Lecture 8 — Sparse Iteration Model II

Stanford CS343D (Winter 2024)
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Course Project

Overview of topics

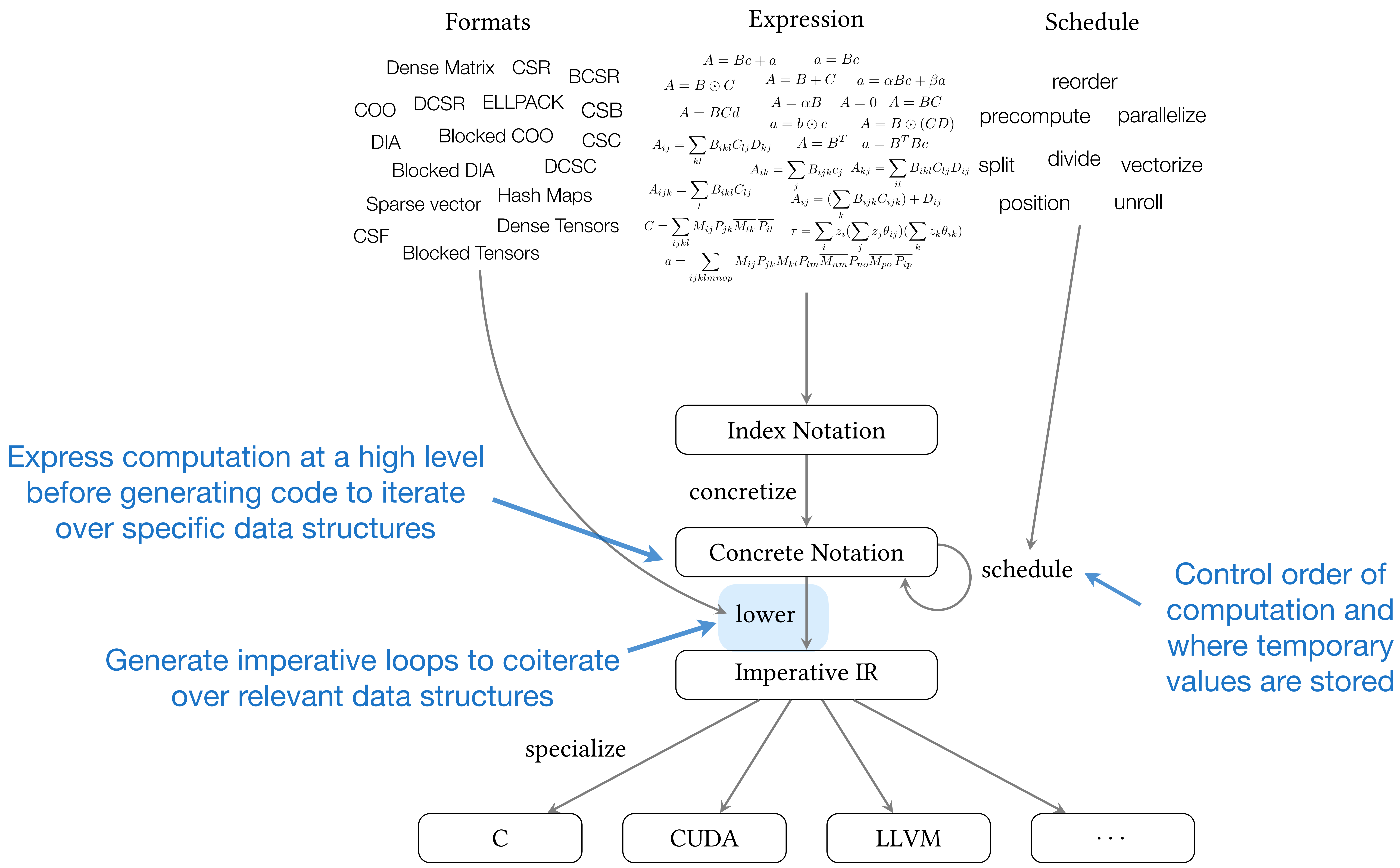
Lecture 7

- Data representation
- Iteration spaces
- Iteration graph IR
- Iteration lattices to represent coiteration

