; y++) {
    _m128i *outPtr = (_m128i *)(&(blurred(xTile, yTile+y)));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_load_si128(tmpPtr+(2*256)/8);
     b = _mm_load_si128(tmpPtr+256/8);
     c = _mm_load_si128(tmpPtr++);
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
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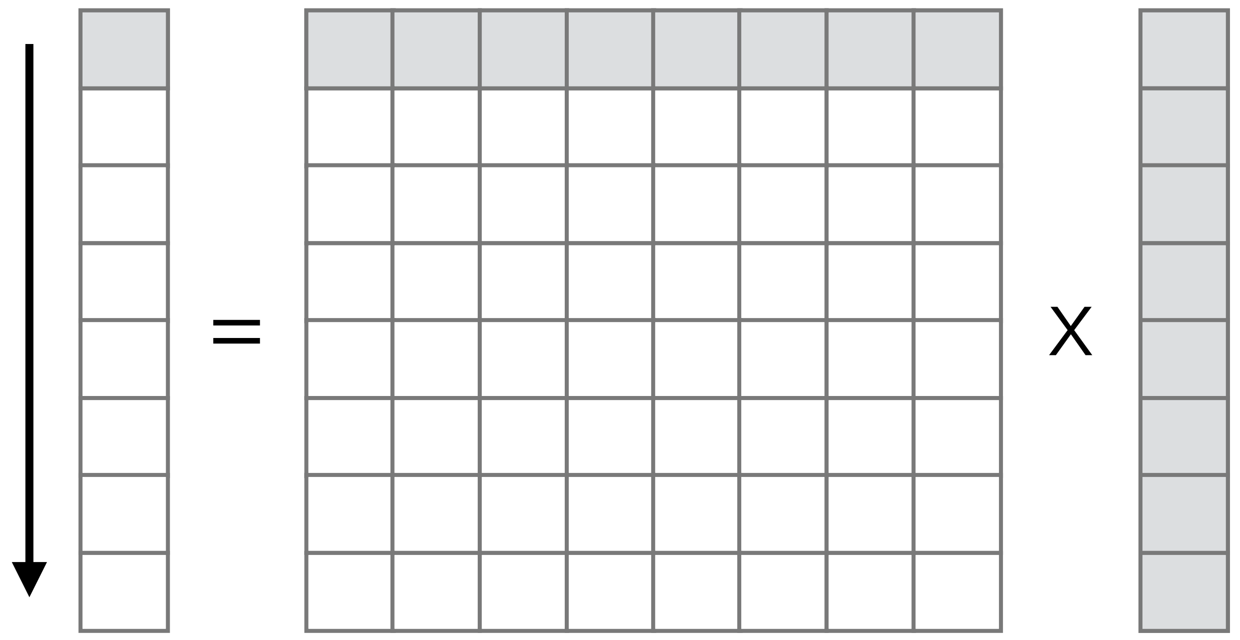
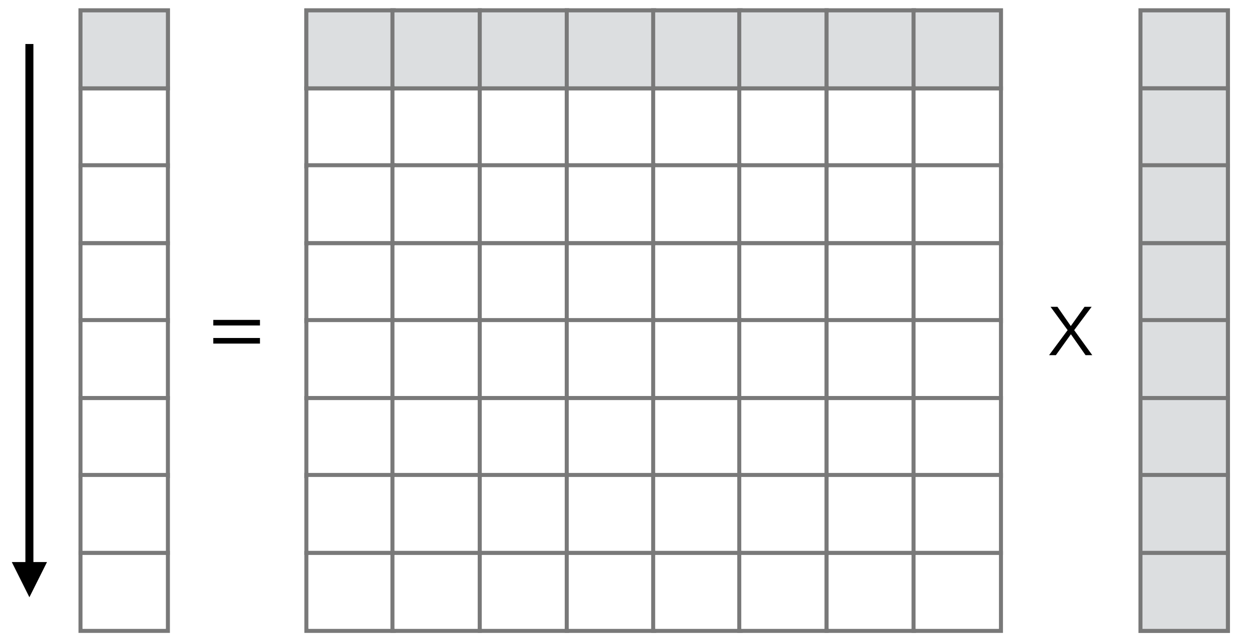
Parallelism in matrix-vector multiplication



```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

Parallelism in matrix-vector multiplication

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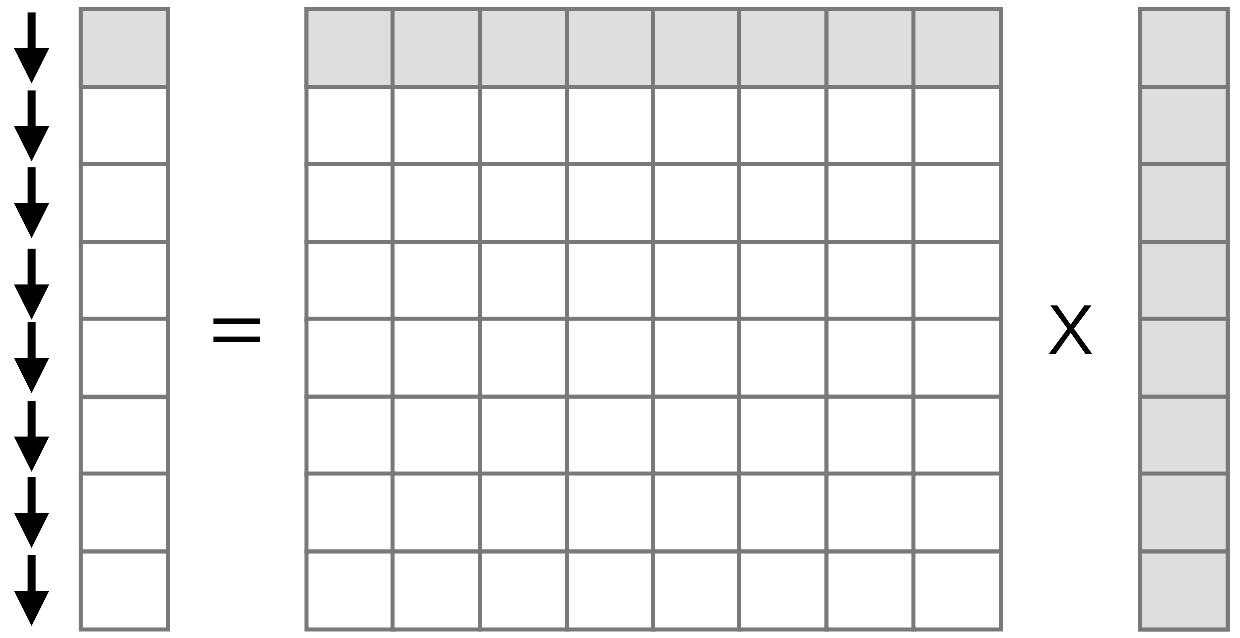


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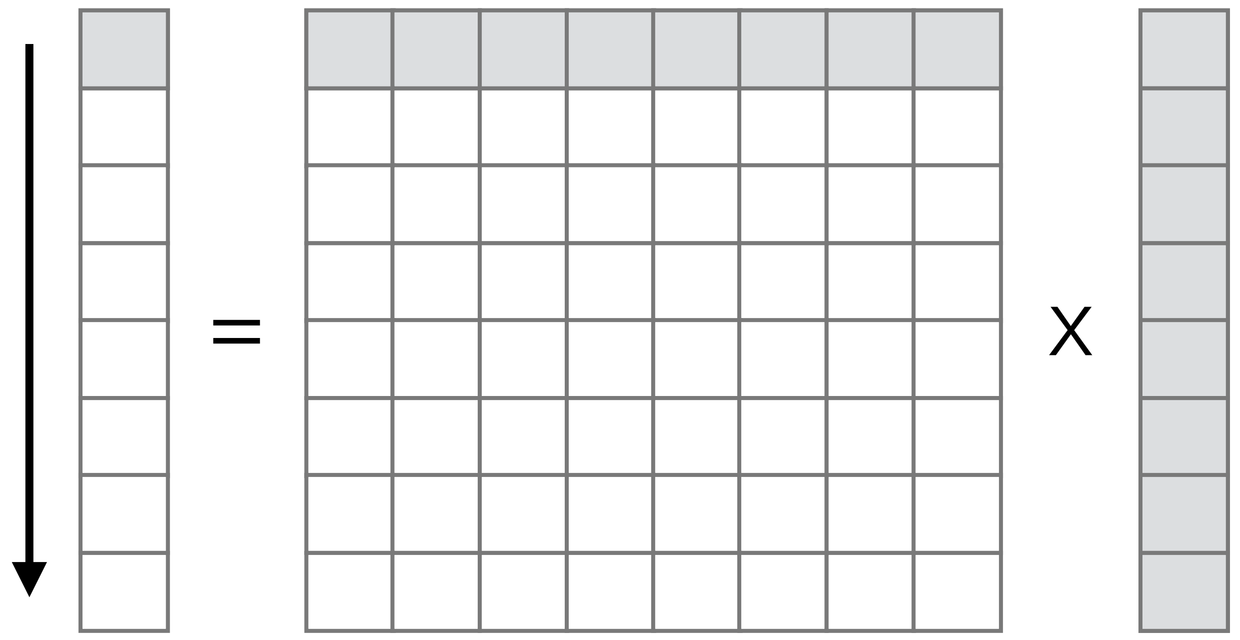

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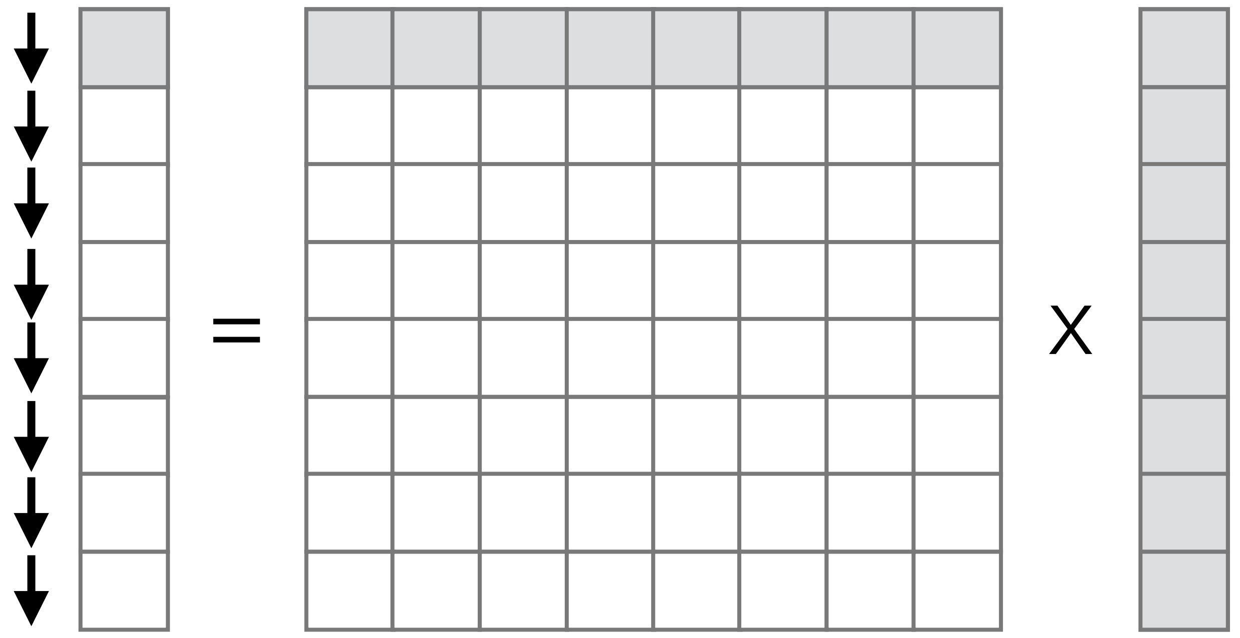

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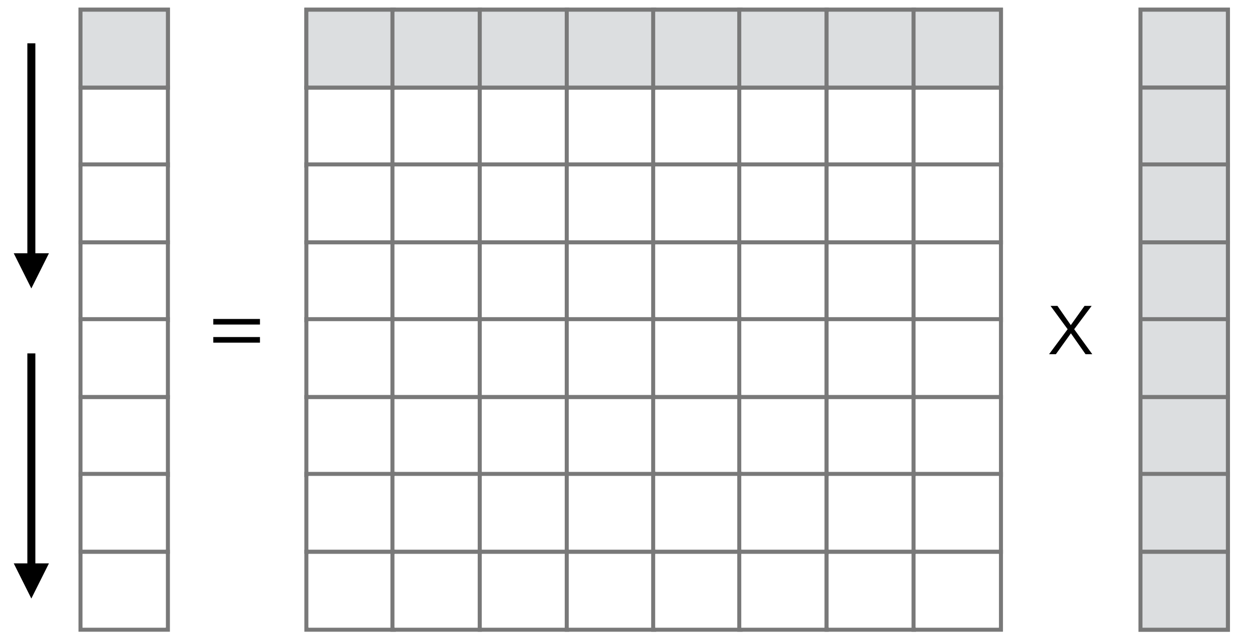
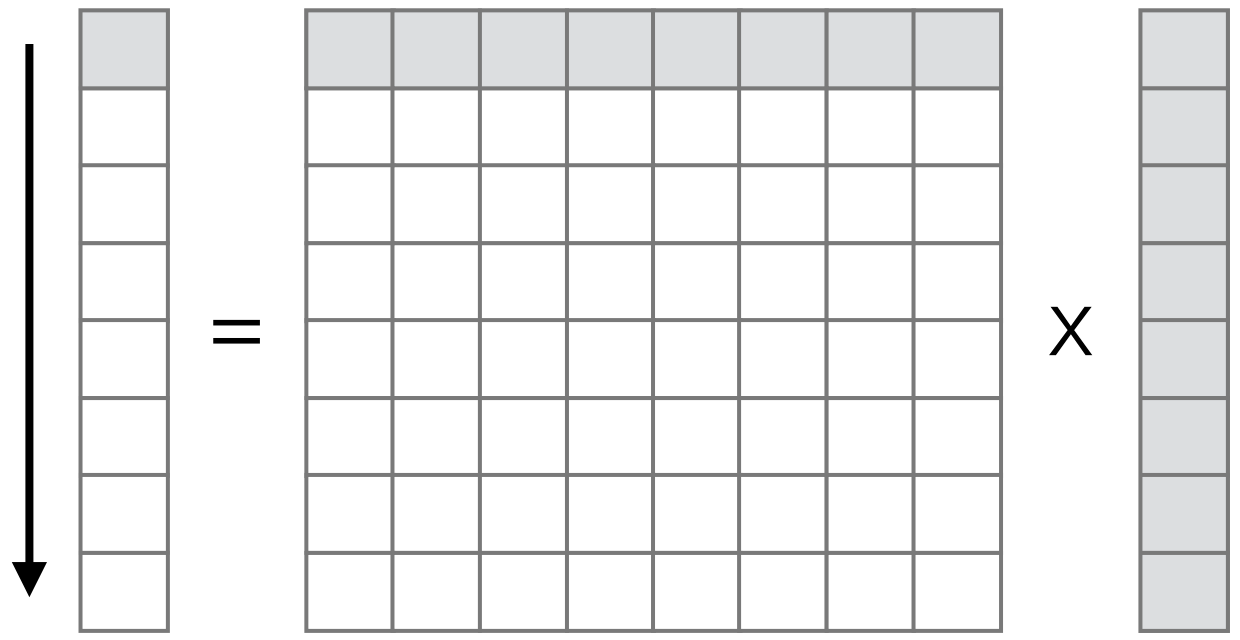