Lecture 8

- Concrete index notation IR
- Code generation algorithm
- Derived iteration spaces
- Optimizing transformations

Overview of compilation stages



Concrete index notation specifies order of computations and location of intermediate values

Index Notation		Concrete Index Notation
$A_{ij} = B_{ij} + C_{ij}$	$\longrightarrow$	$\forall_i \forall_j \, A_{ij} = B_{ij} + C_{ij}$
$\alpha = \sum_i b_i c_i$	$\longrightarrow$	$\forall_i \, \alpha \mathrel{+}= b_i c_i$
$a_i = \sum_j B_{ij} c_j$	$\longrightarrow$	$\forall_i \, a_i = t \; \textbf{where} \; \forall_j \, t \mathrel{+}= B_{ij} c_j$

Concrete index notation grammars

Assignment statement

$$A_{i\ldots} = \text{expr}$$

Forall statement

$$\forall_i \ \text{stmt}$$

Where statement

$$\text{stmt}_c \ \textbf{where} \ \text{stmt}_p$$

Environment

$$\text{index index "} \xrightarrow{\text{"collapse"}} \text{" index}$$

$$\text{index "} \xrightarrow{\text{"split(" d "," s ")"}} \text{" index index}$$

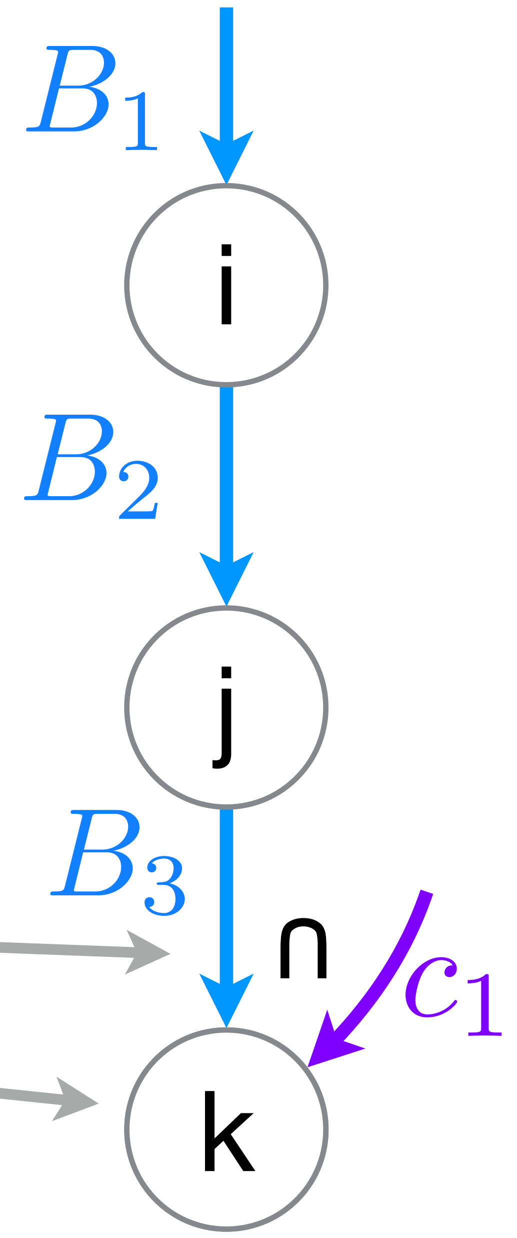
“bound(” index “,” b “)”

“parallelize(” index “,” p “,” r “)”

“unroll(”index “,” u“)”

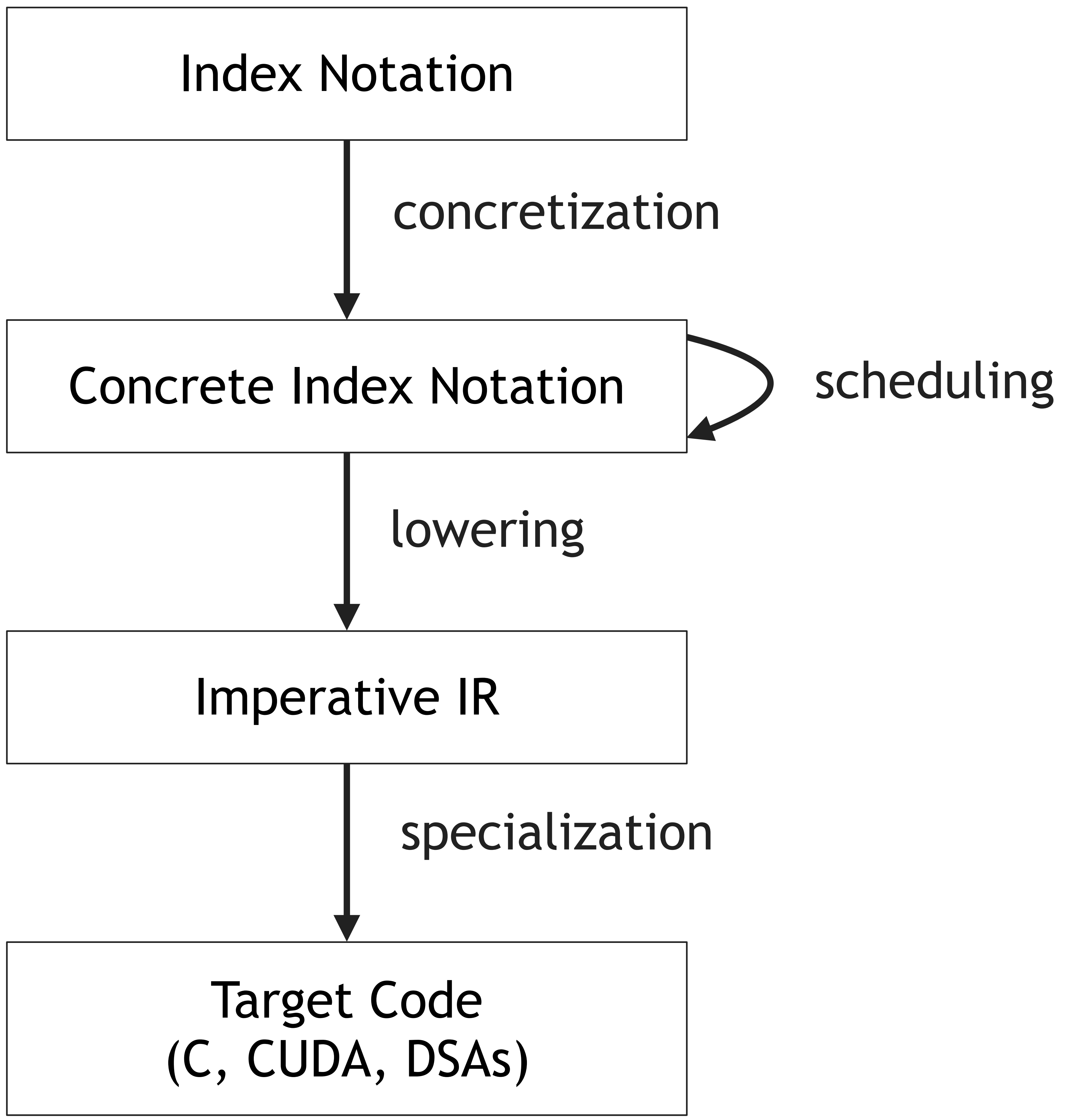
Concrete index notation contains iteration graphs