```
#pragma omp parallel for
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    for (int j=0; j<n; j++)
      y[i] += A[i*n+j] * x[j];
```

Parallelism in matrix-vector multiplication

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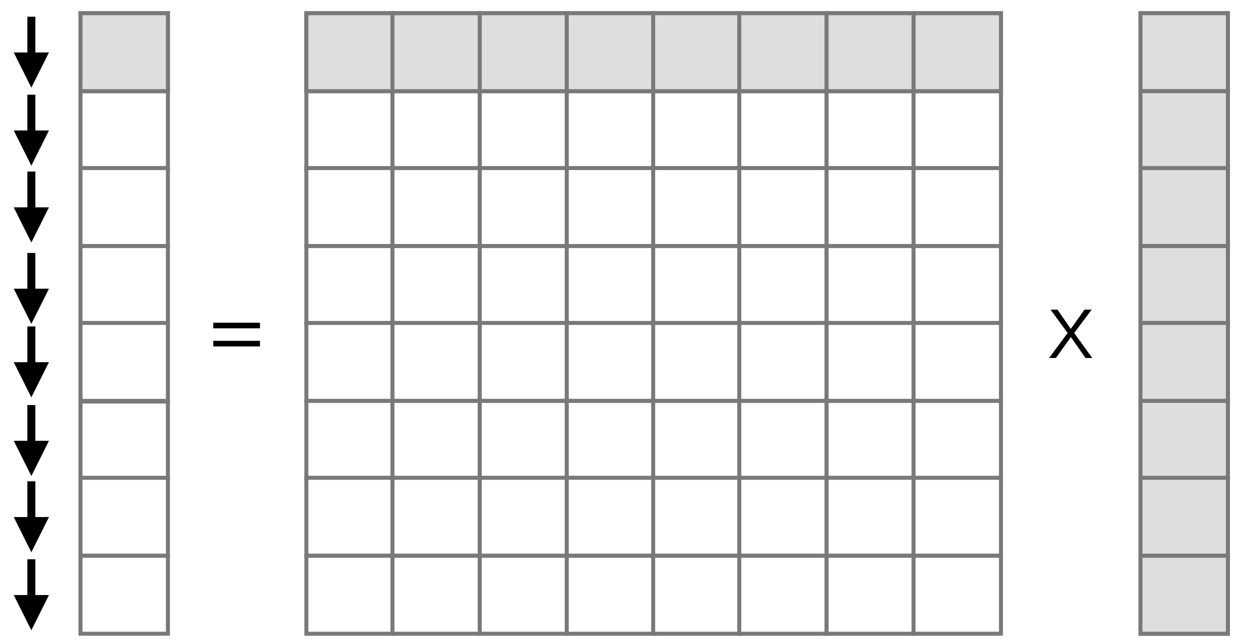


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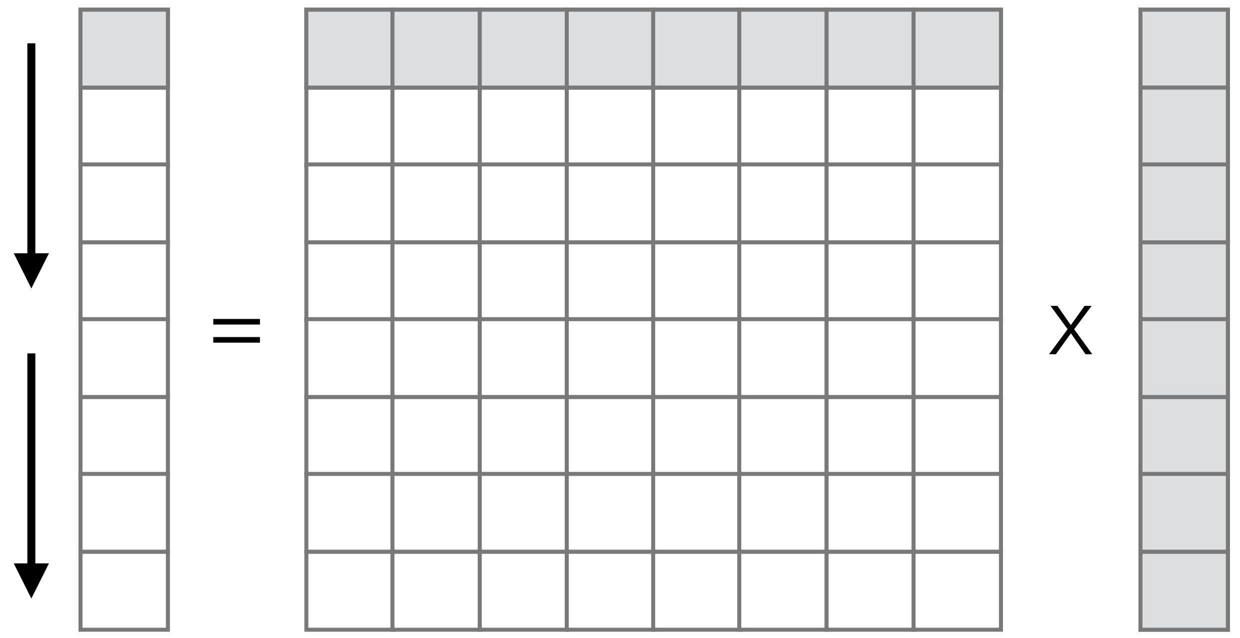


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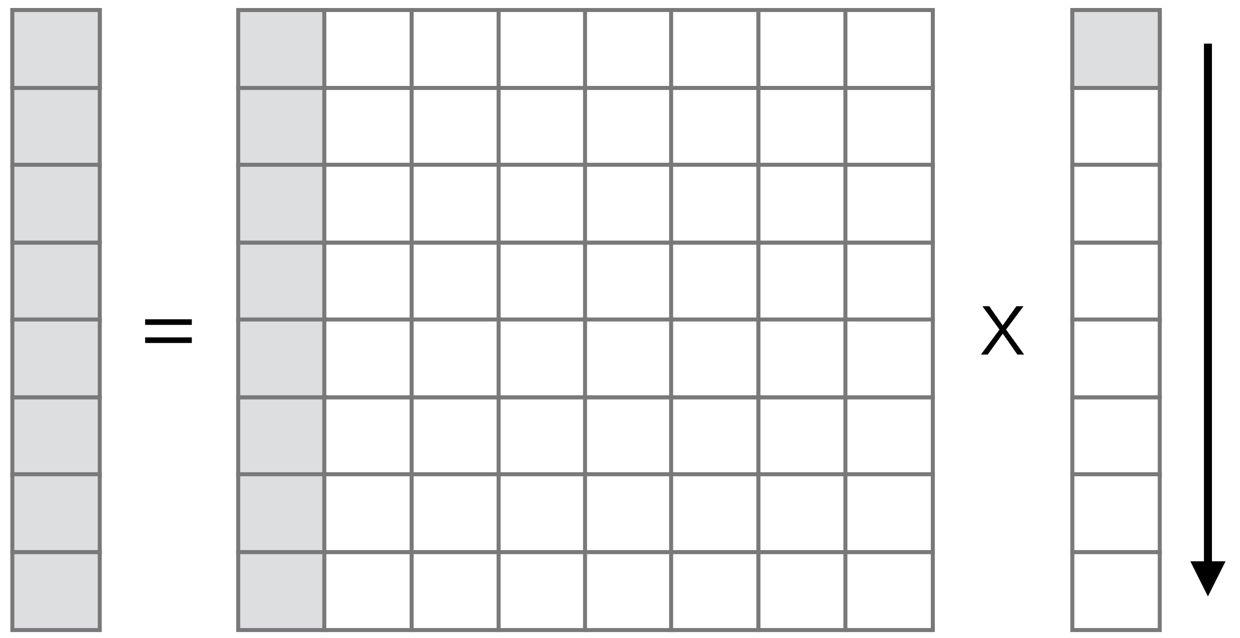


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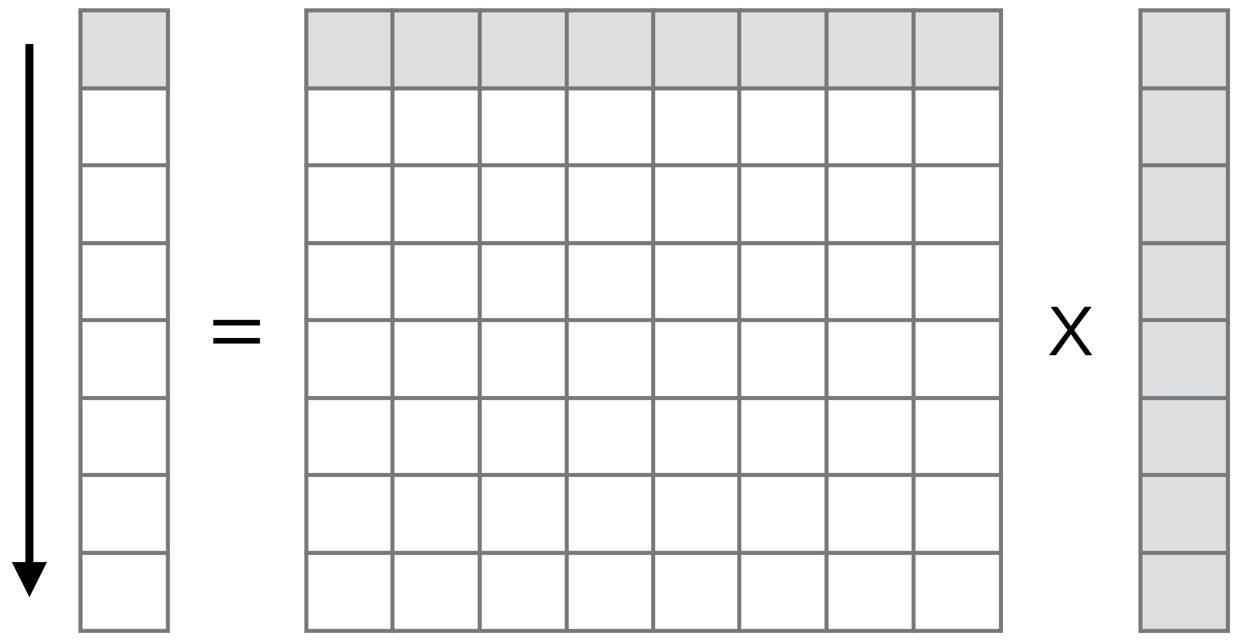
Parallelism in matrix-vector multiplication

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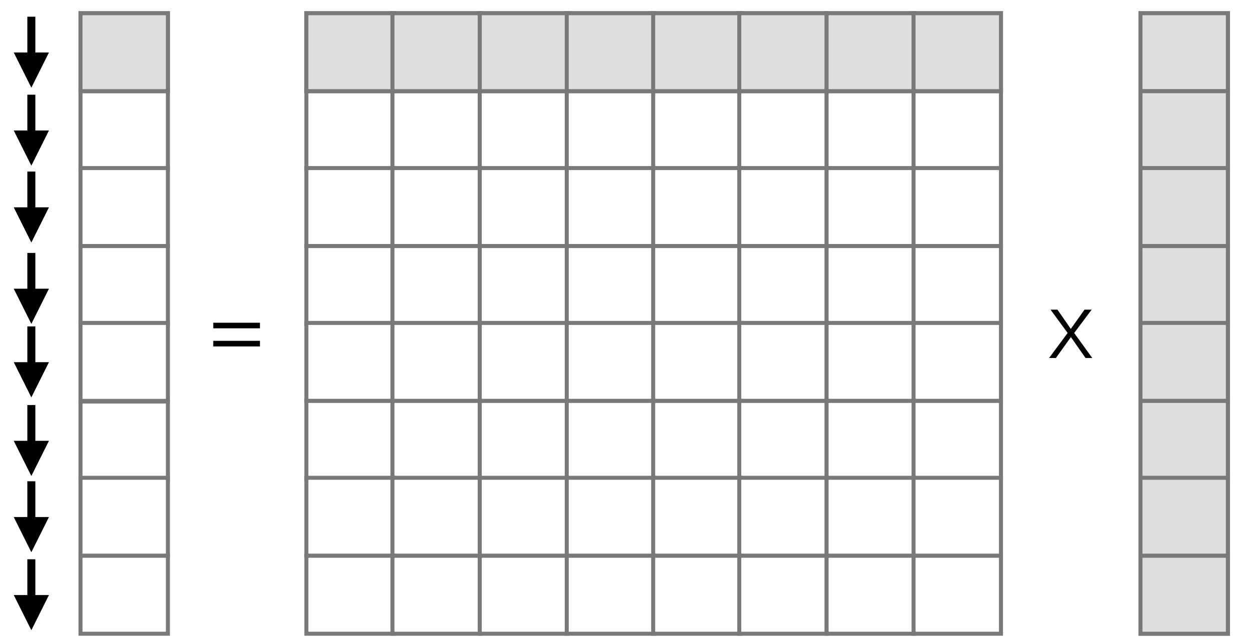


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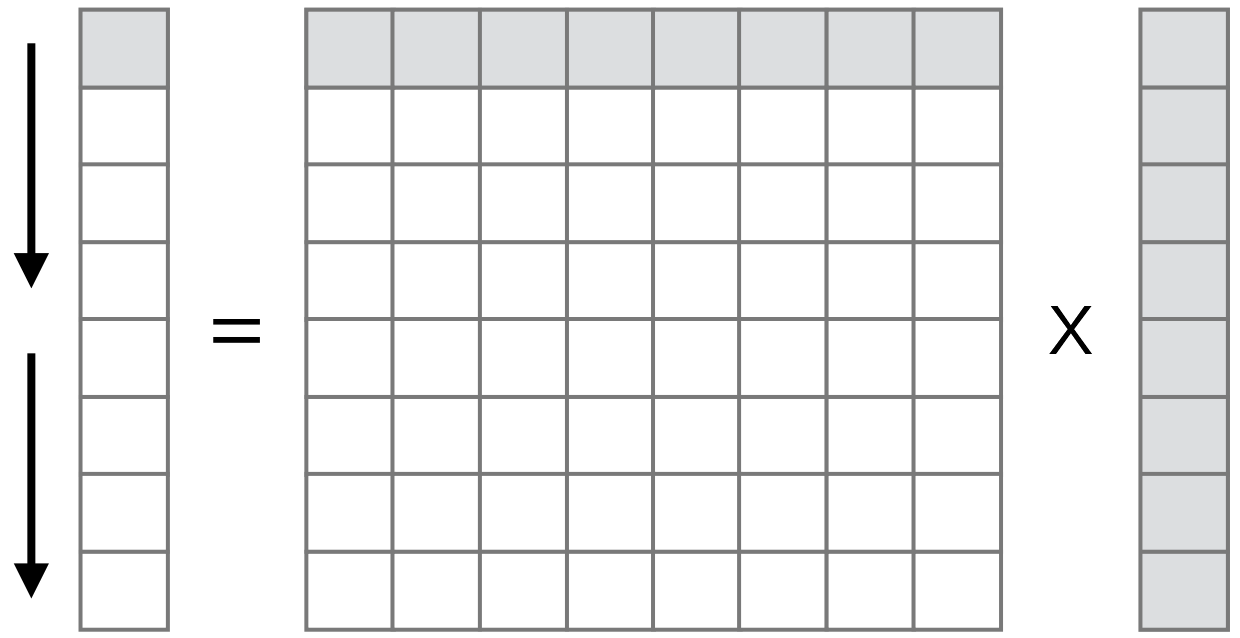


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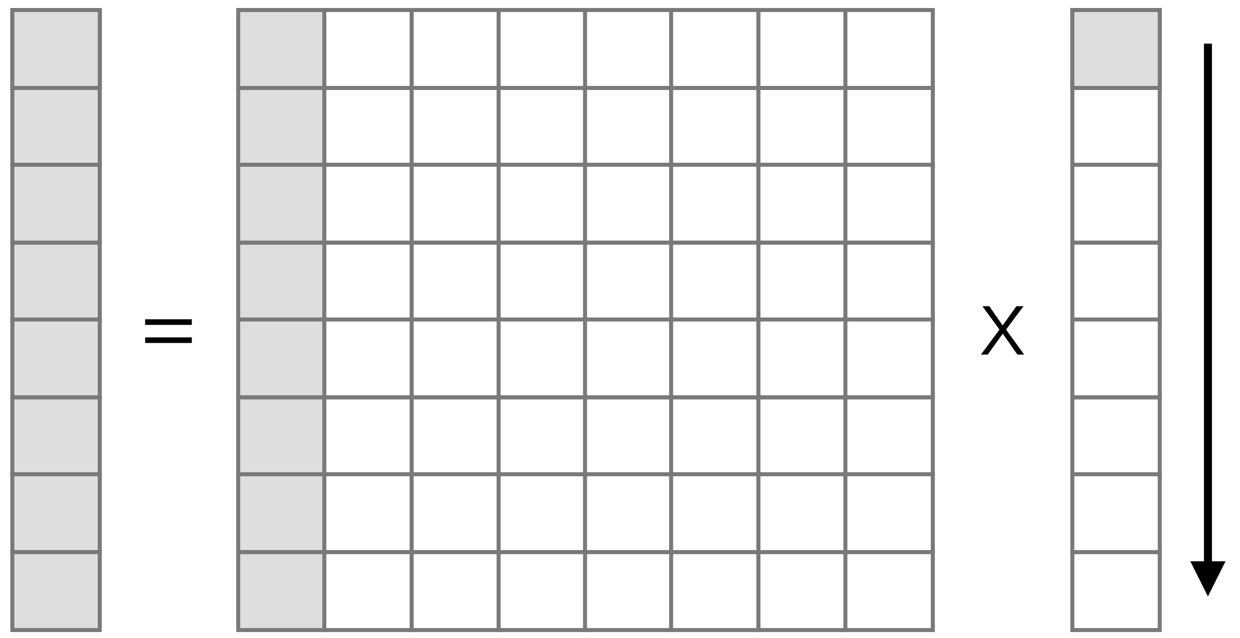


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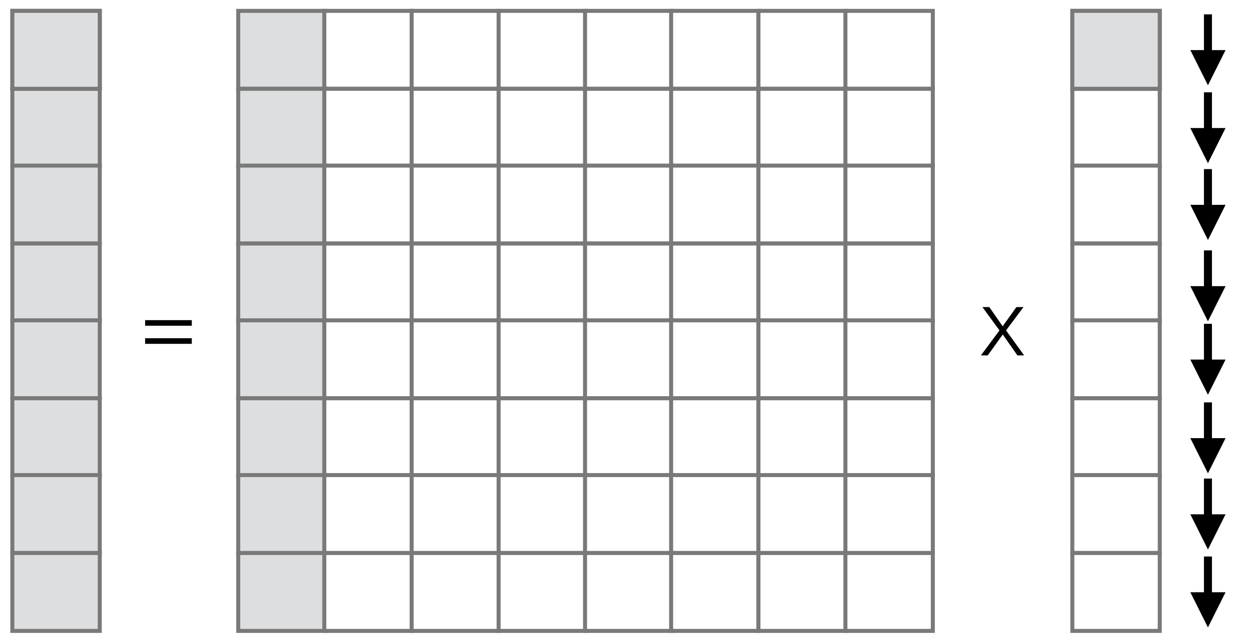
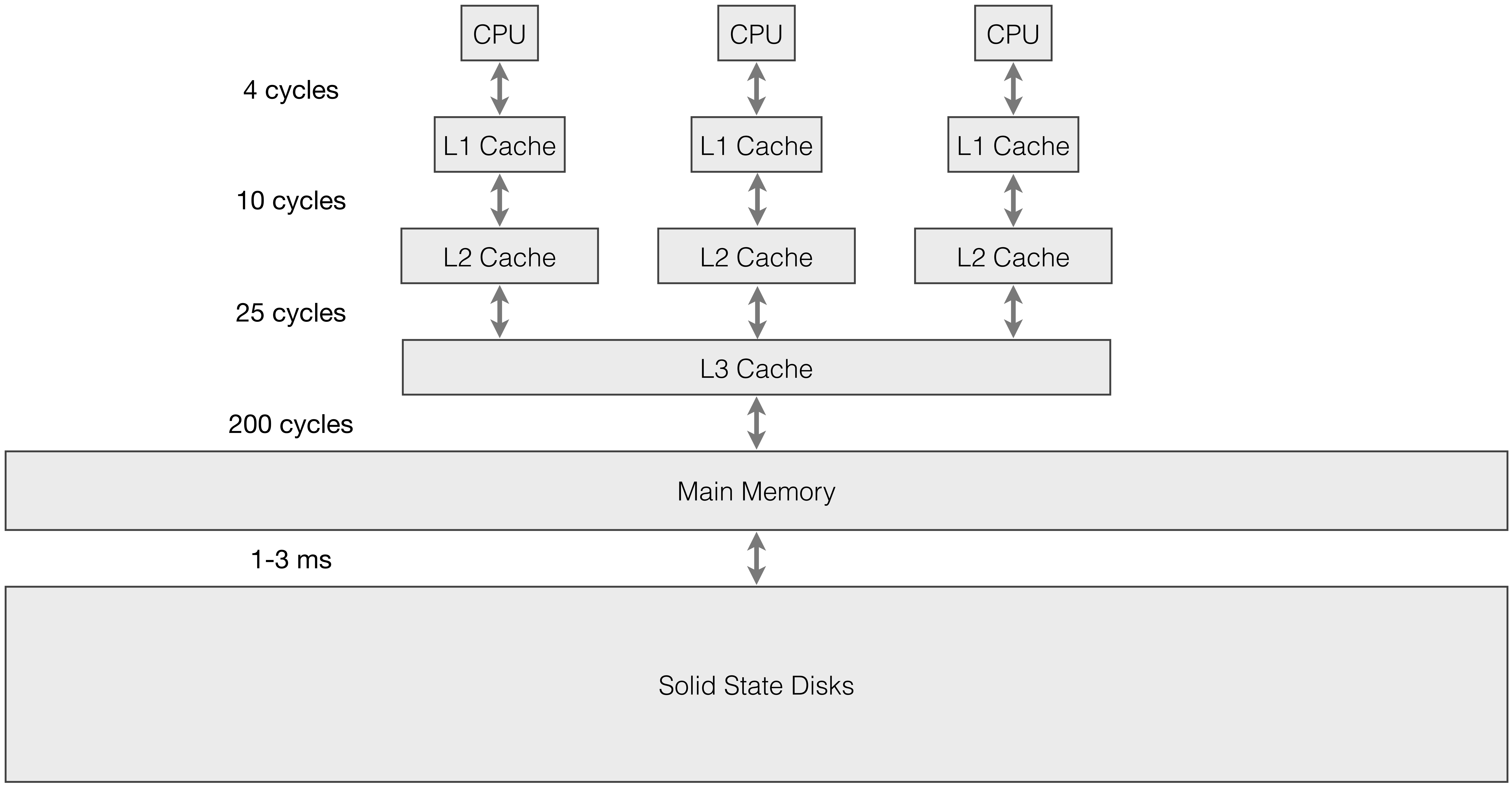
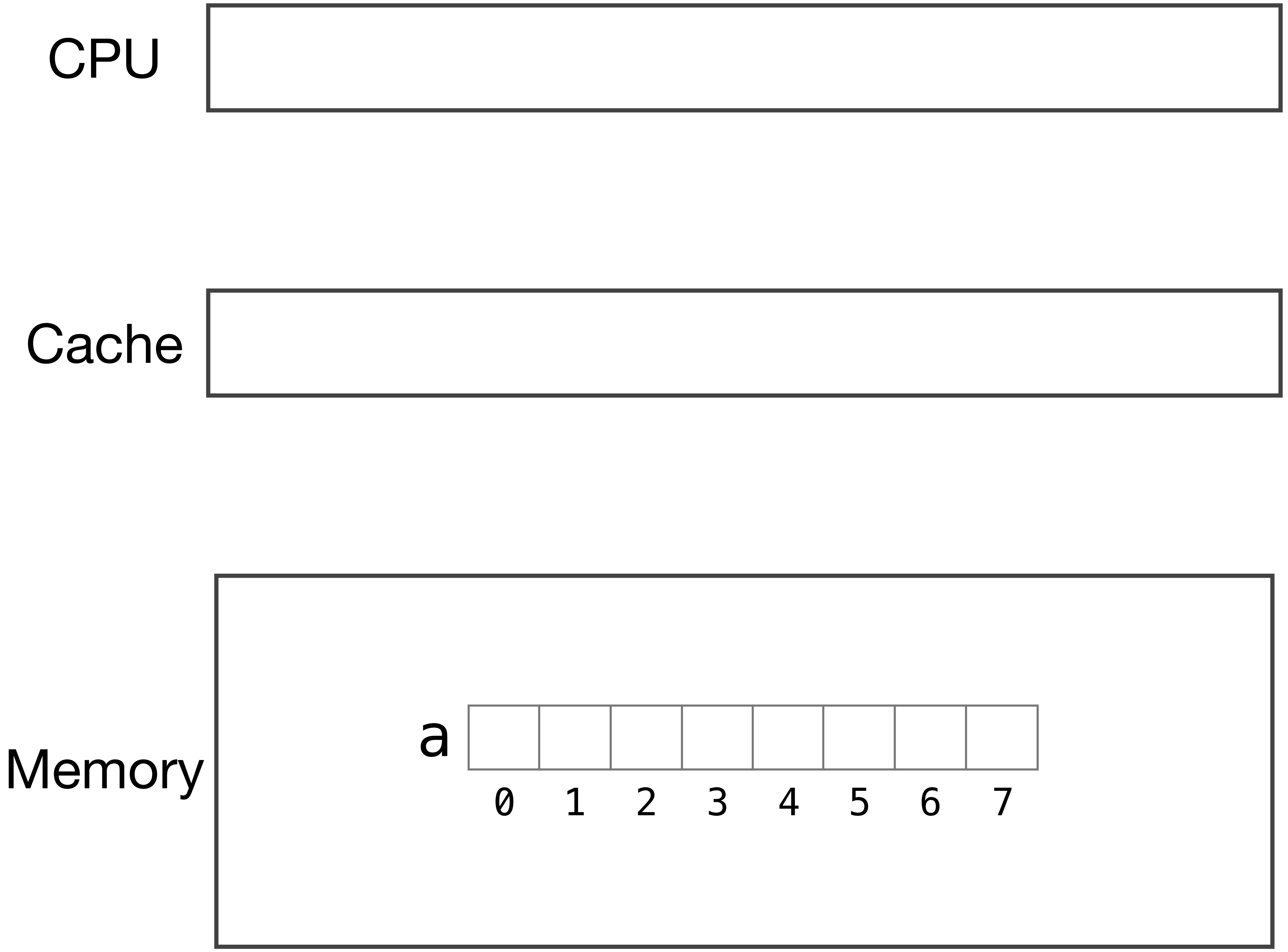



```
#pragma omp parallel for
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    #pragma omp atomic
    y[i] += A[i*n+j] * x[j];
```

Cache Hierarchies with typical latencies



Spatial locality



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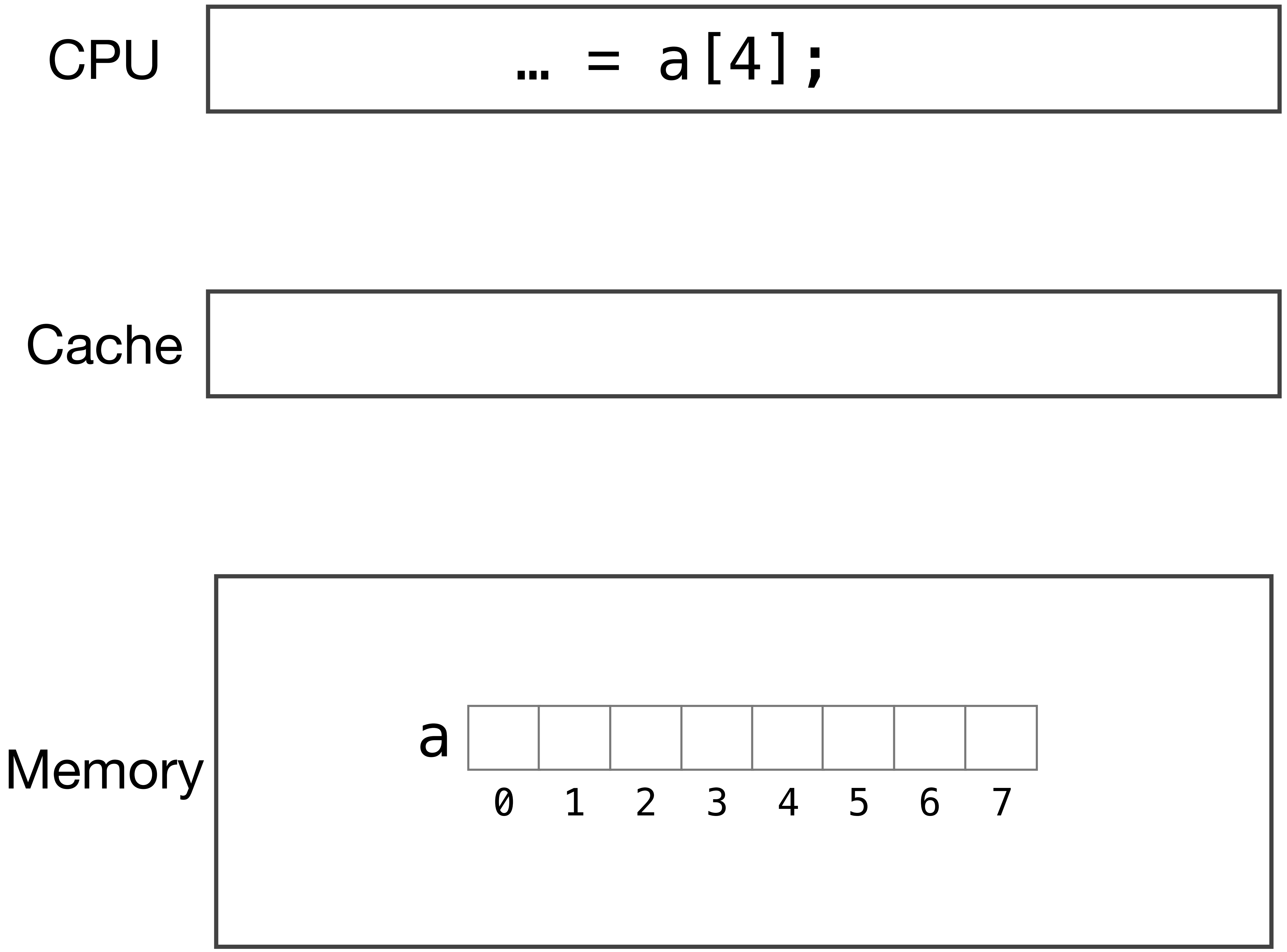

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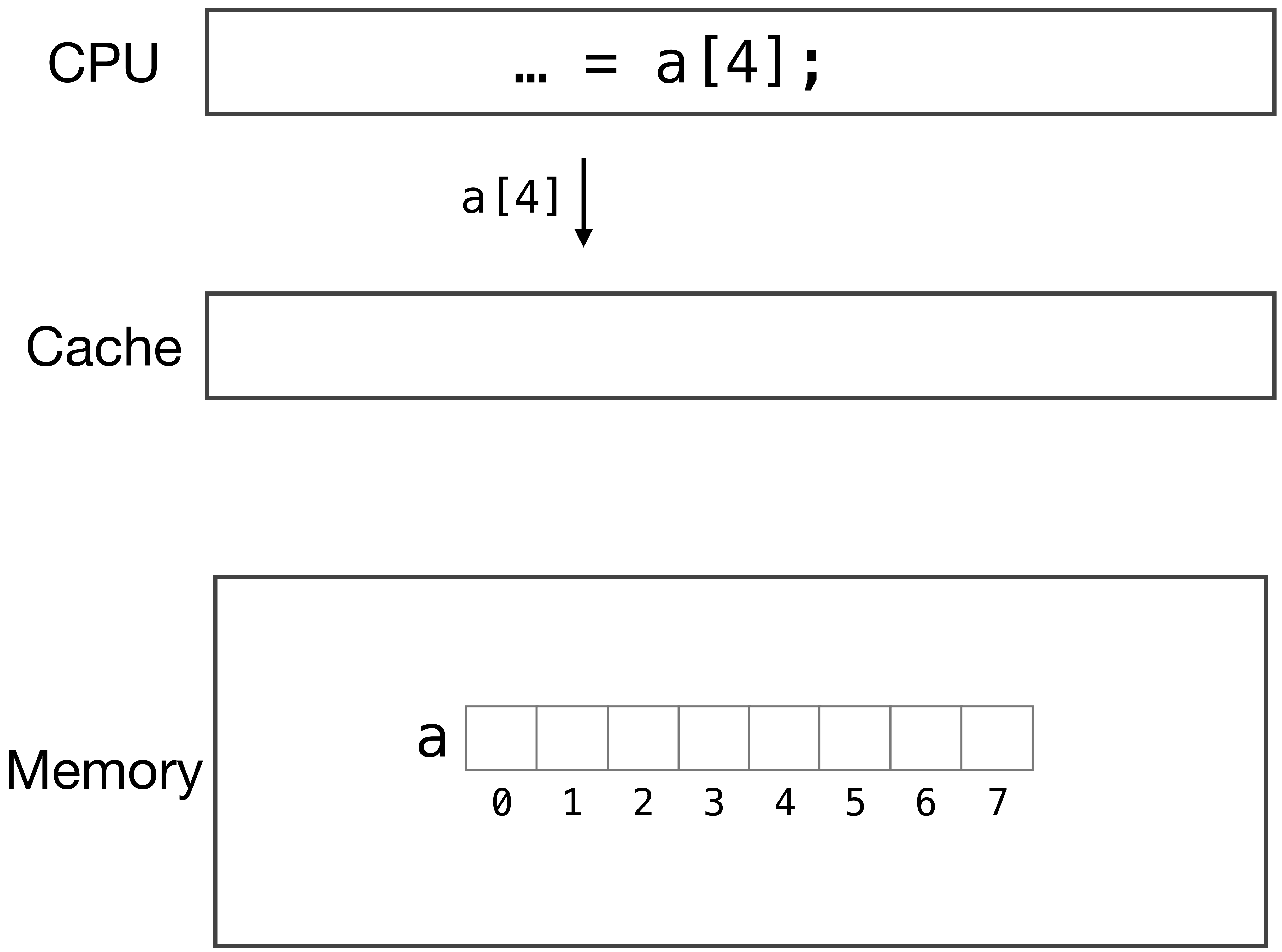

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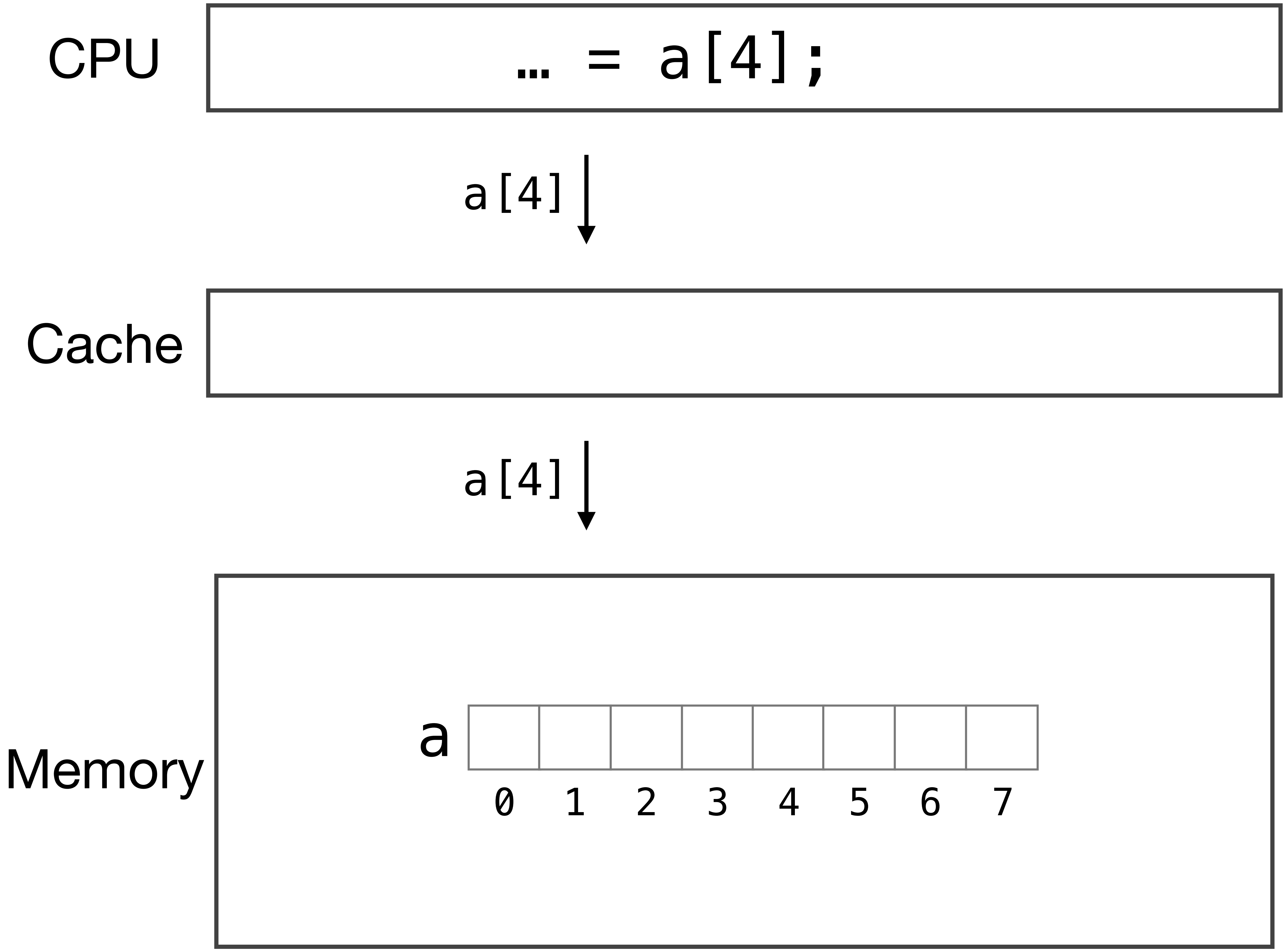

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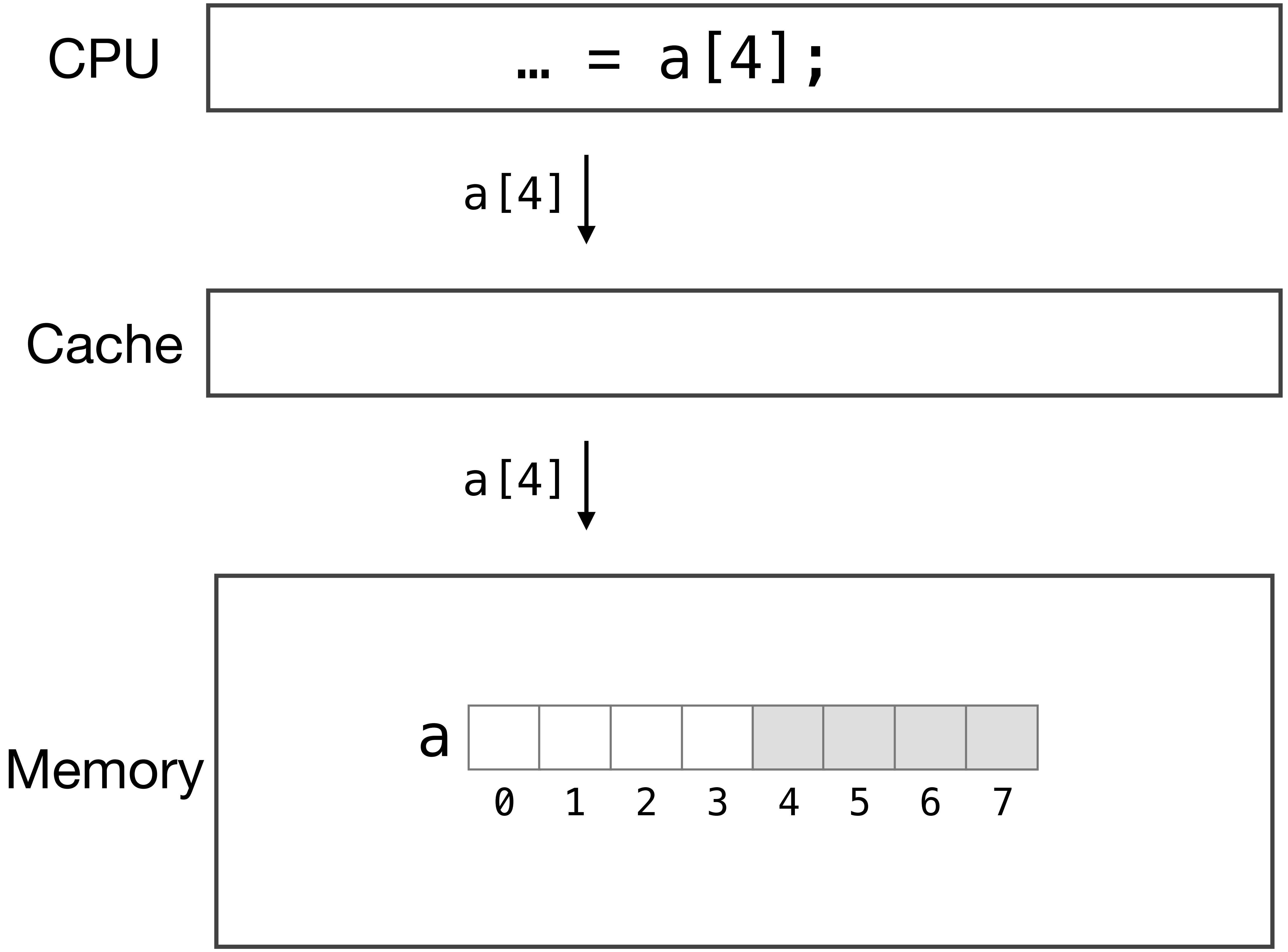

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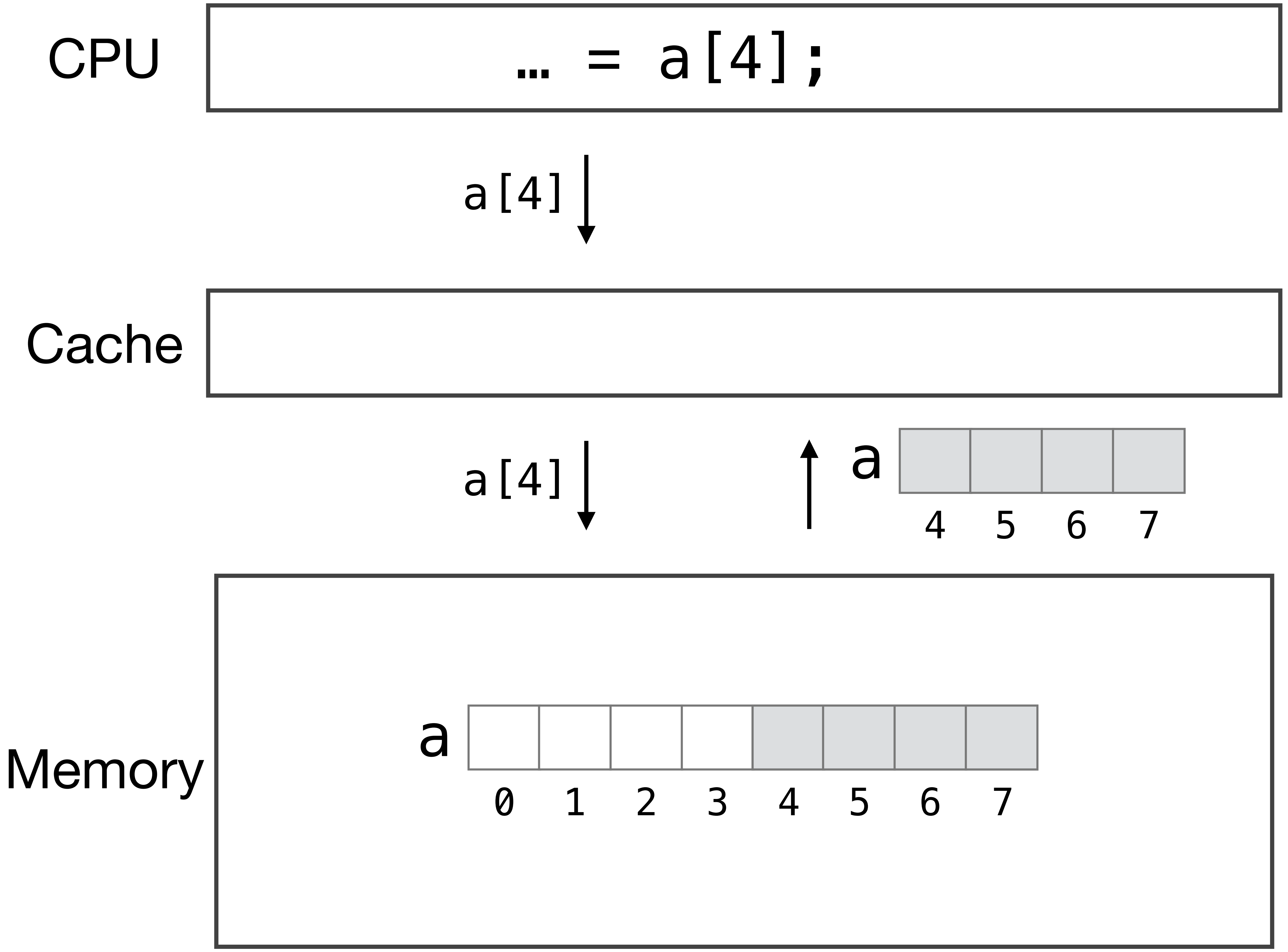

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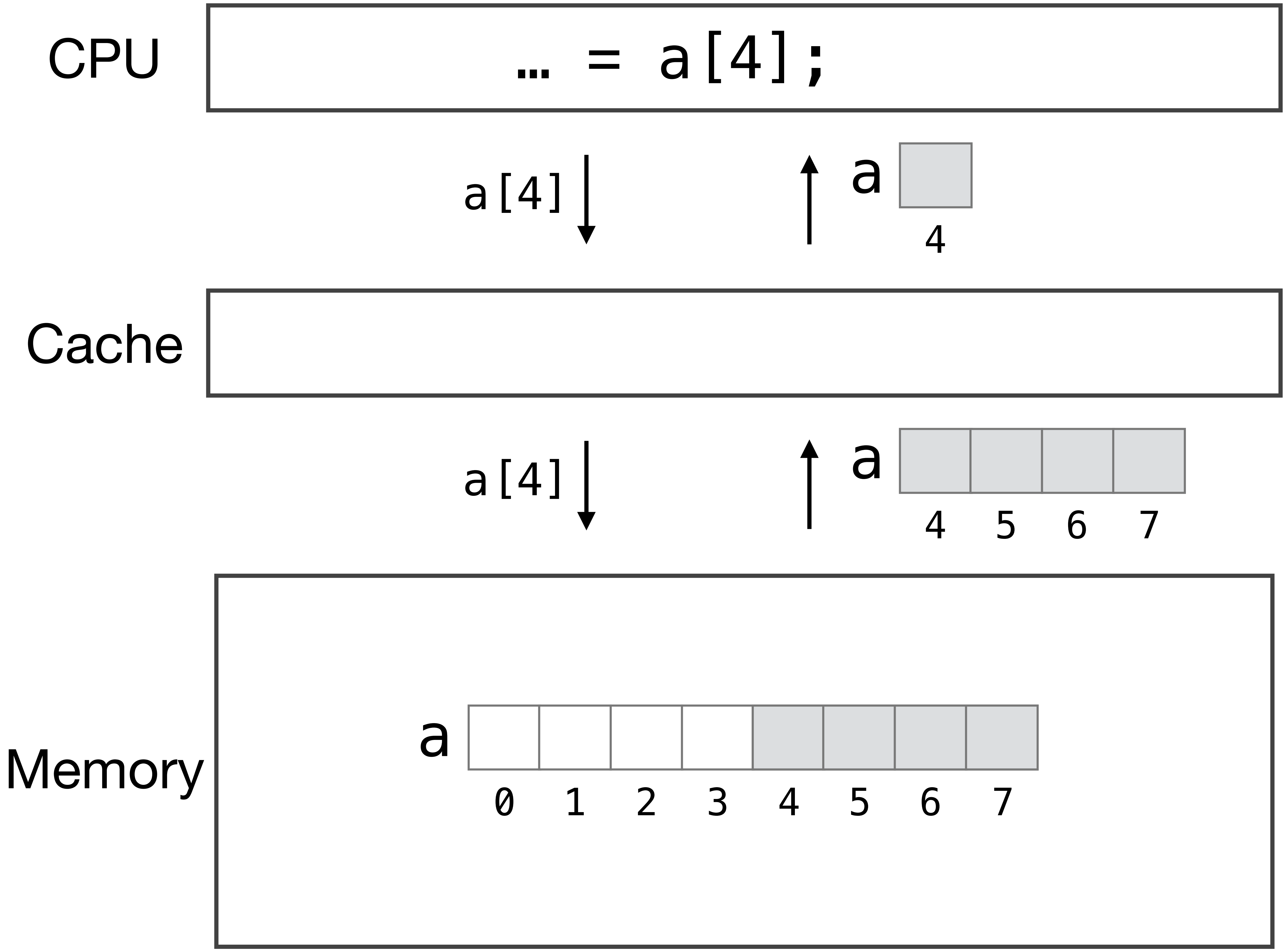

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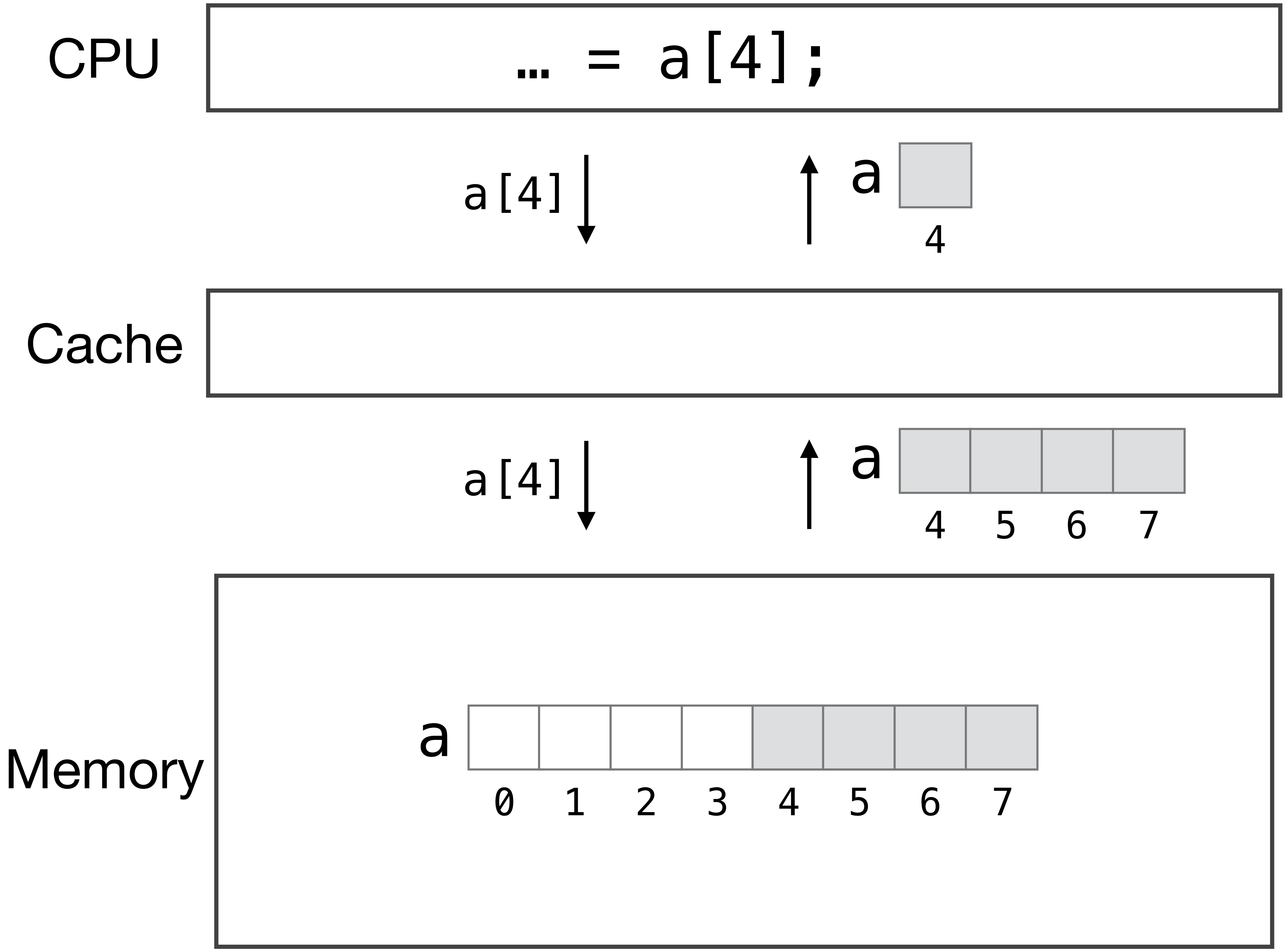

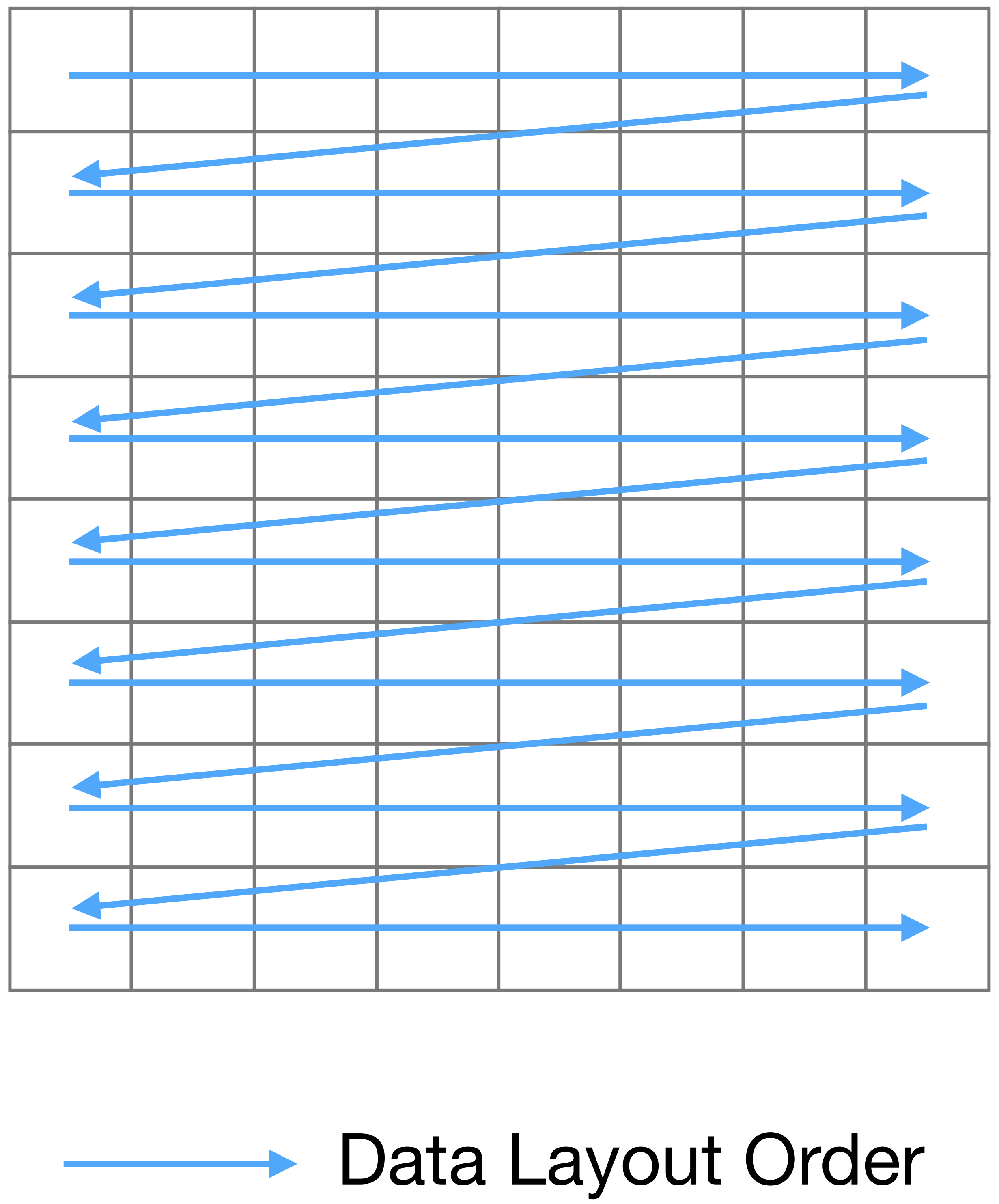

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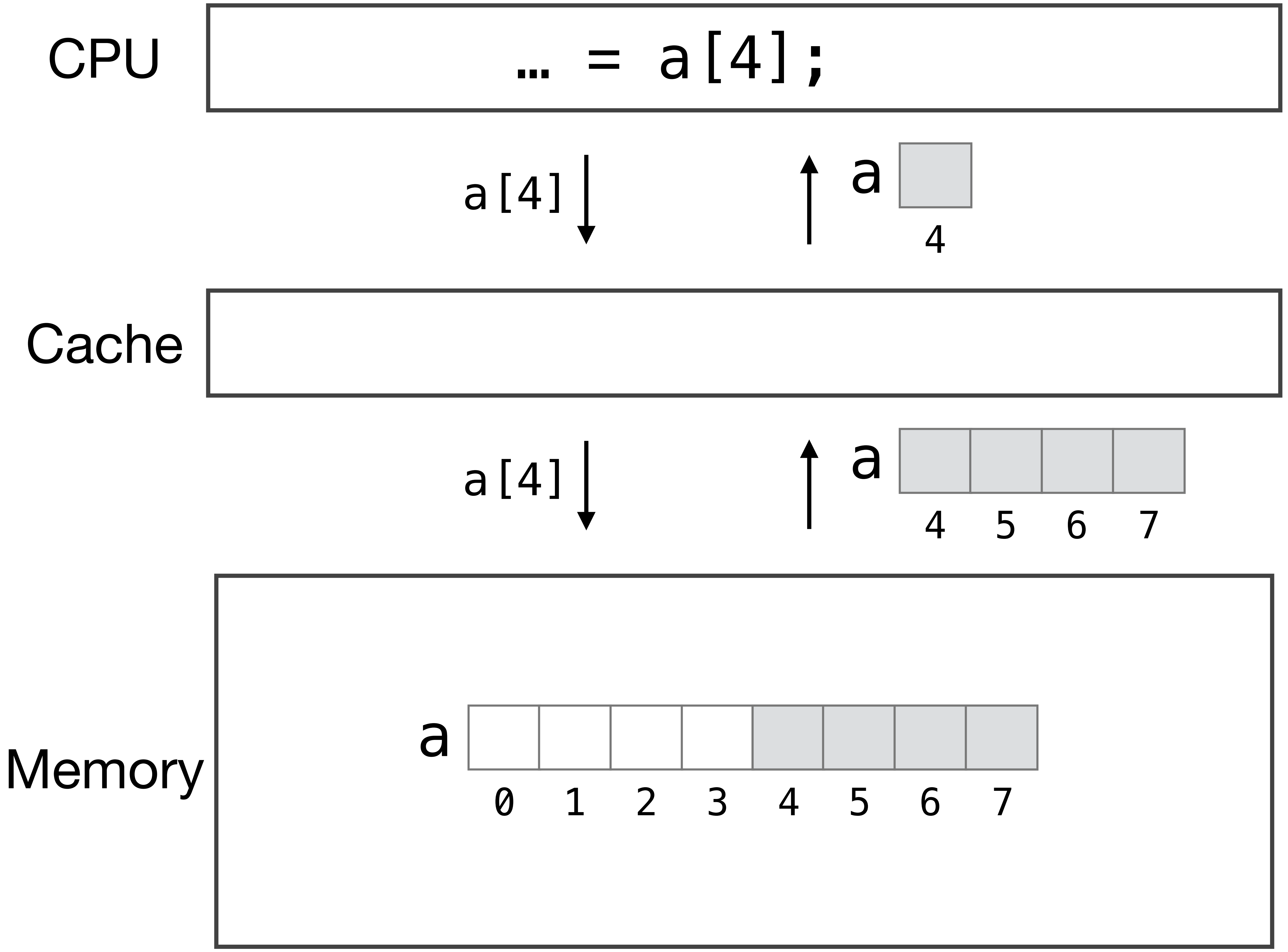
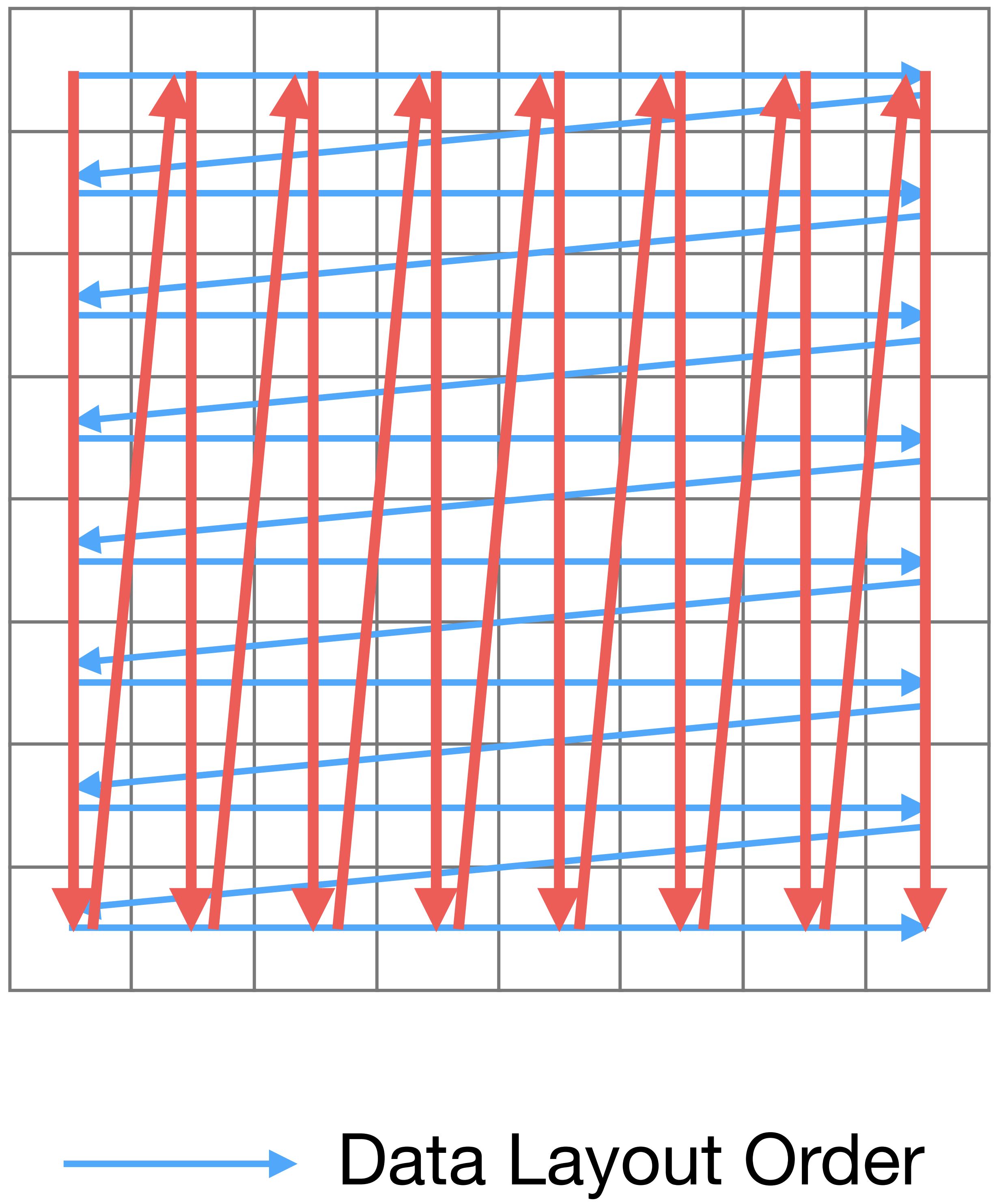


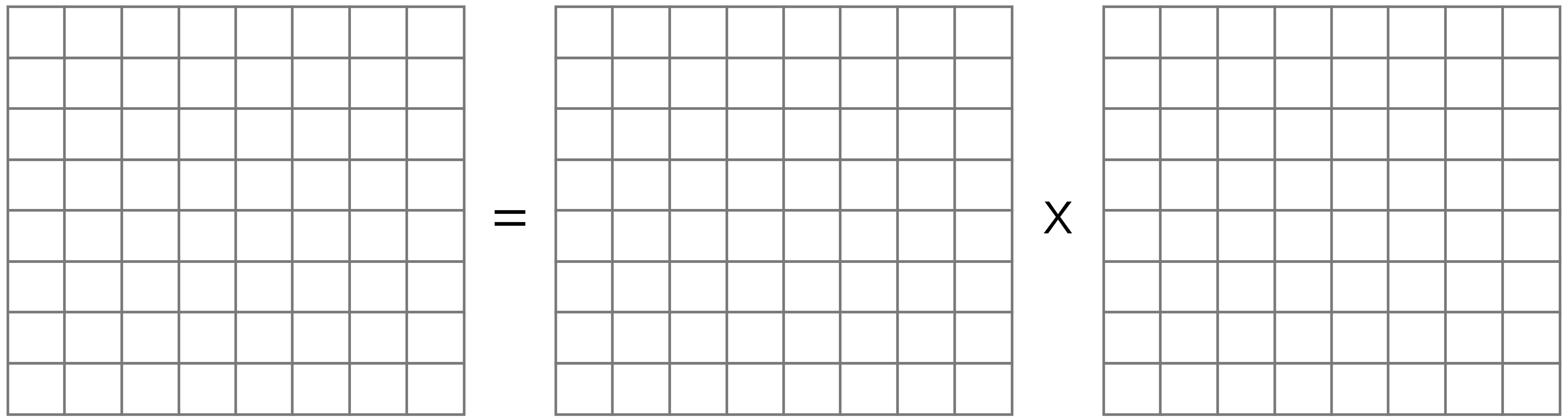
Avoid jumping around the address space

by not iterating along the data layout



Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$



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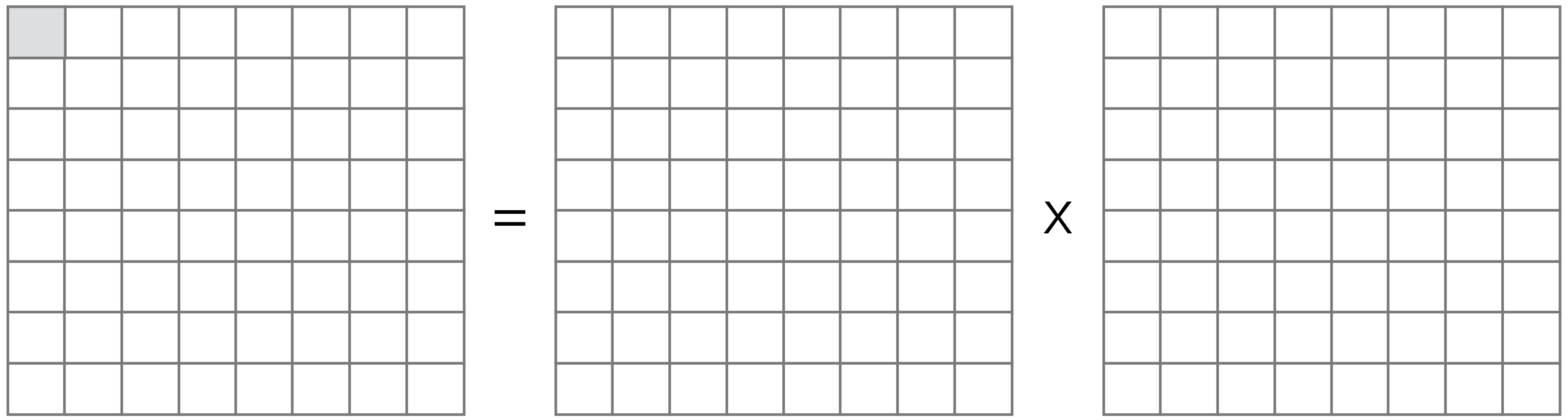

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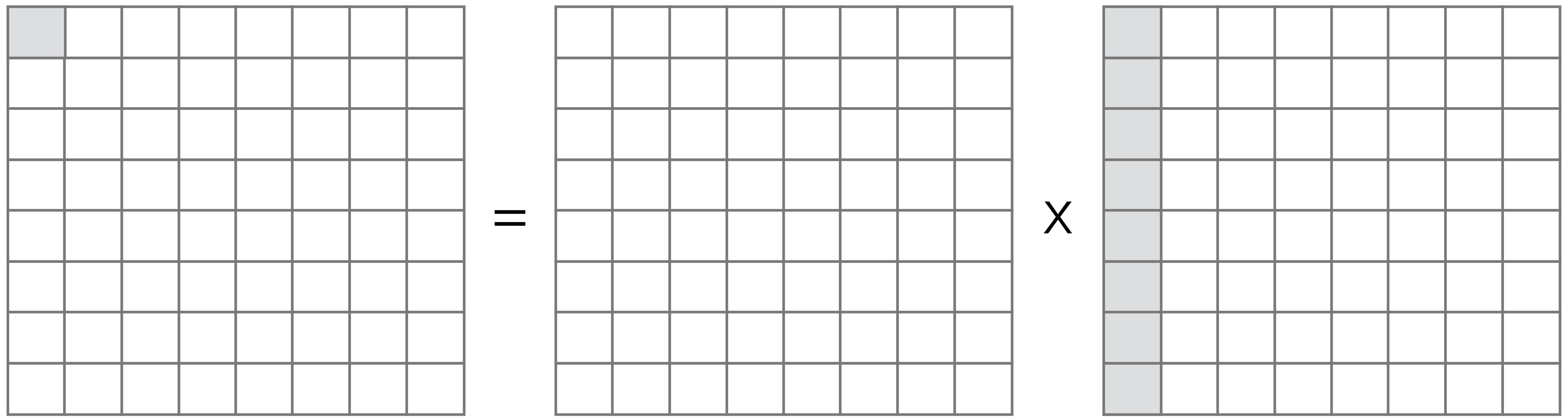

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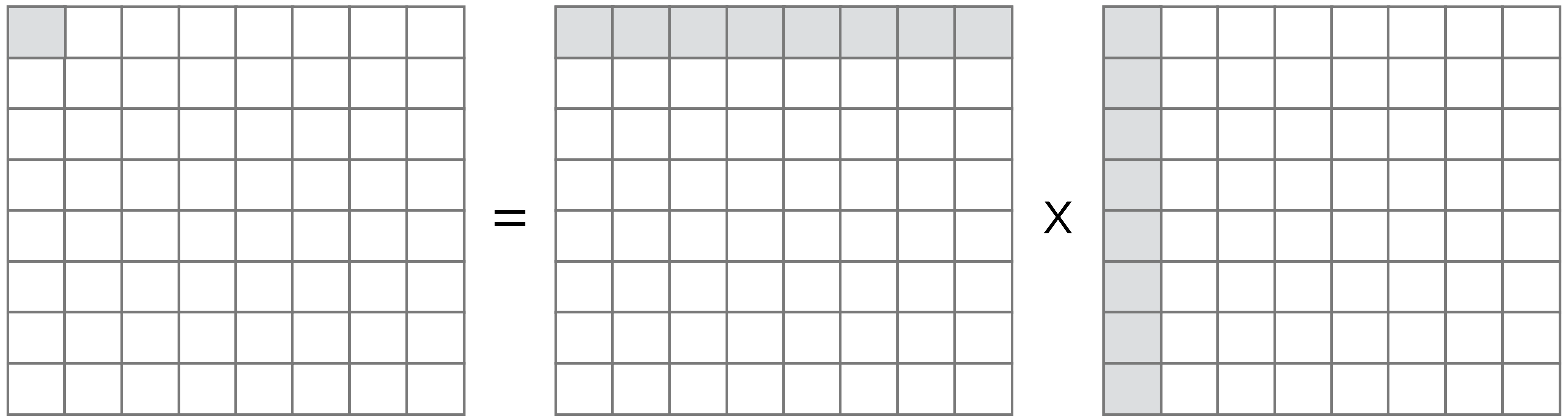

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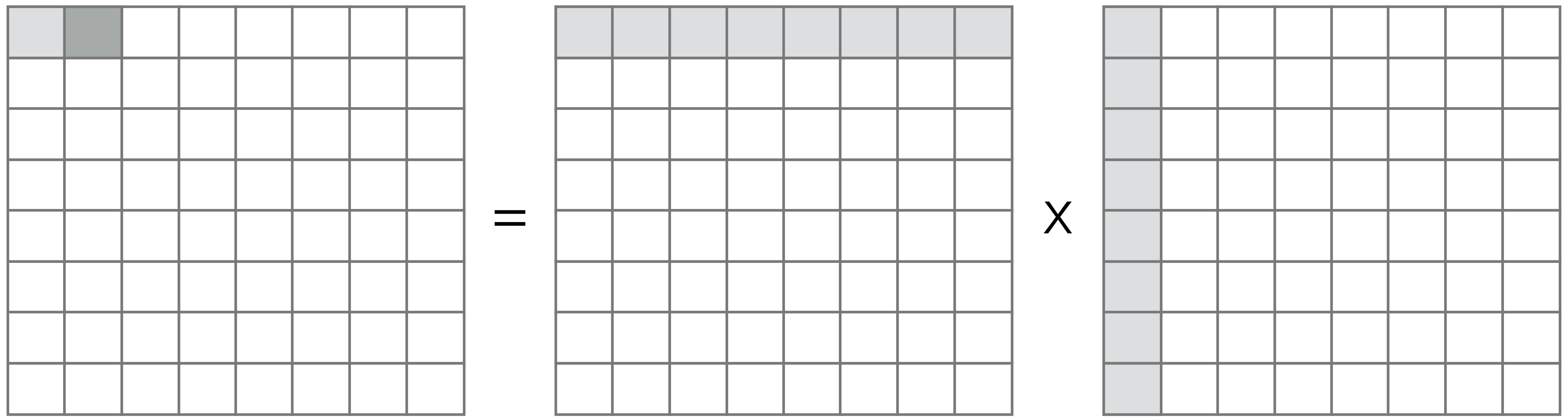

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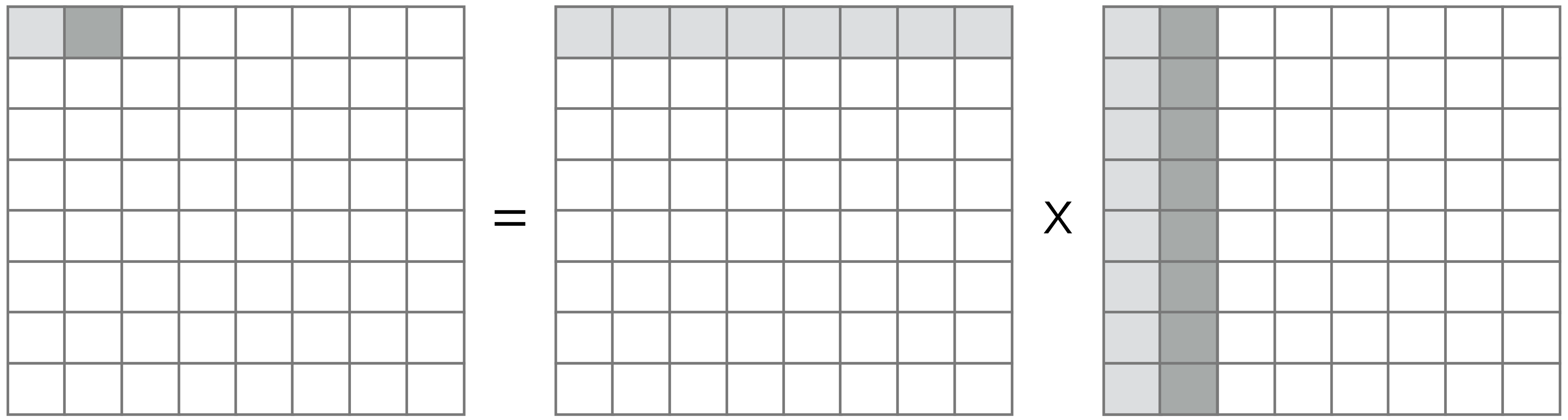

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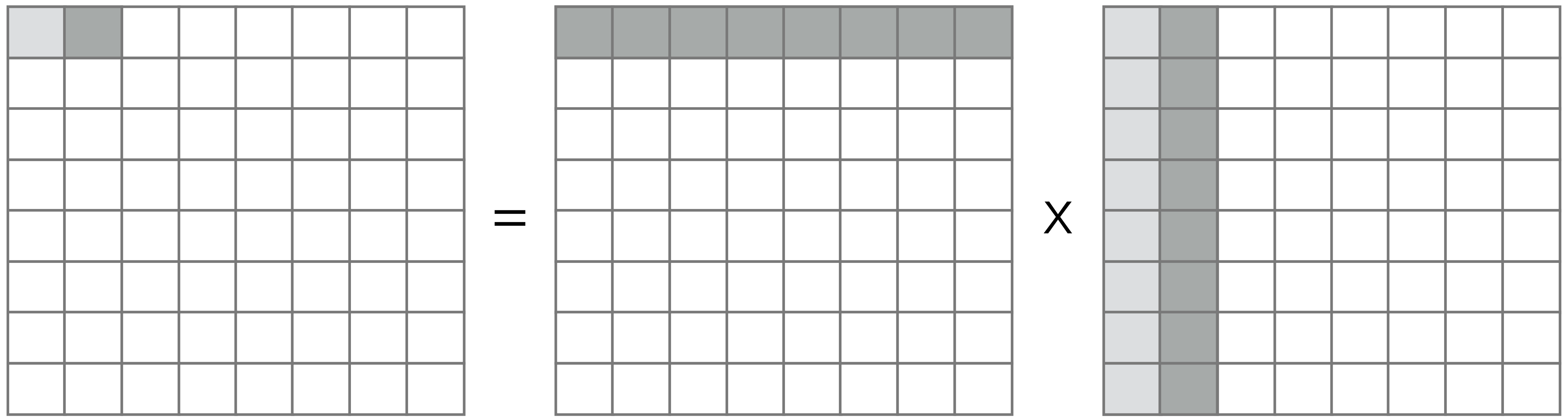

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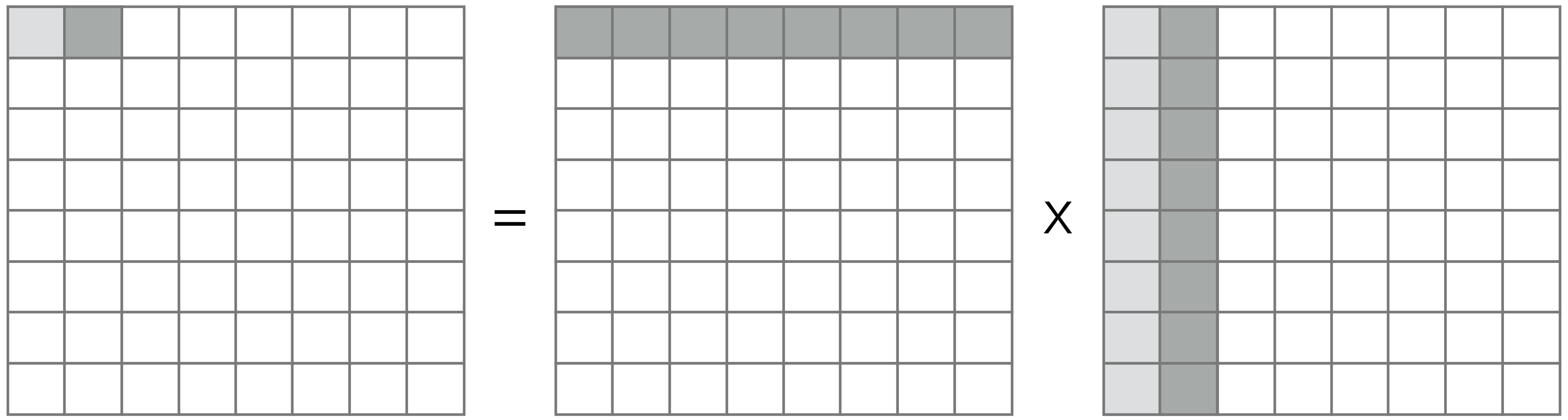
if matrix is large, row will have left the cache



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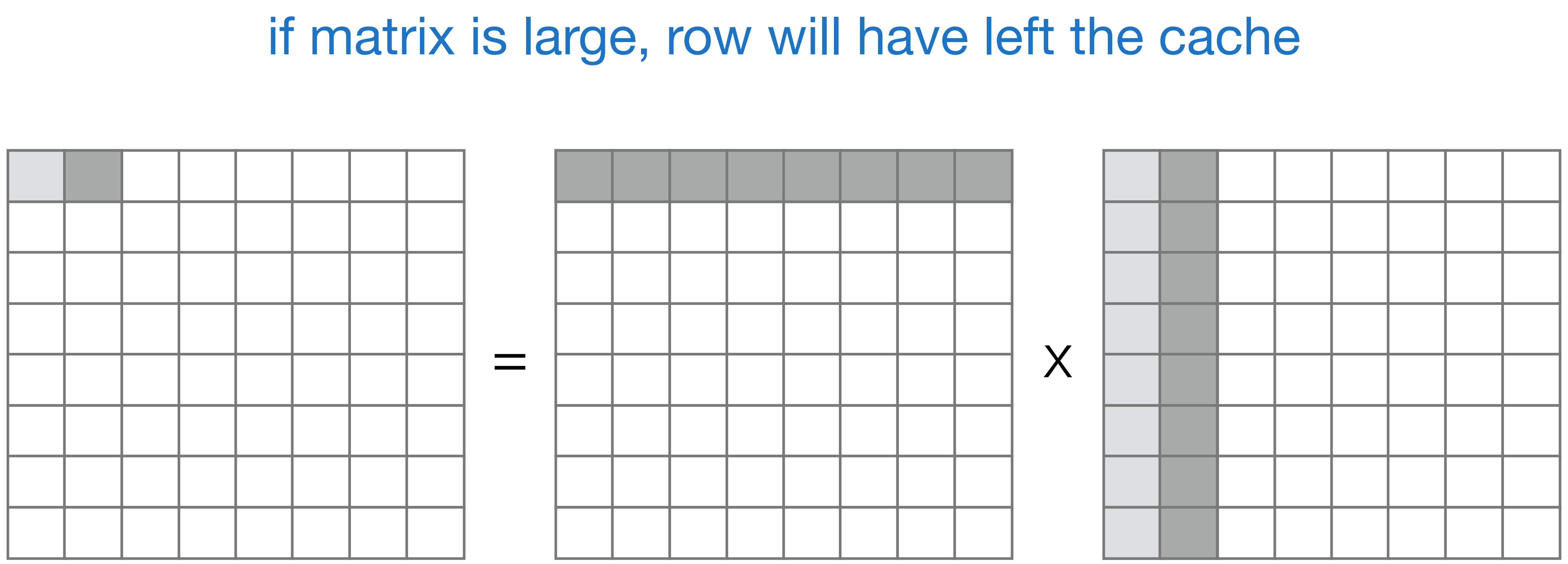
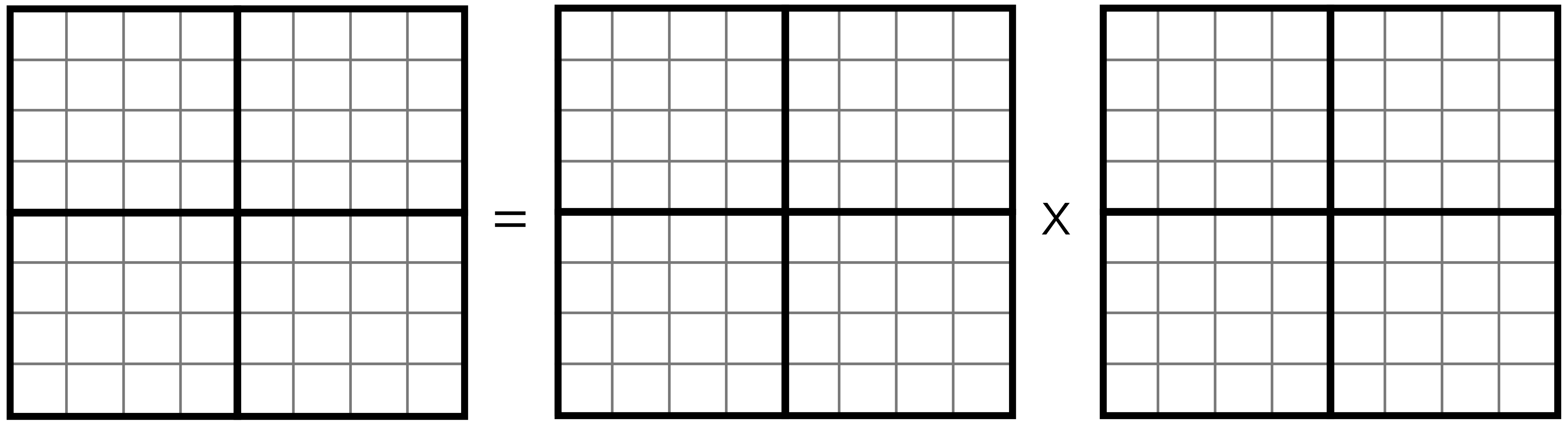


2x2 matrix multiply,
where the operations are
4x4 matrix multiplies



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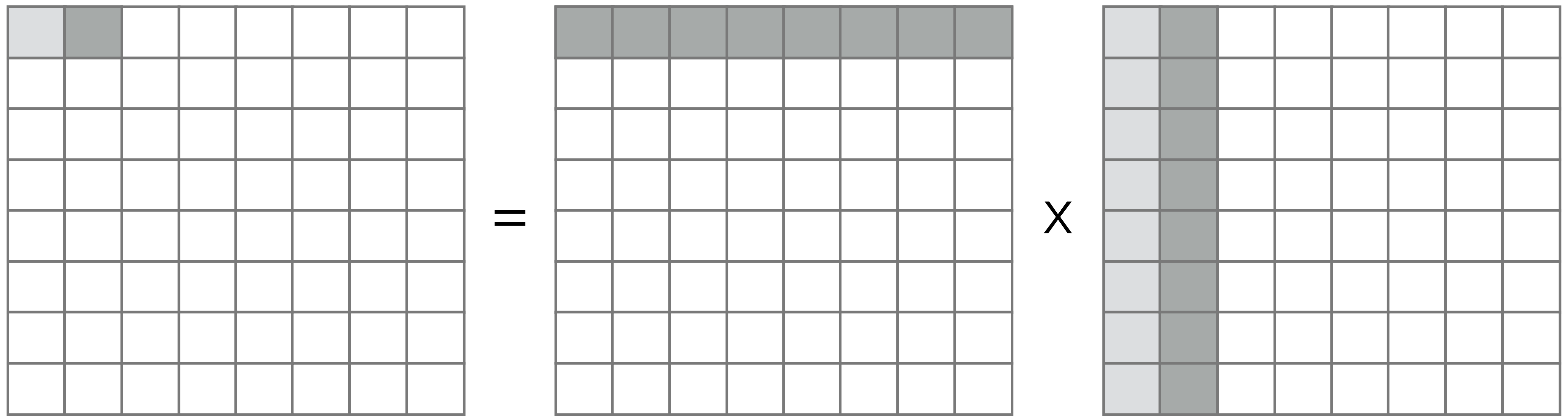

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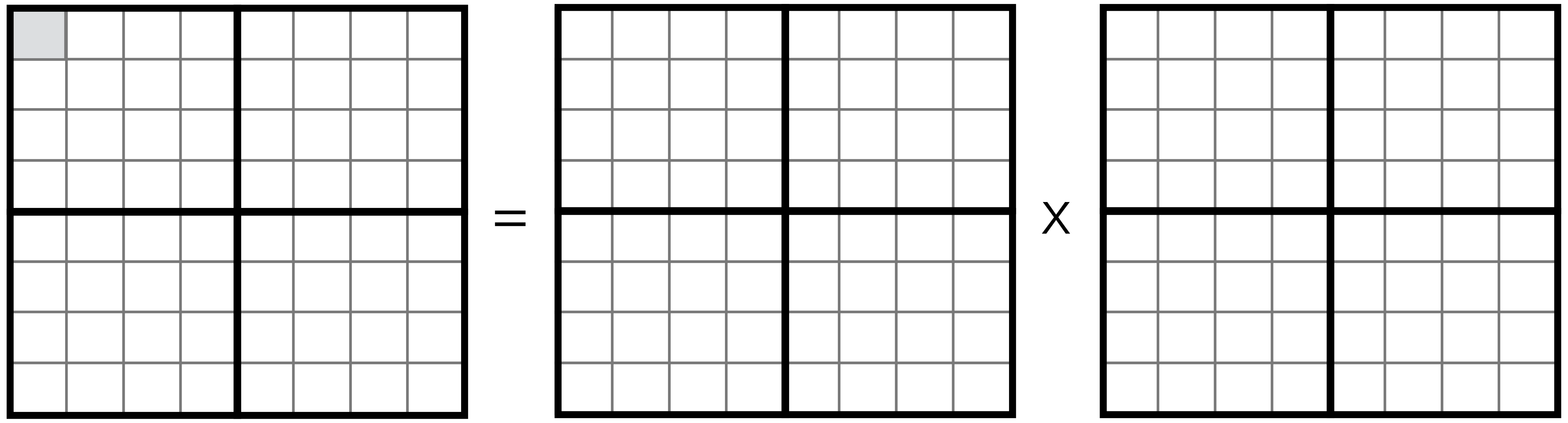

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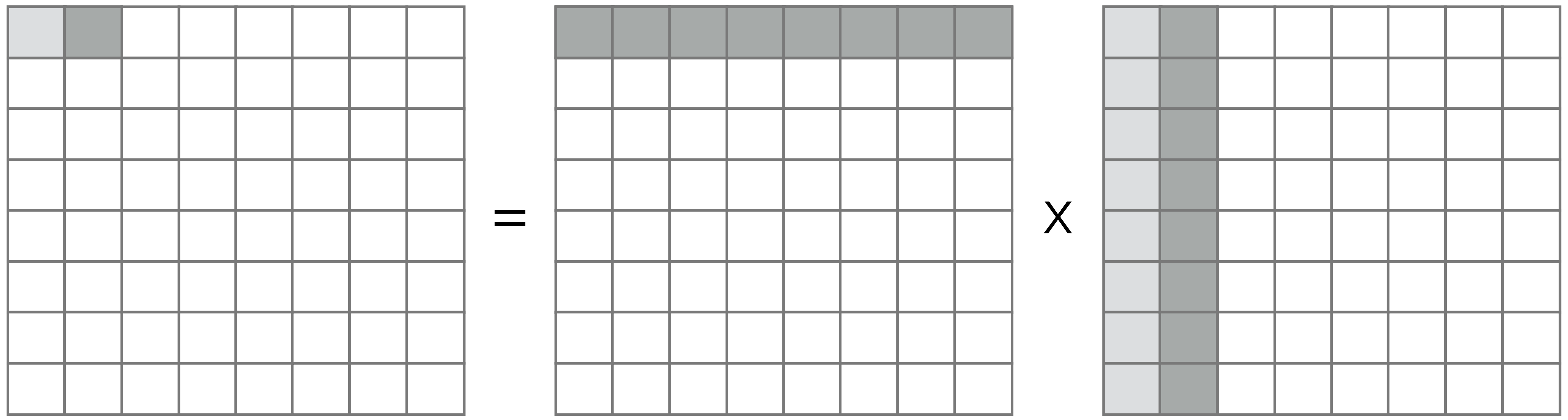

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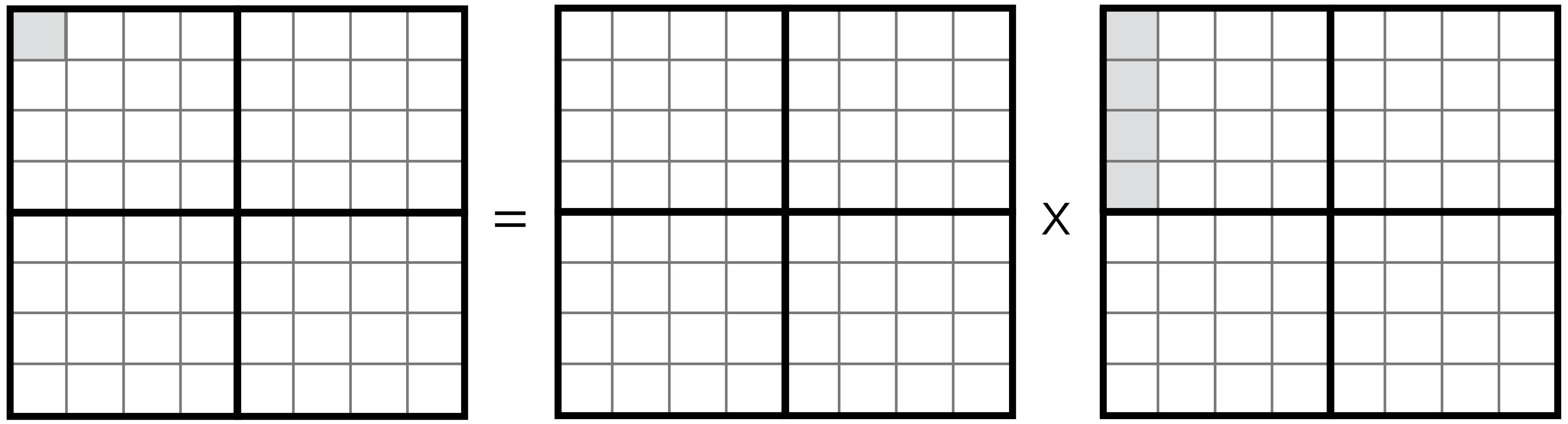

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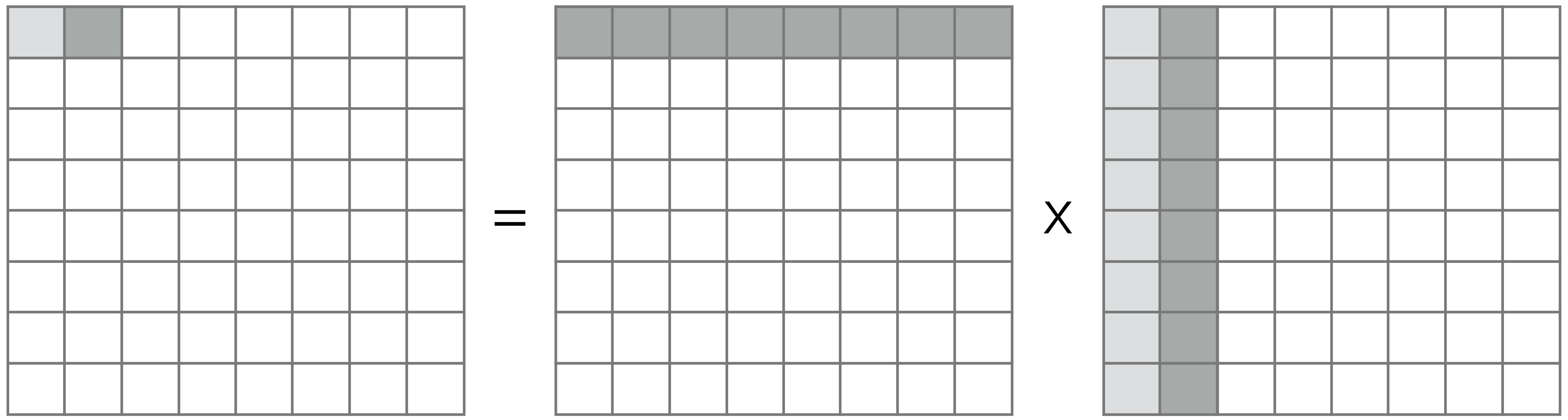

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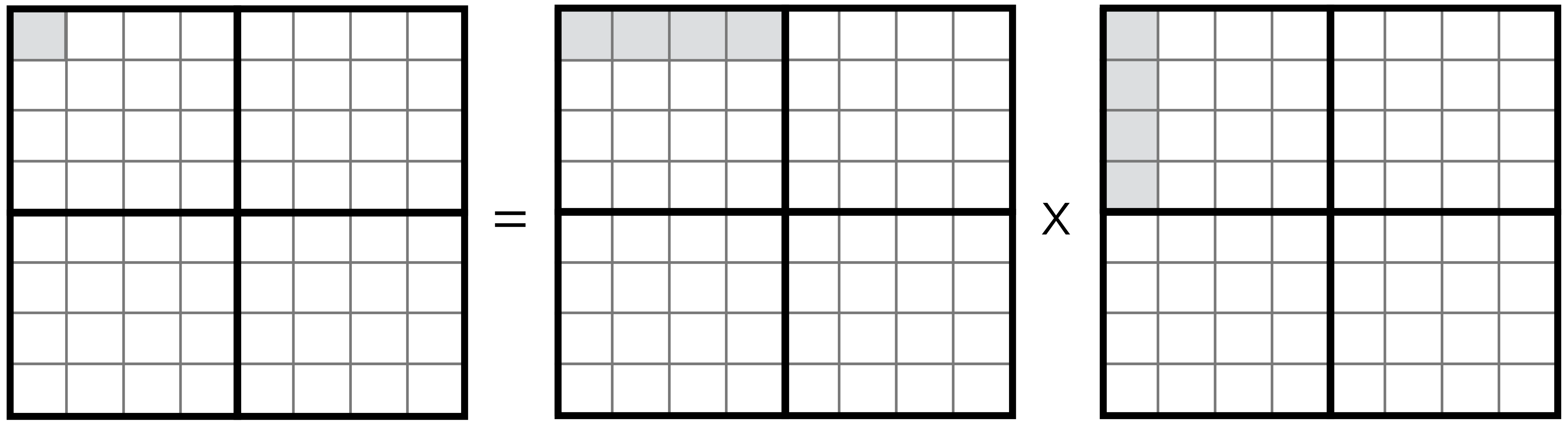

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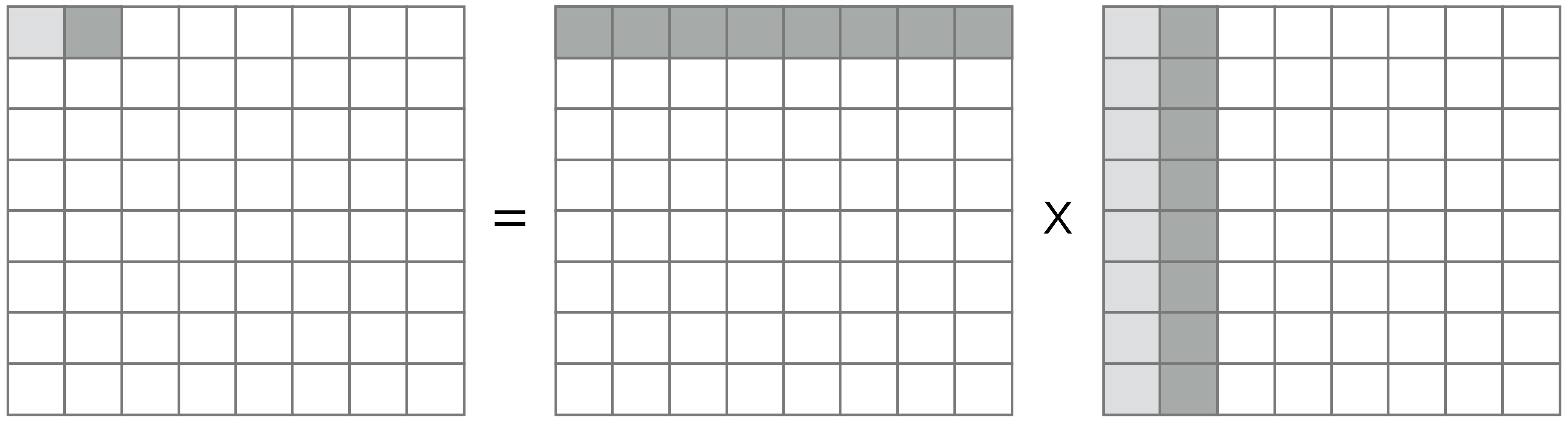

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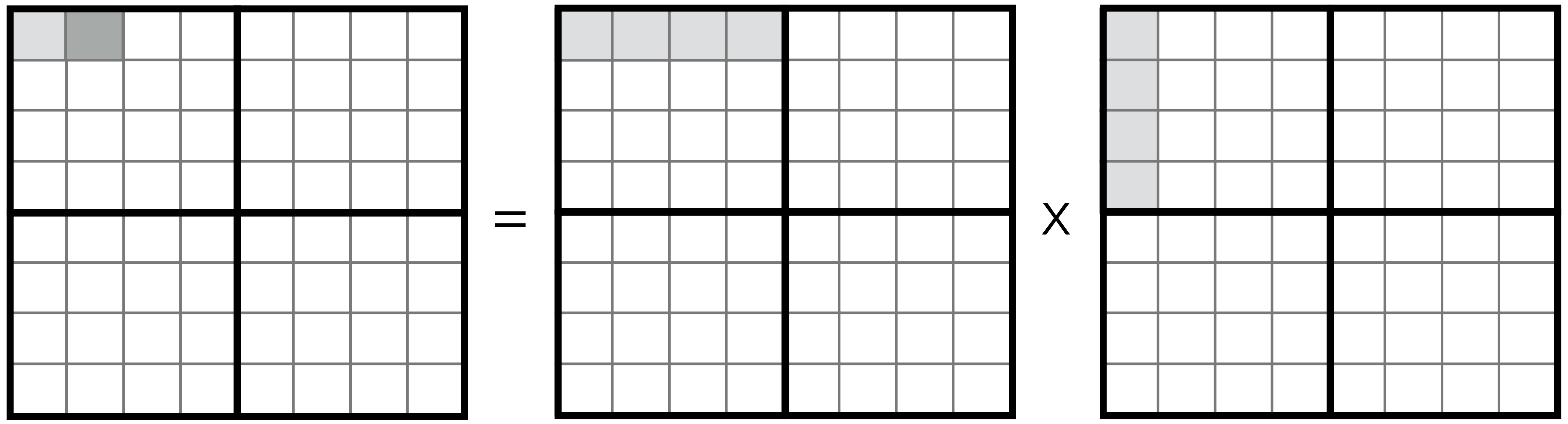

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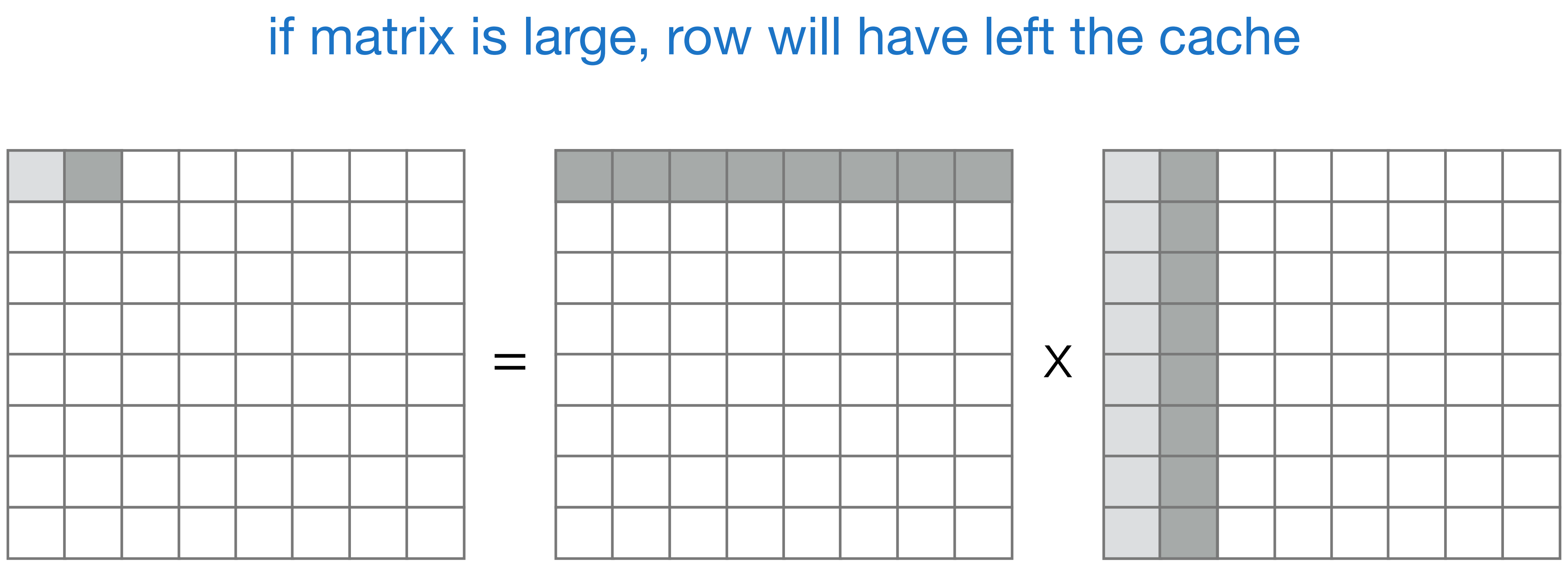

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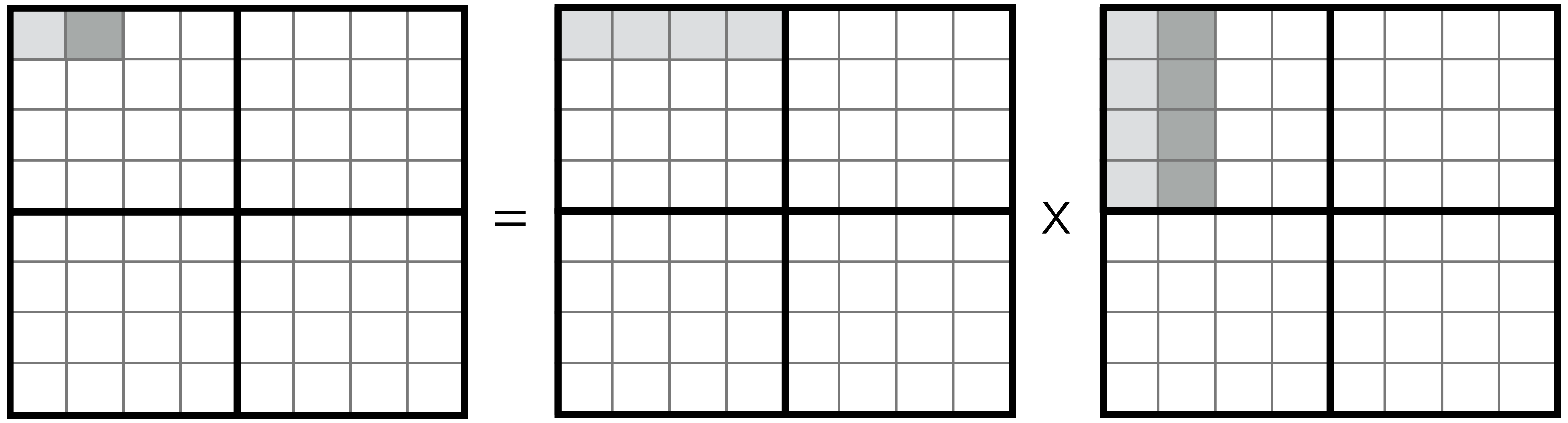

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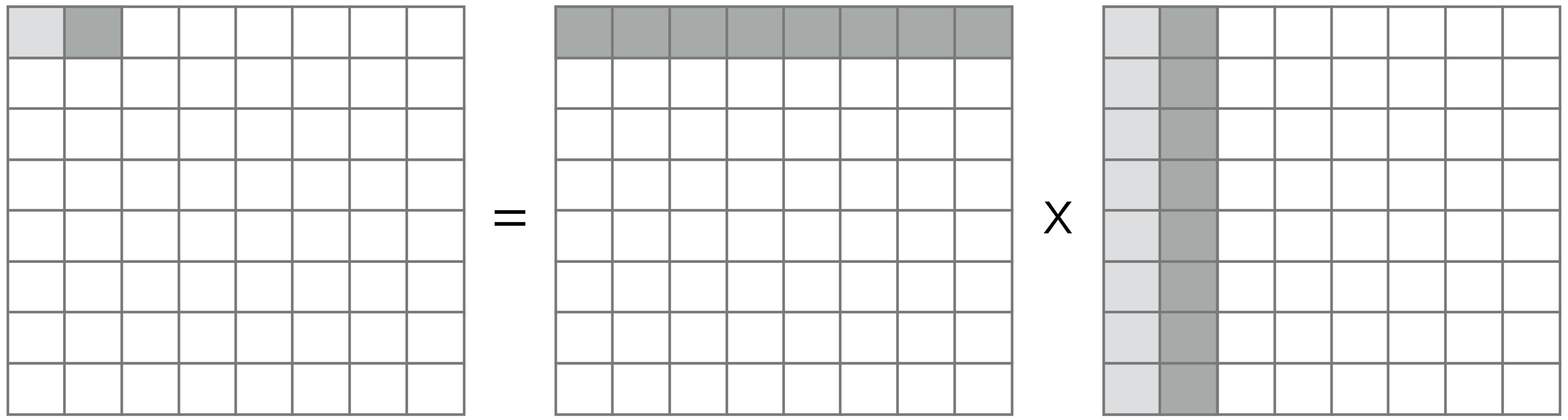

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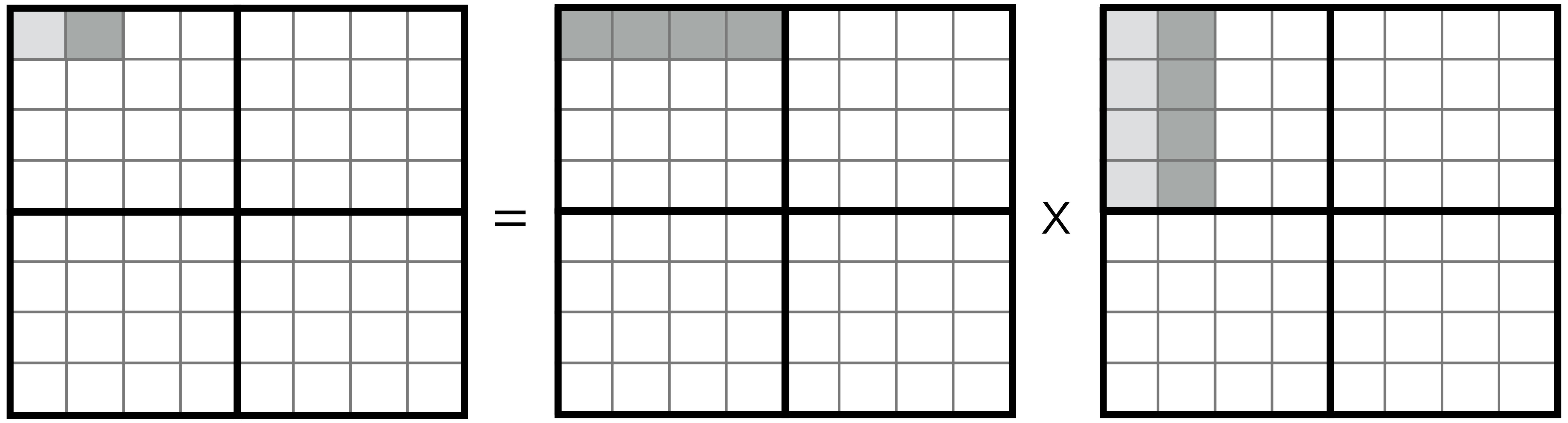

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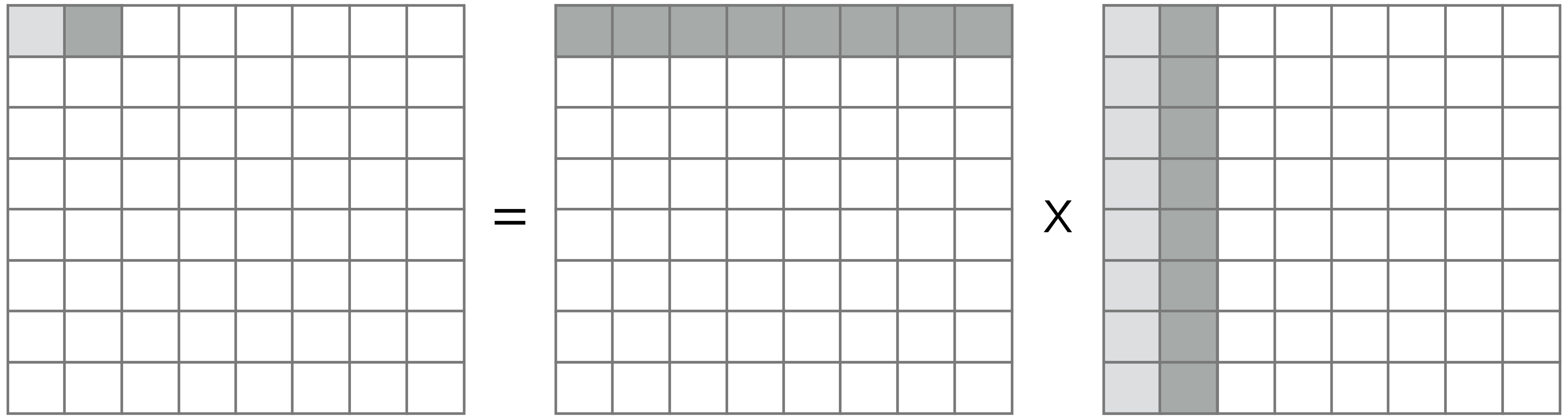

shorter reuse distance

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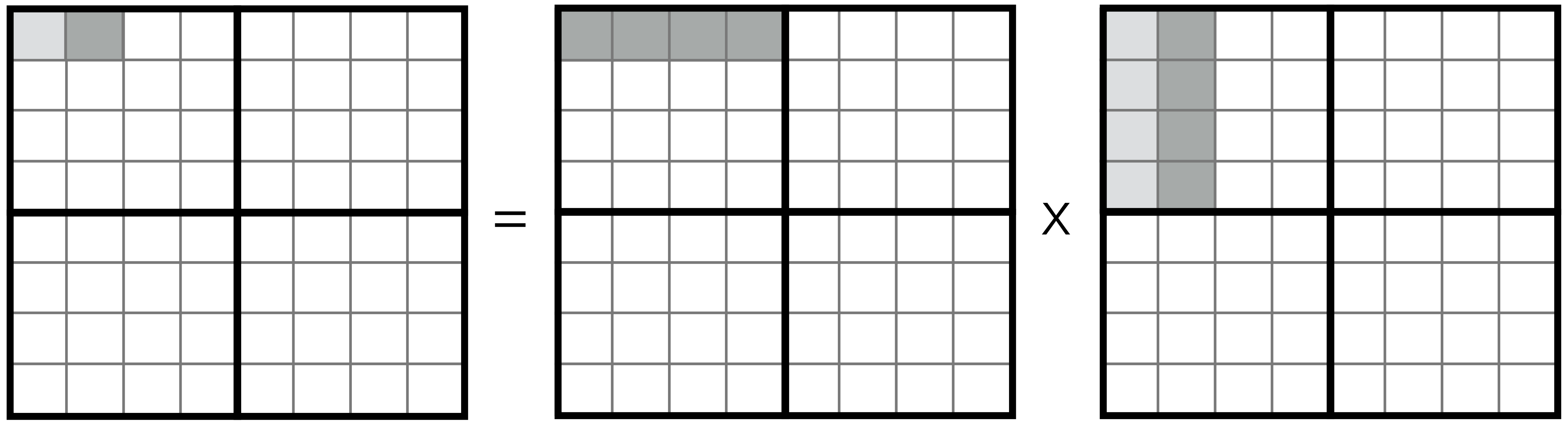

Buying locality with redundant work in fused stencils

Stencil loops

```
for (int j=0; j<4; i++)
  tmp[j] = (input[j-1] + input[j] + input[j+1]) / 3;

for (int i=1; i<3; i++)
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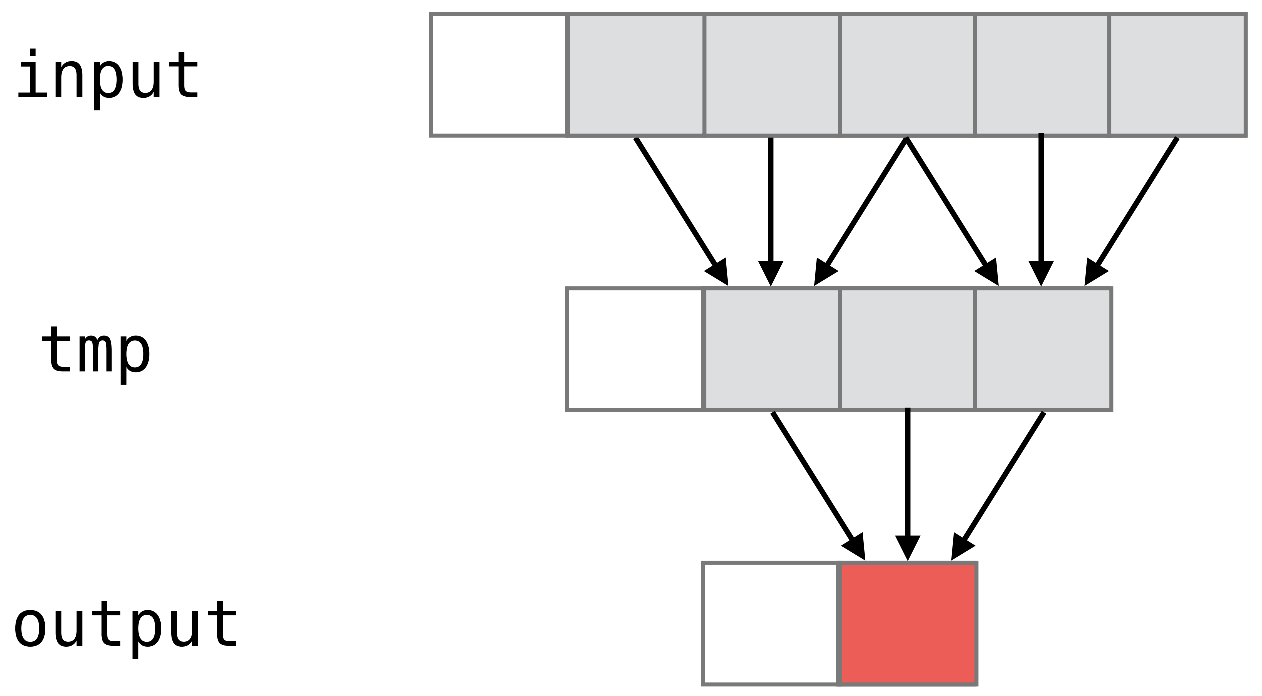

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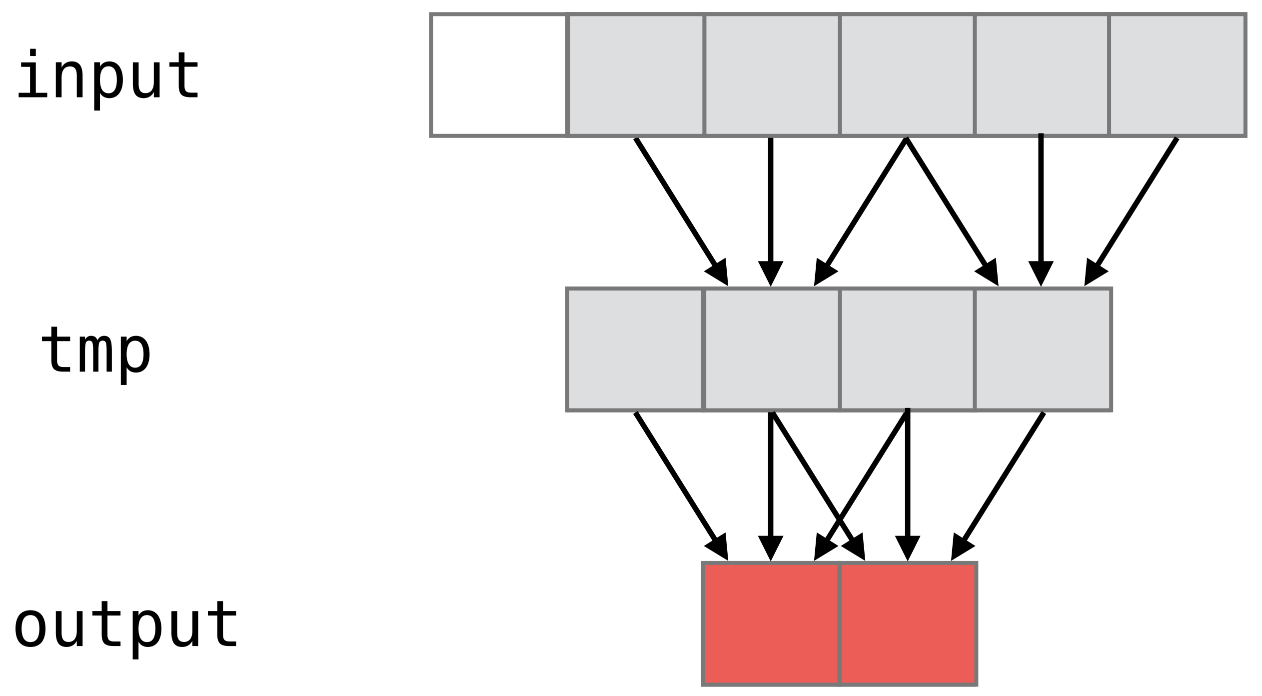

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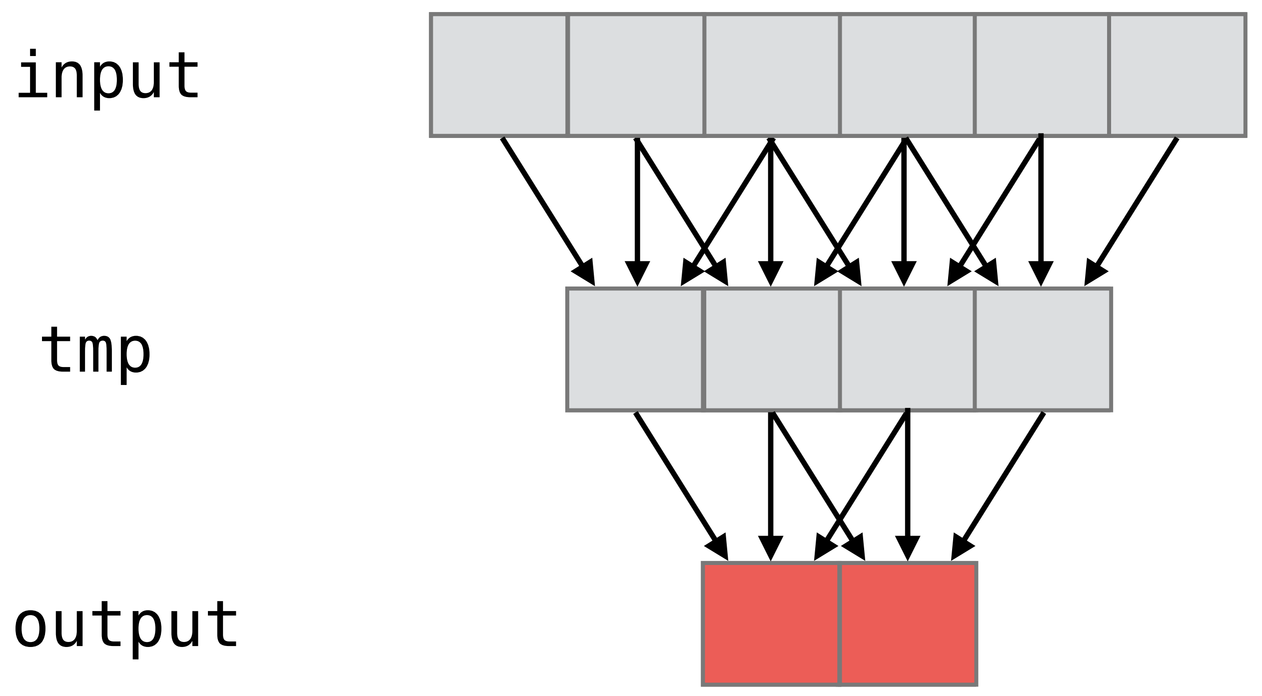

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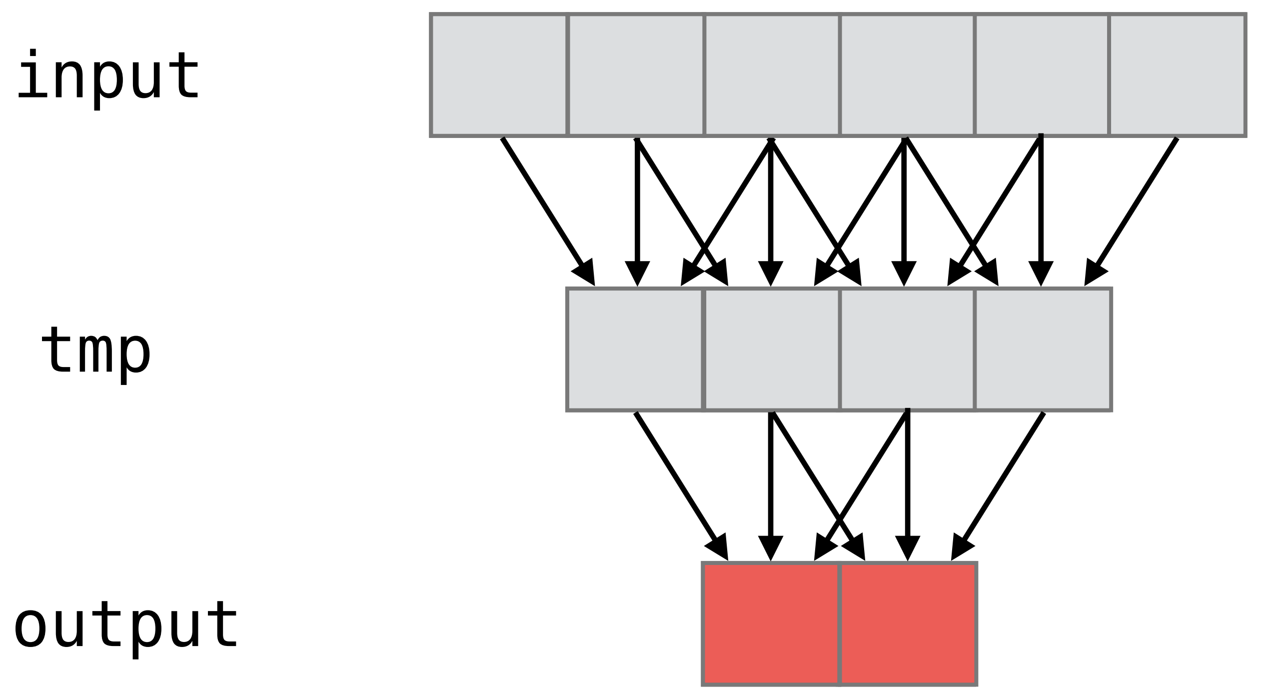

Fused stencil loops

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for (int i=1; i<3; i++)
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                + (input[i-1] + input[i]   + input[i+1]) / 3
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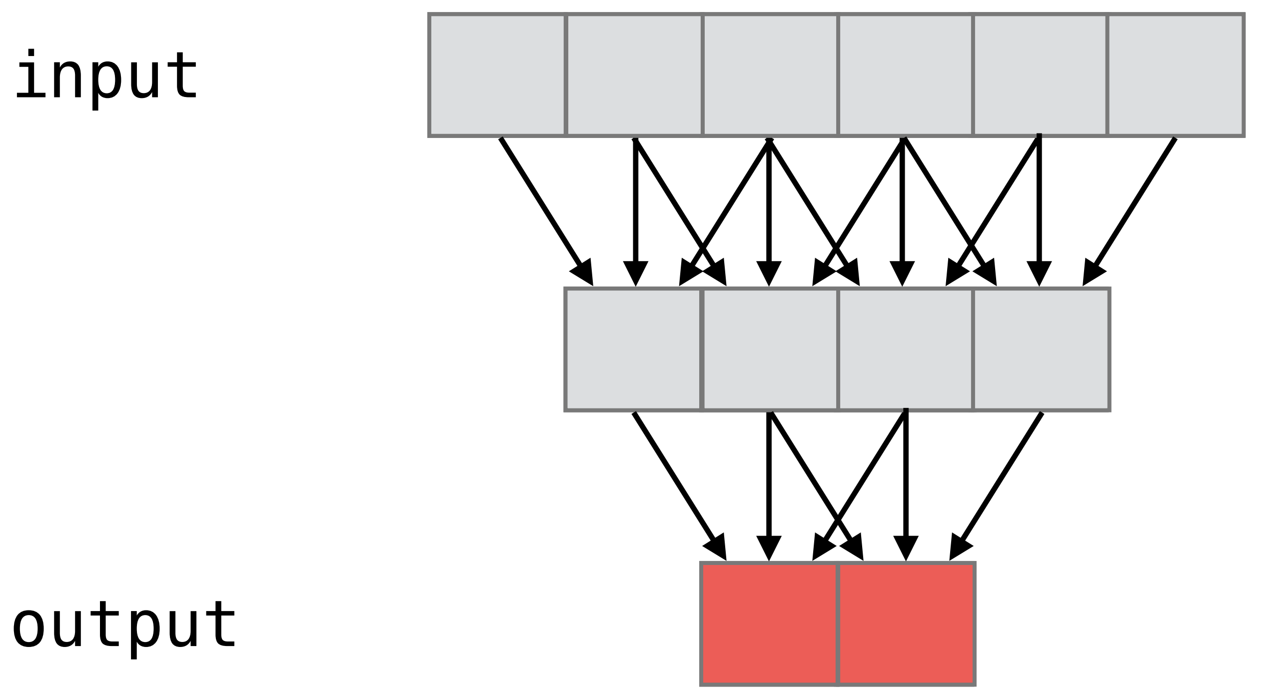

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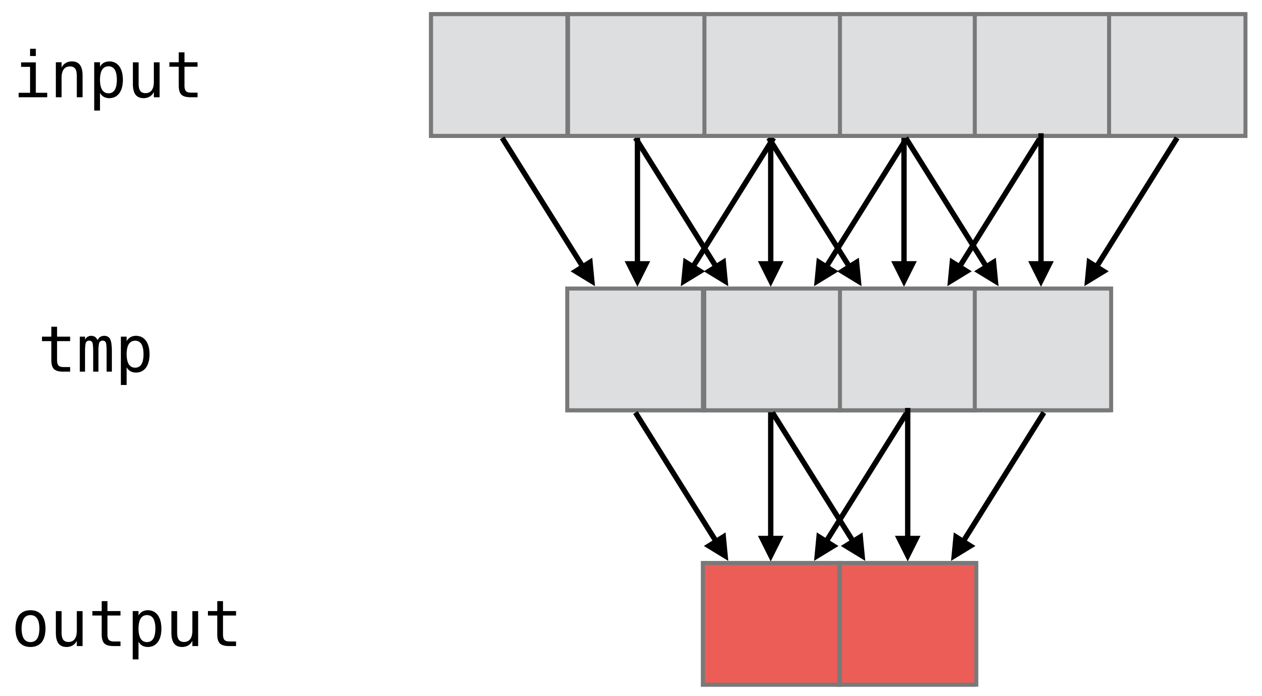

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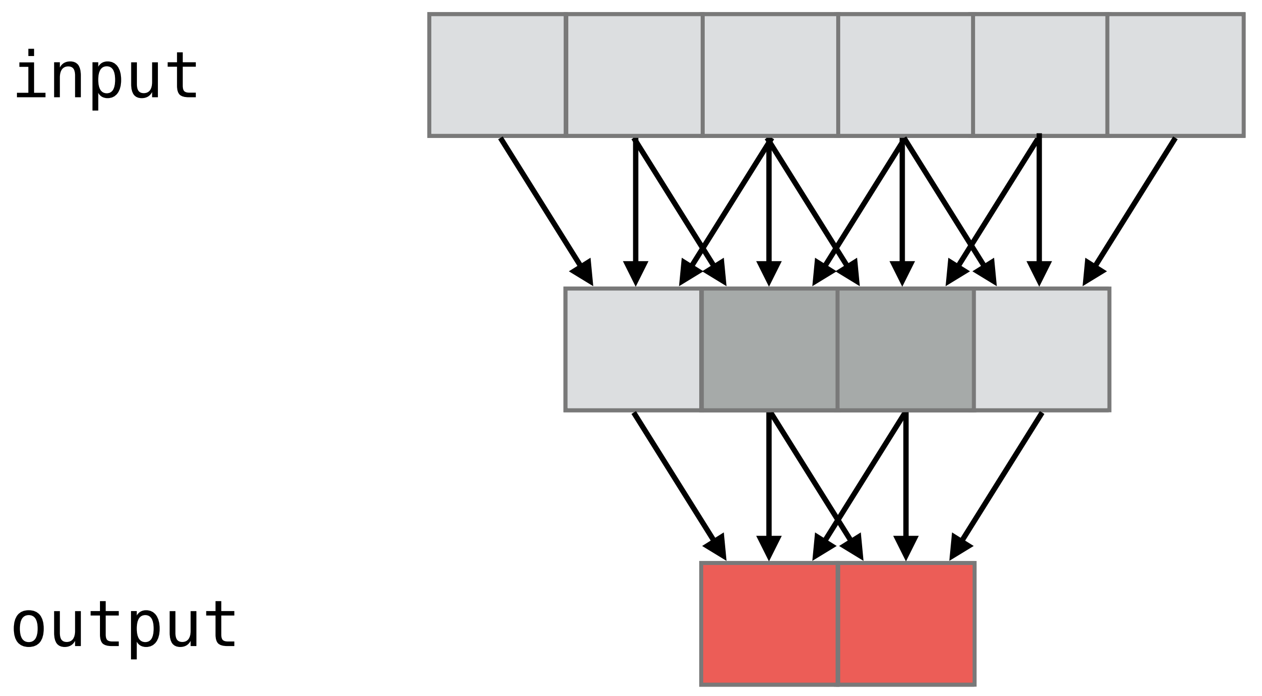

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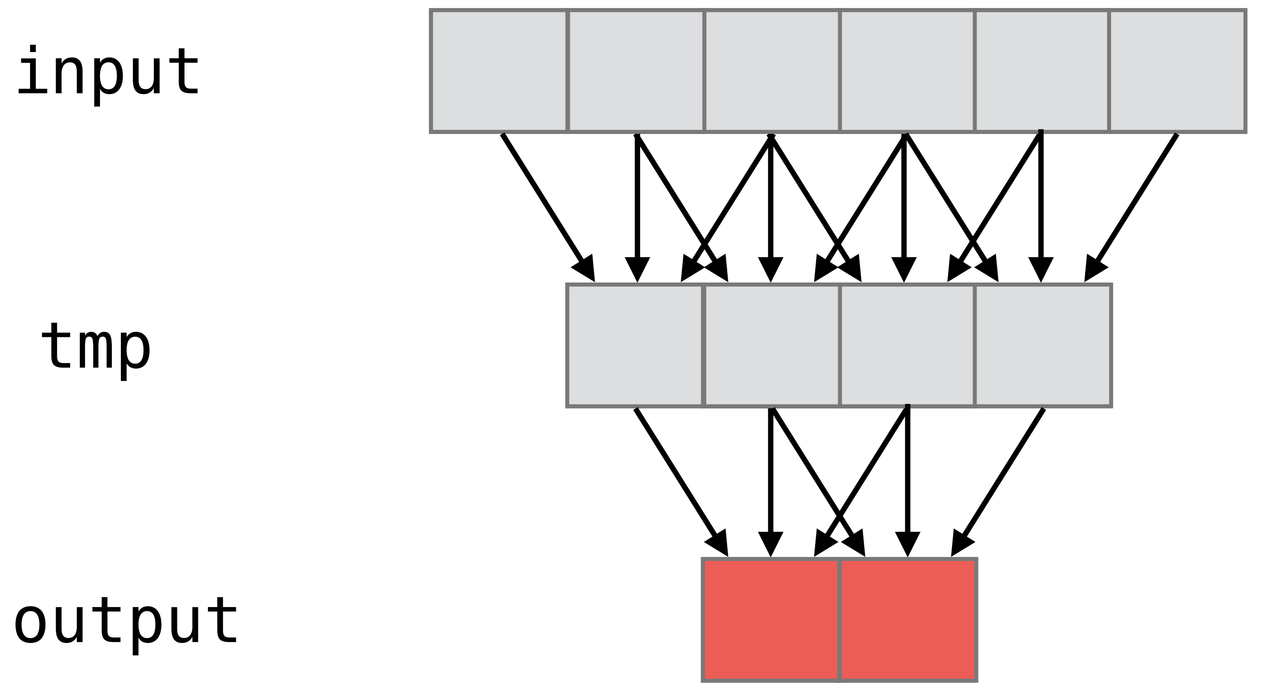

8 additions and
4 divides

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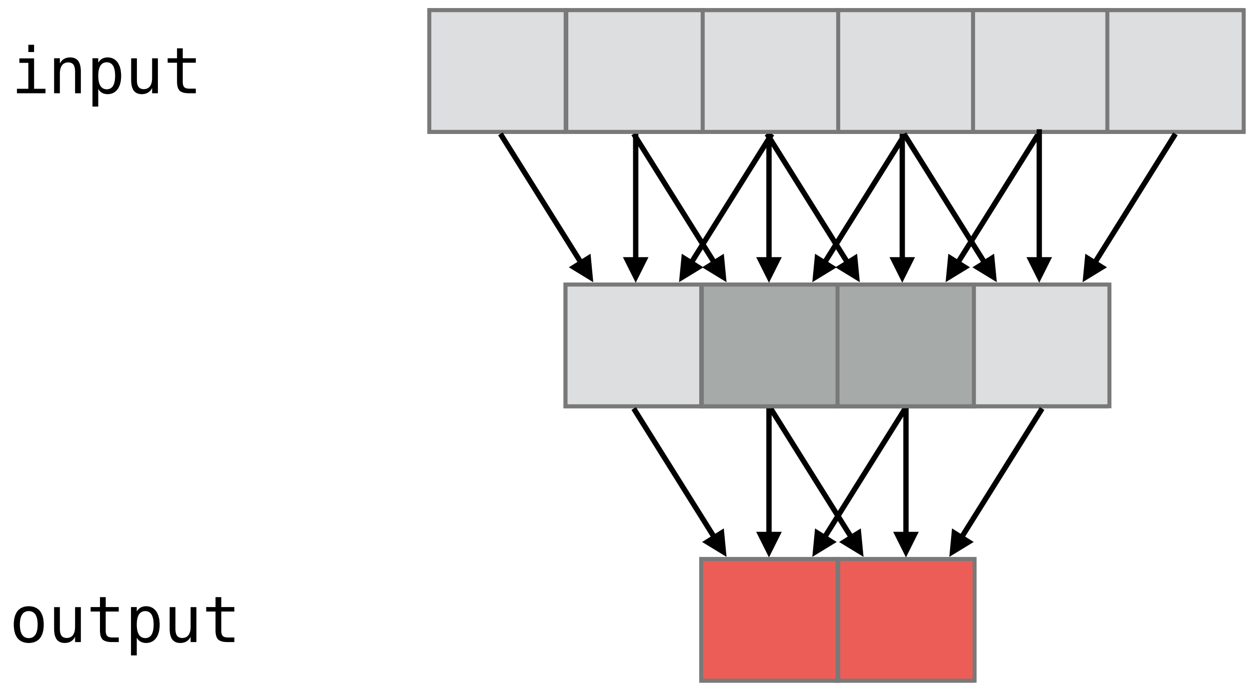

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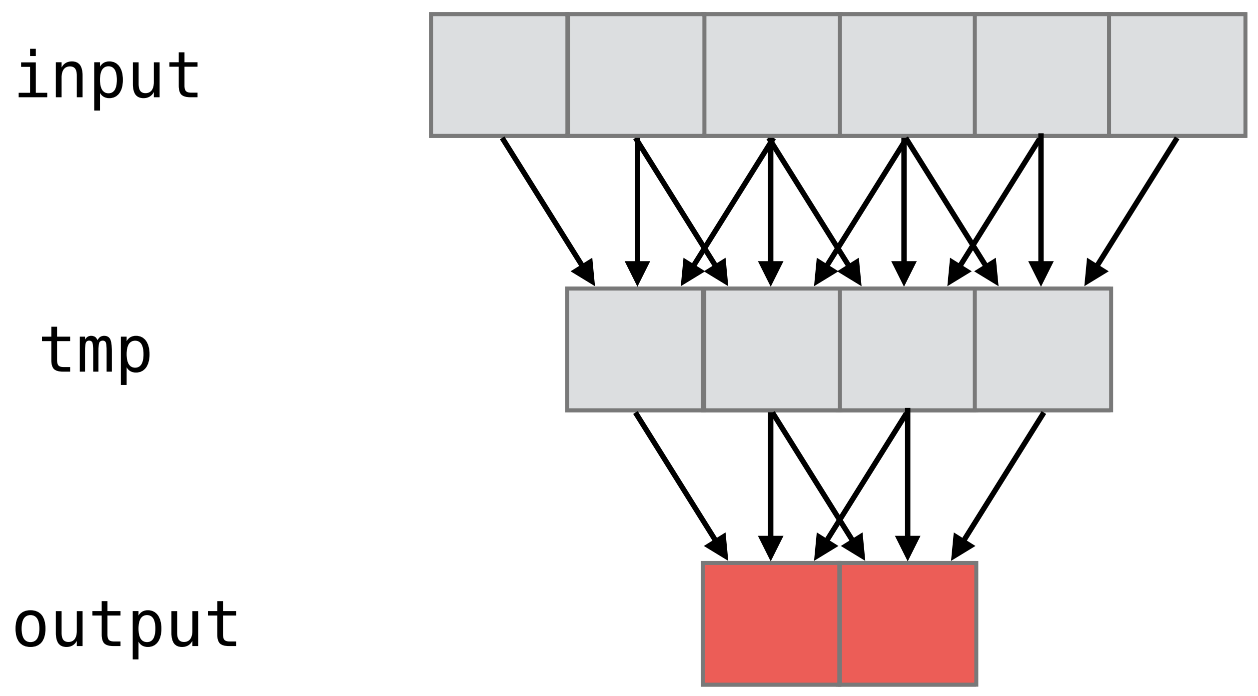

8 additions and
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4 additions and
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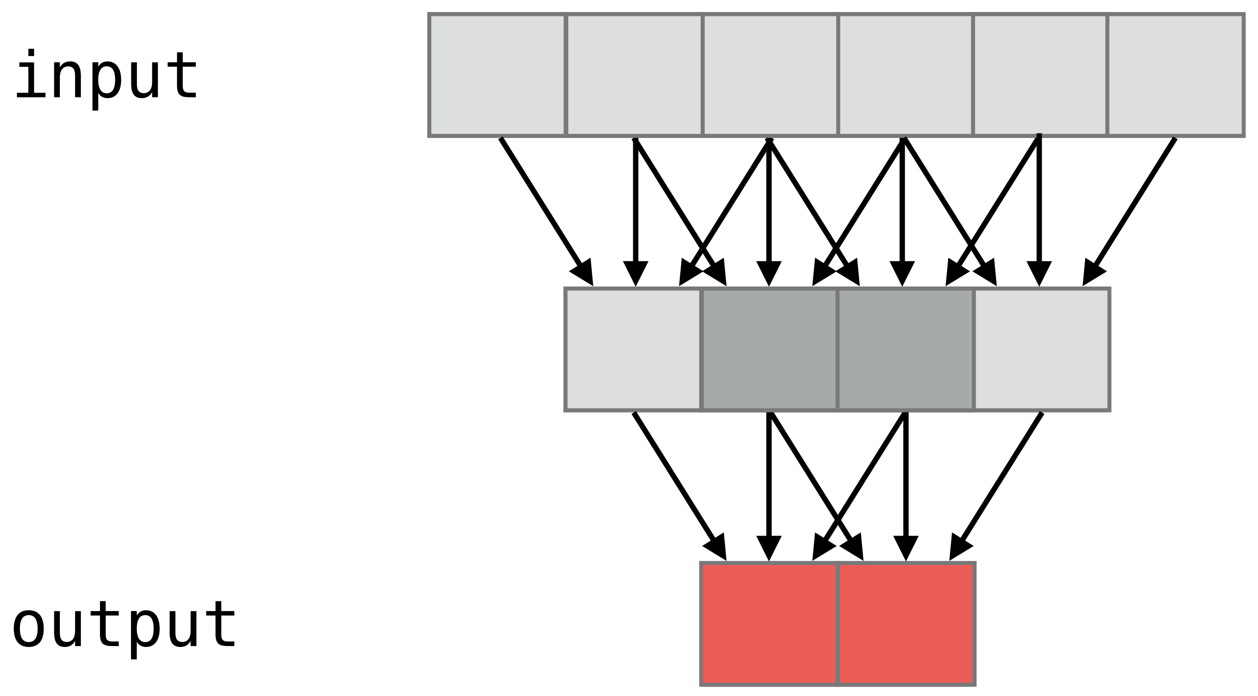

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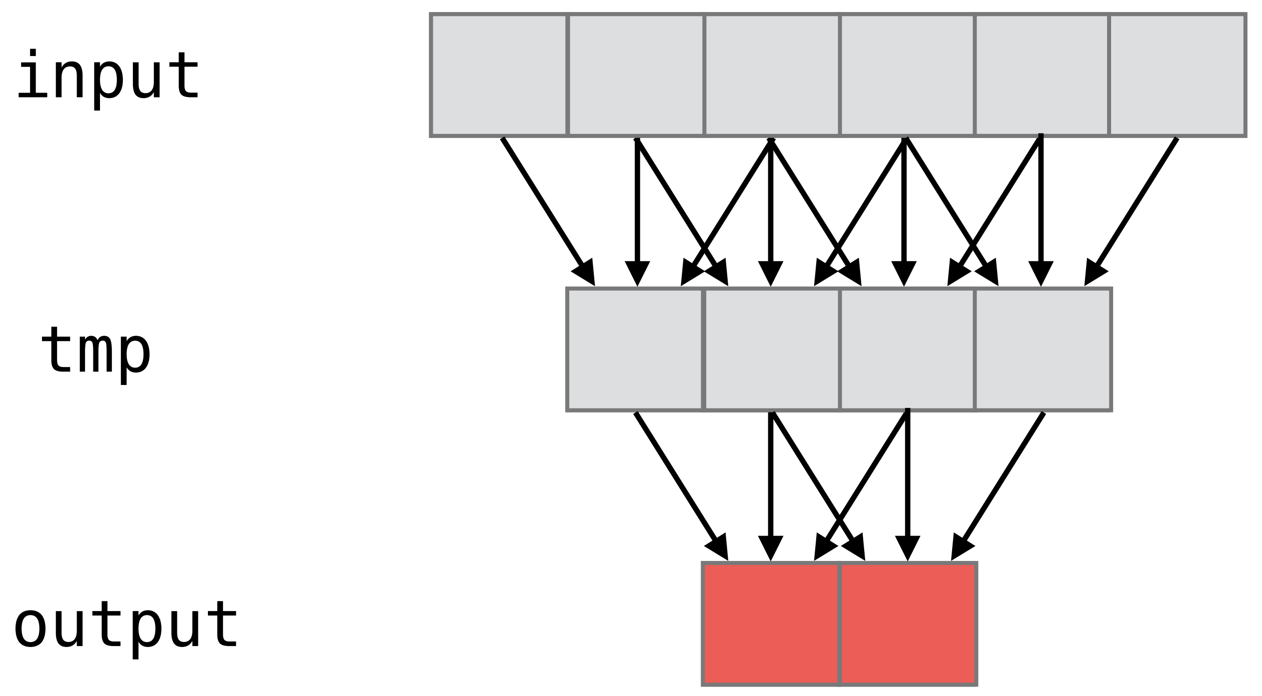

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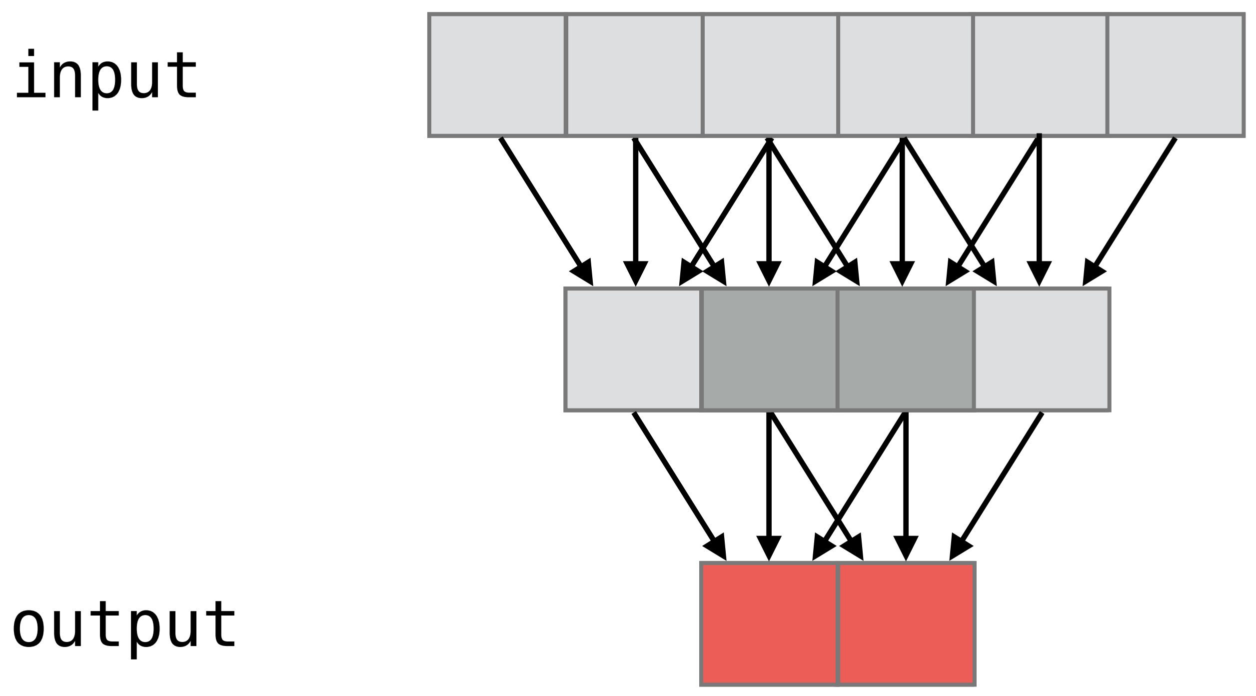
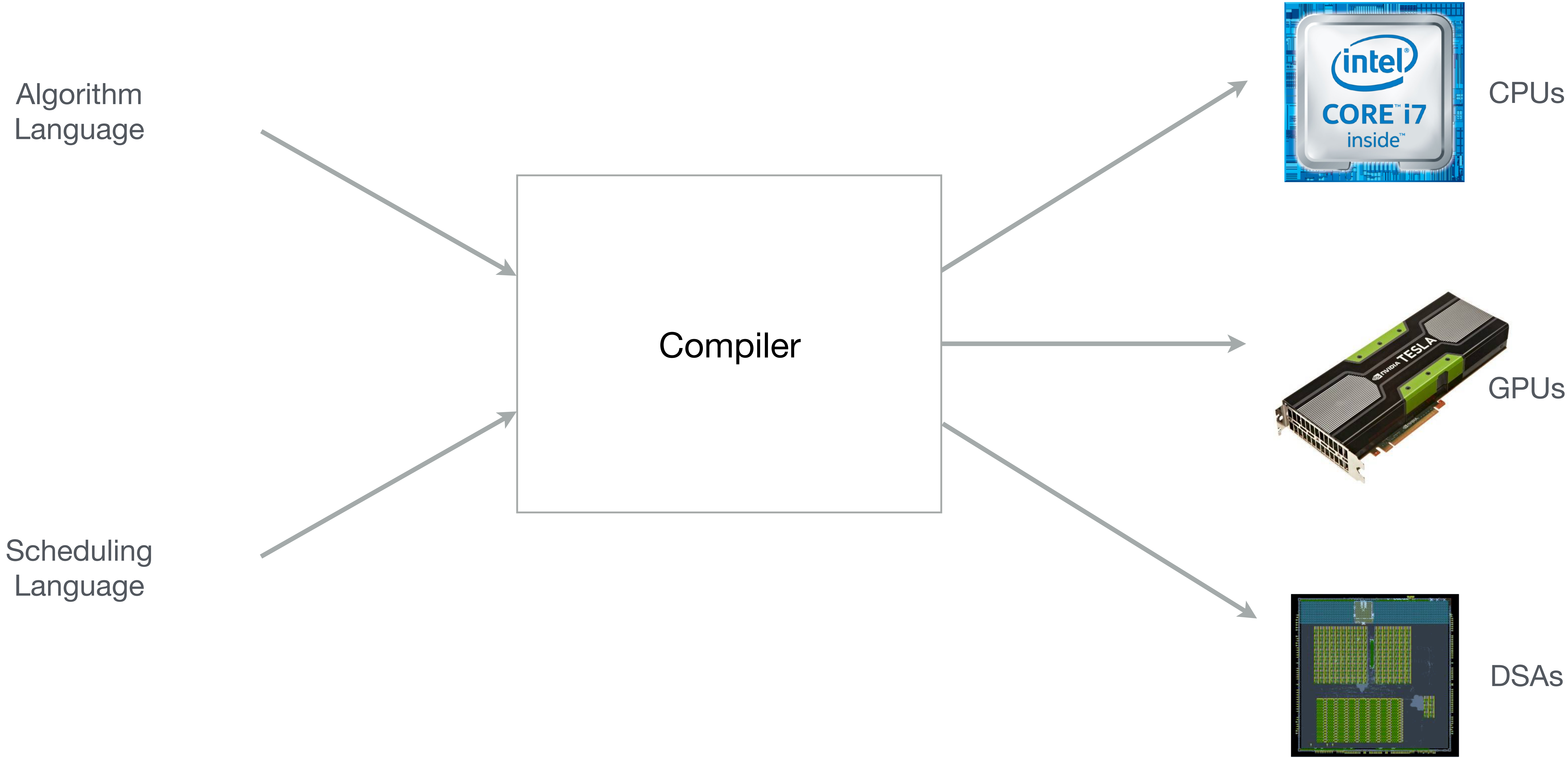


16 additions and
8 divides

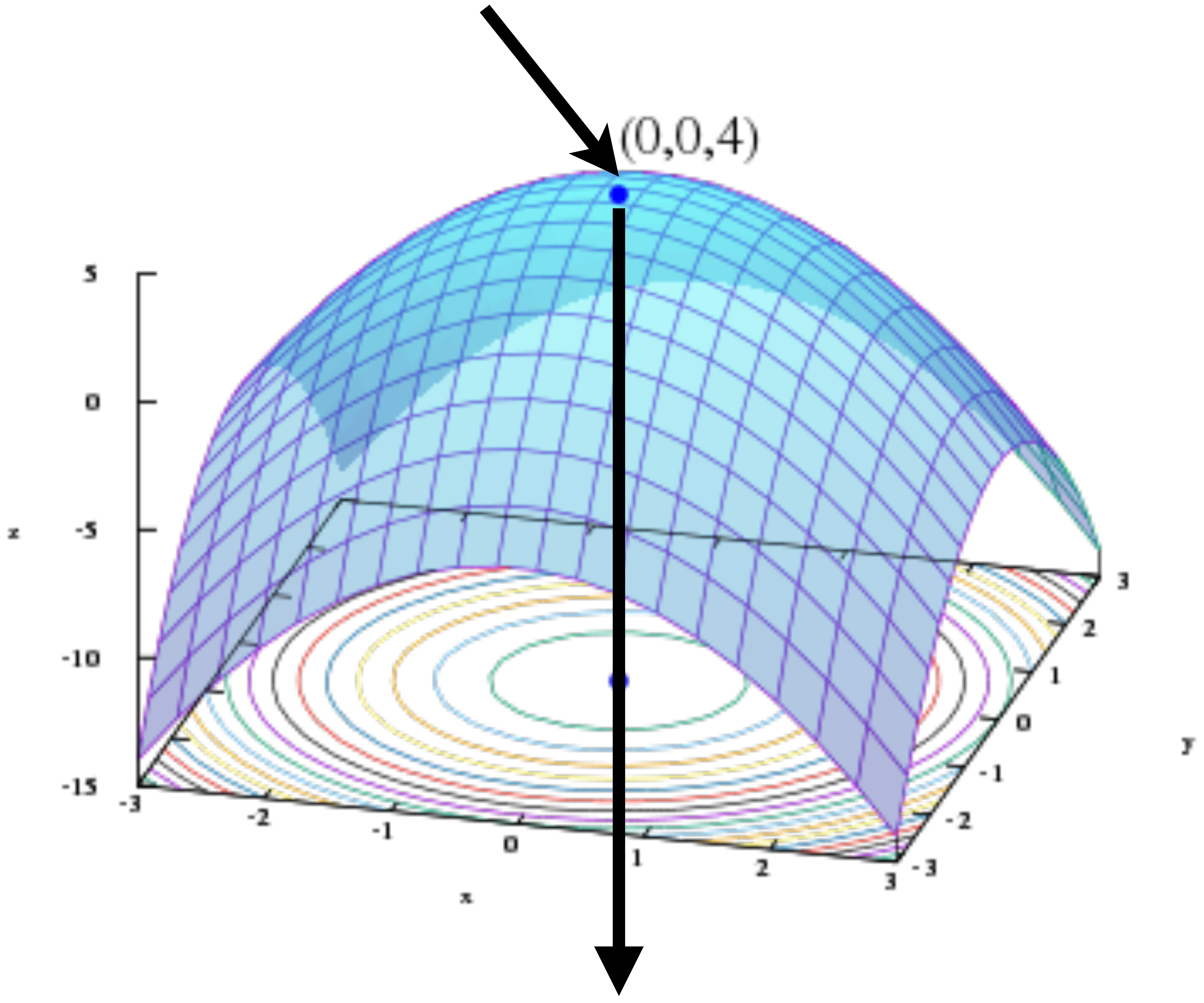
Separation of algorithm from schedules



This idea was most clearly demonstrated in the Halide system

General Principle: Separation of policy and mechanism

Policy is deciding what to do
(decide what transformations to apply)



Mechanism is doing it
(generate code)

Separate by a clean API/language to:

- Solve one complex problem at a time
- Experiment with automatic policy systems without reimplementing mechanism
- Allow users to override default decisions with their own
- Policy tends to evolve faster than mechanism

The Nucleus of a Multiprogramming System. *P. Brinch Hansen* (1970)

Optimization strategies in compilers

Optimization strategies in compilers

1. Greedy or heuristic rewrites

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Optimization strategies in compilers

1. Greedy or heuristic rewrites
2. Integer-linear programming
3. Beam search combined with ML
4. Autotuning with hill climbing, genetic algorithms, etc.
5. Or pick your favorite optimization strategy and
    - Define an optimization space and a cost function
    - Implement a search procedure

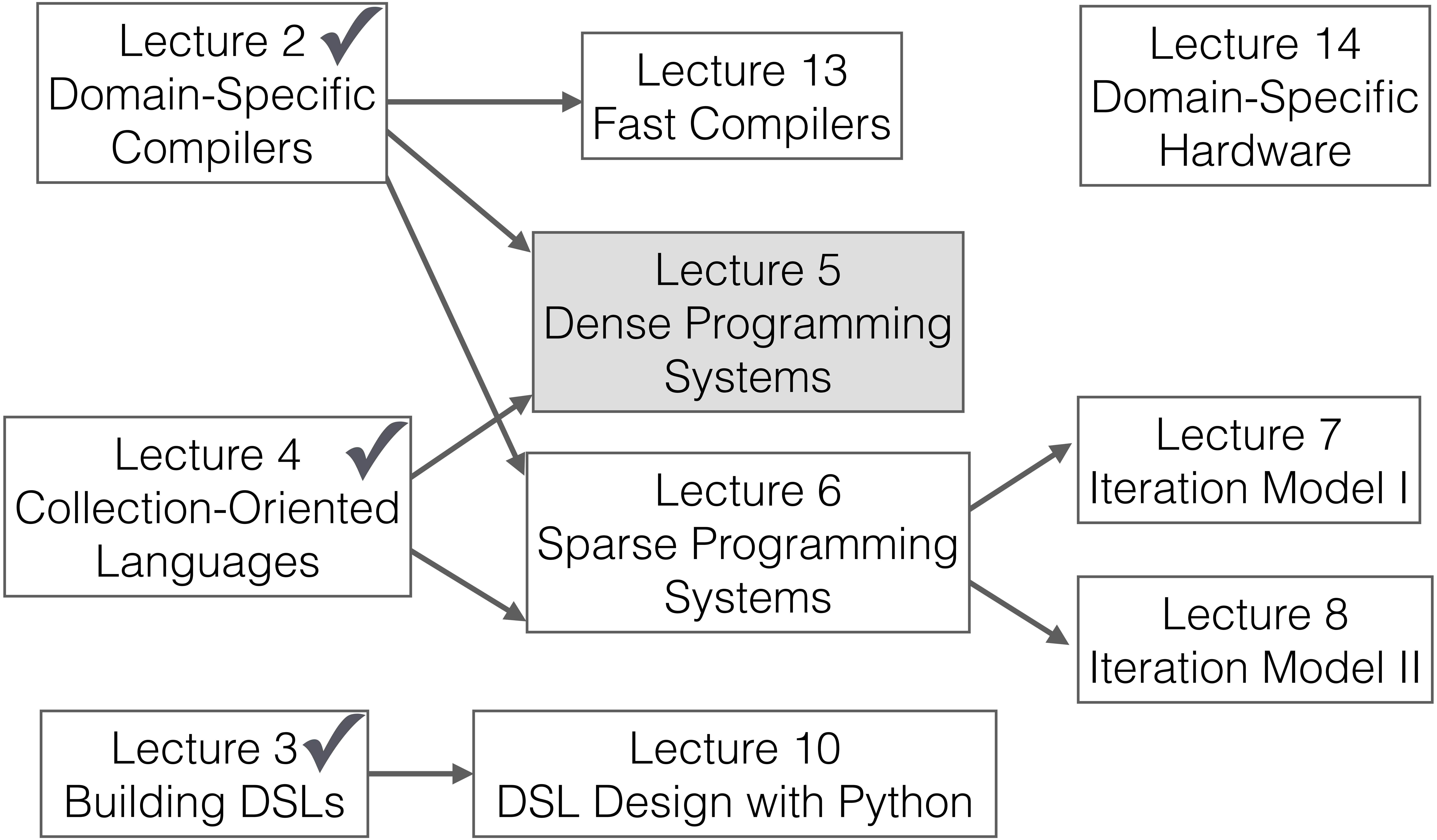
Example: Halide

```
Func halide_blur(Func in) {
 Func tmp, blurred;
 Var x, y, xi, yi;

 // The algorithm
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;

 // The schedule
 blurred.tile(x, y, xi, yi, 256, 32)
        .vectorize(xi, 8).parallel(y);
 tmp.chunk(x).vectorize(x, 8);

 return blurred;
}
```

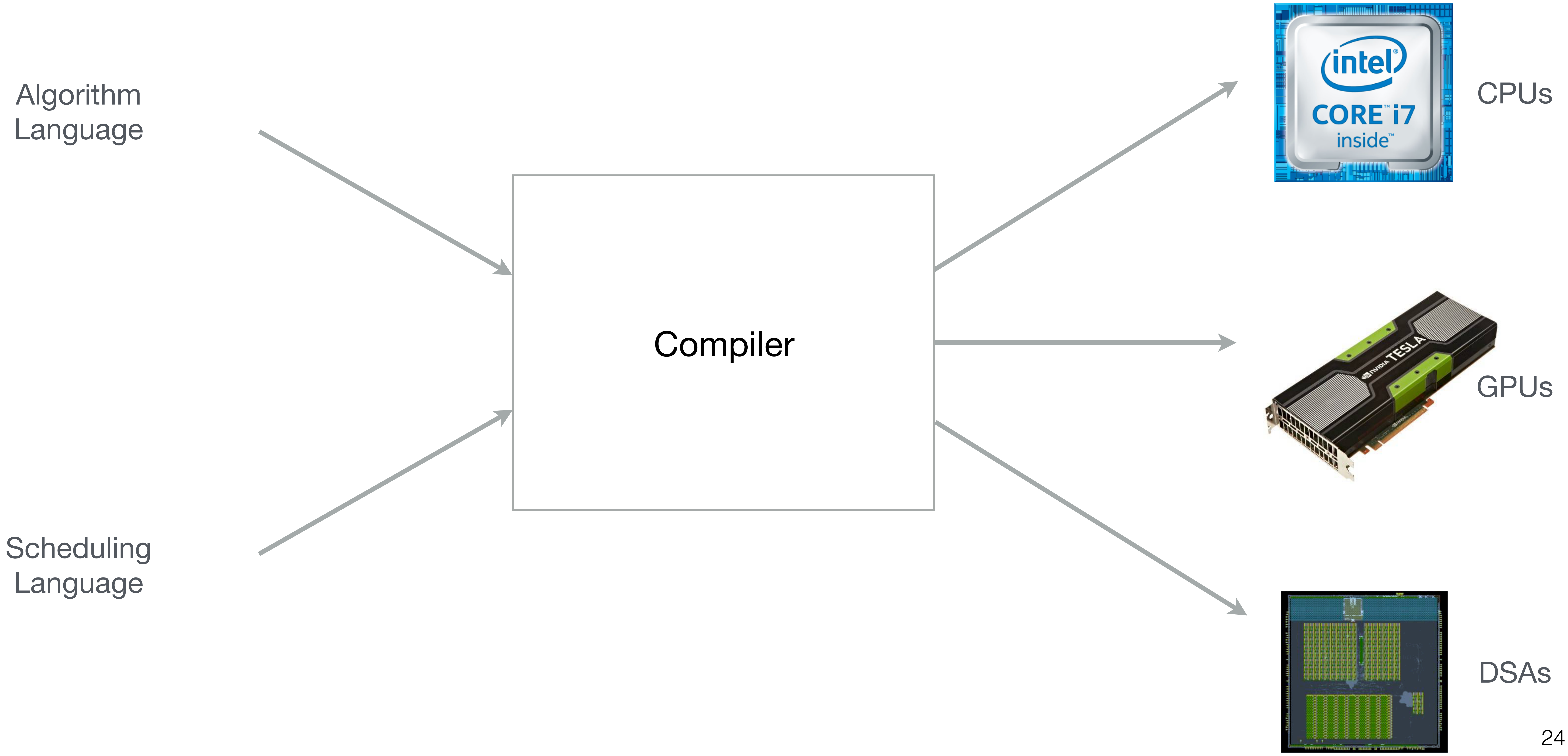
Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines. *Ragan-Kelley et al.* (2012)

Several auto-schedulers have been developed; a recent autoscheduler uses beam-search

Lecture Overview



Next up: separation of Algorithm, Schedule, and Data Representation



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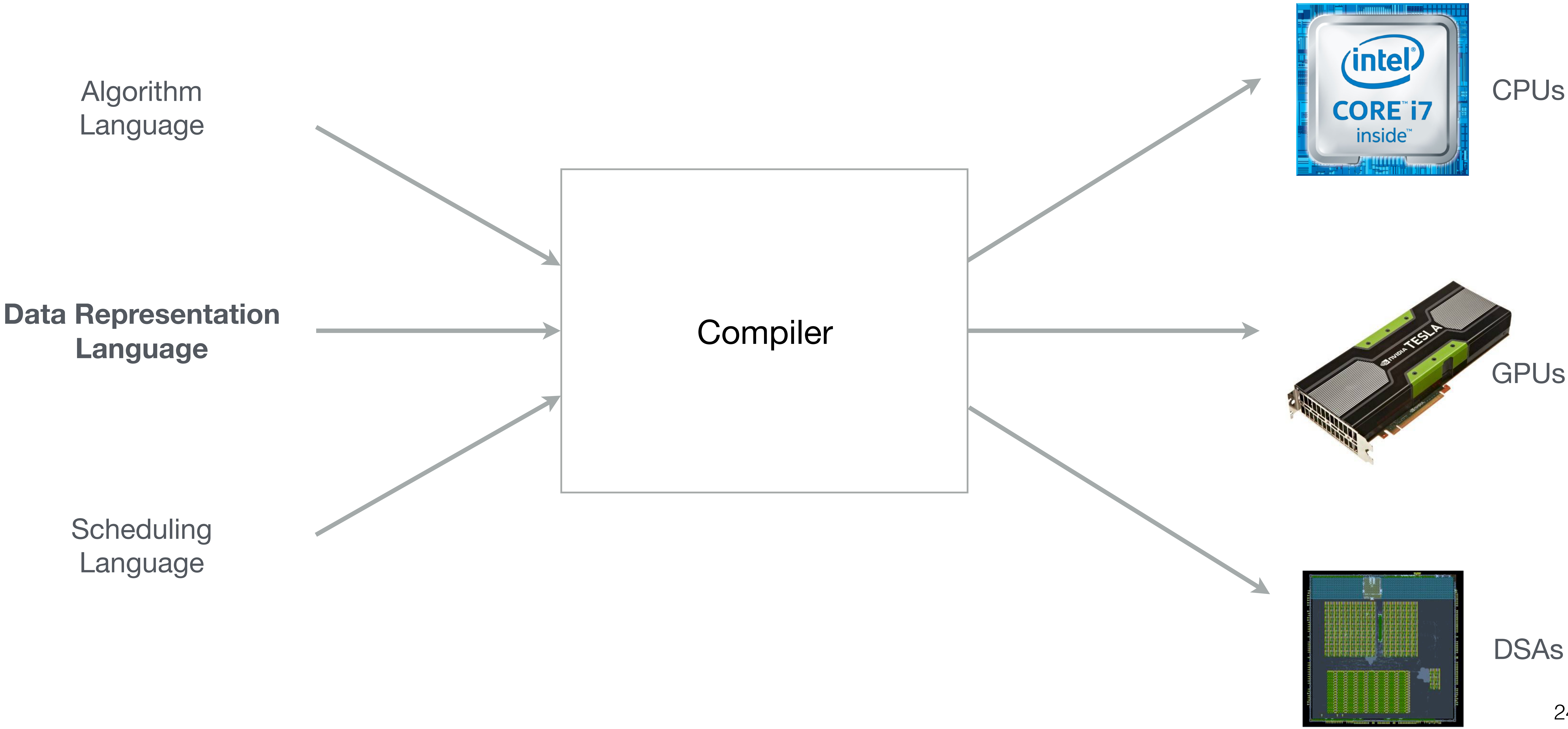

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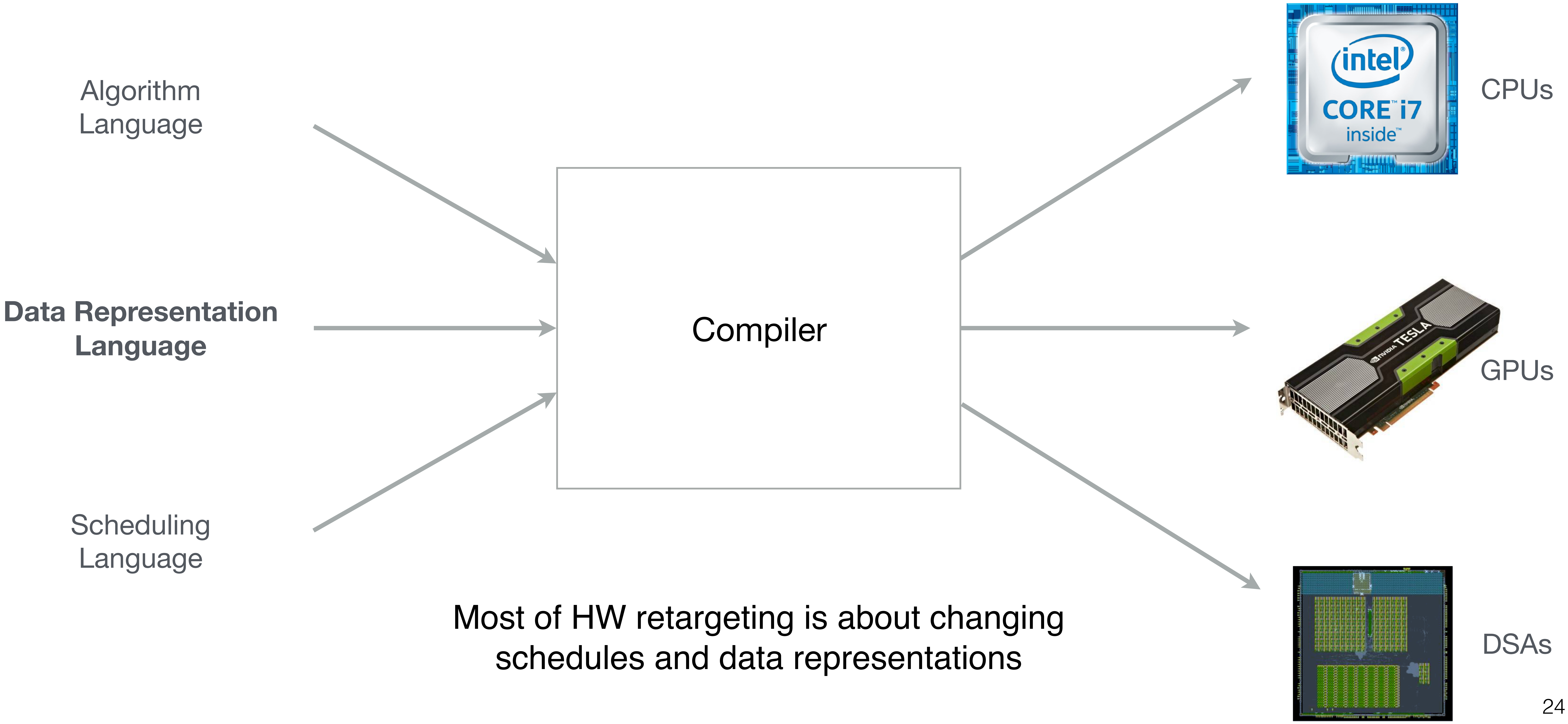

Most of HW retargeting is about changing schedules and data representations