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} \, c_k$$



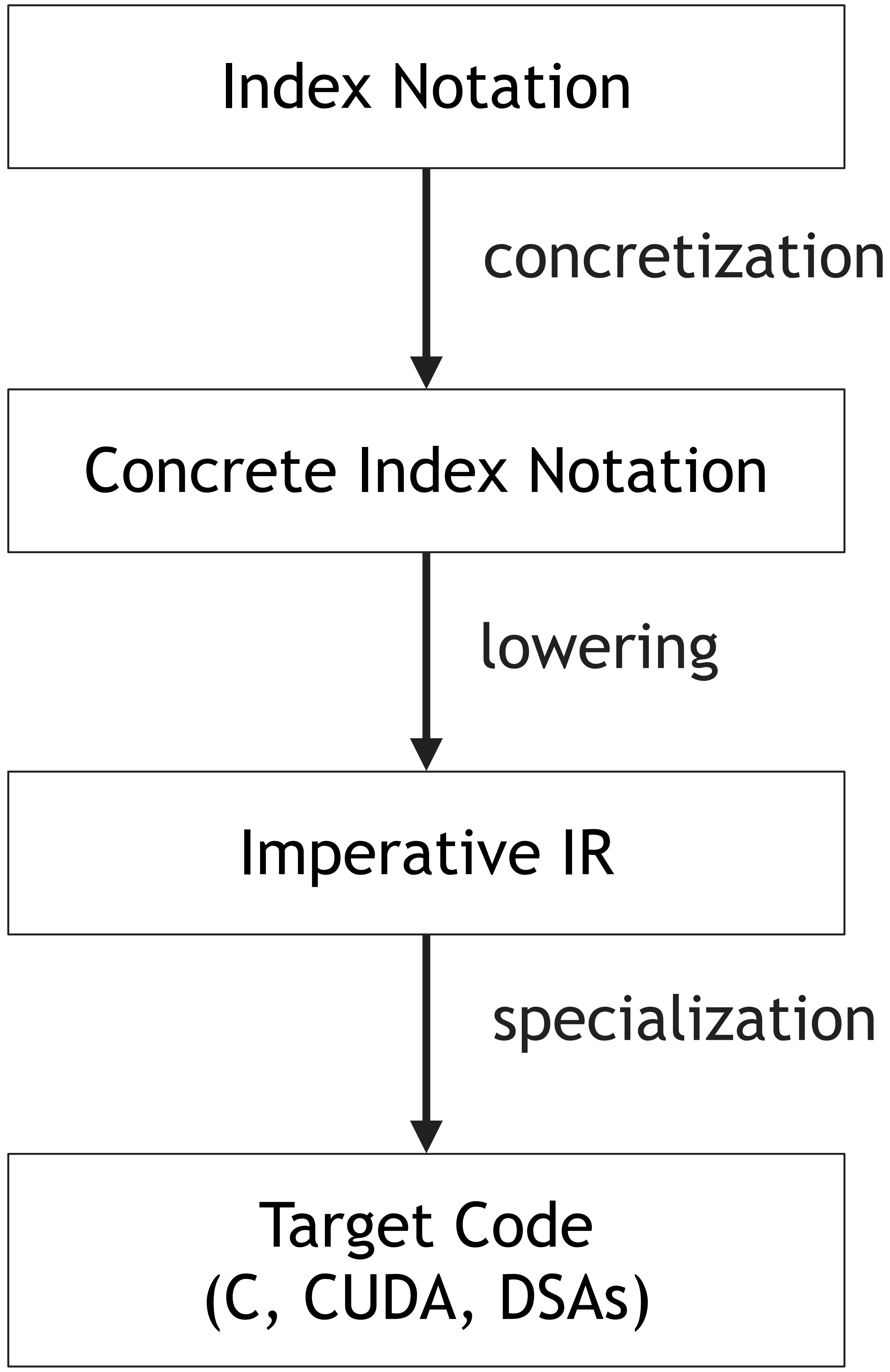
$$B_{ijk} \cap c_k$$

Concrete index notation as an optimization IR



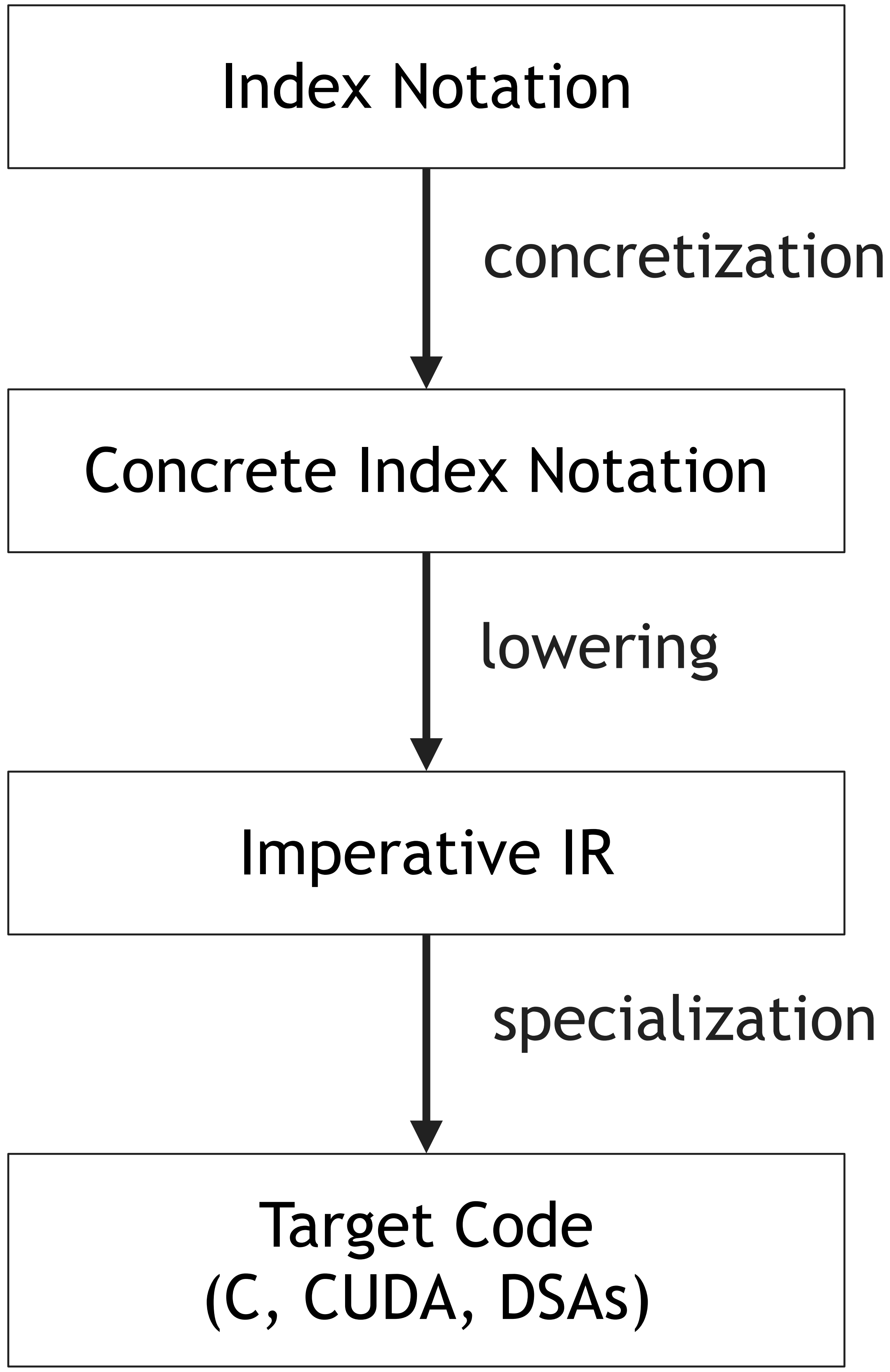
Optimizing transformations are applied before coiteration code is generated. Thus it applies equally to dense and sparse computation

Concrete index notation example



$$A_{ij} = \sum_k B_{ijk} c_k$$

Concrete index notation example

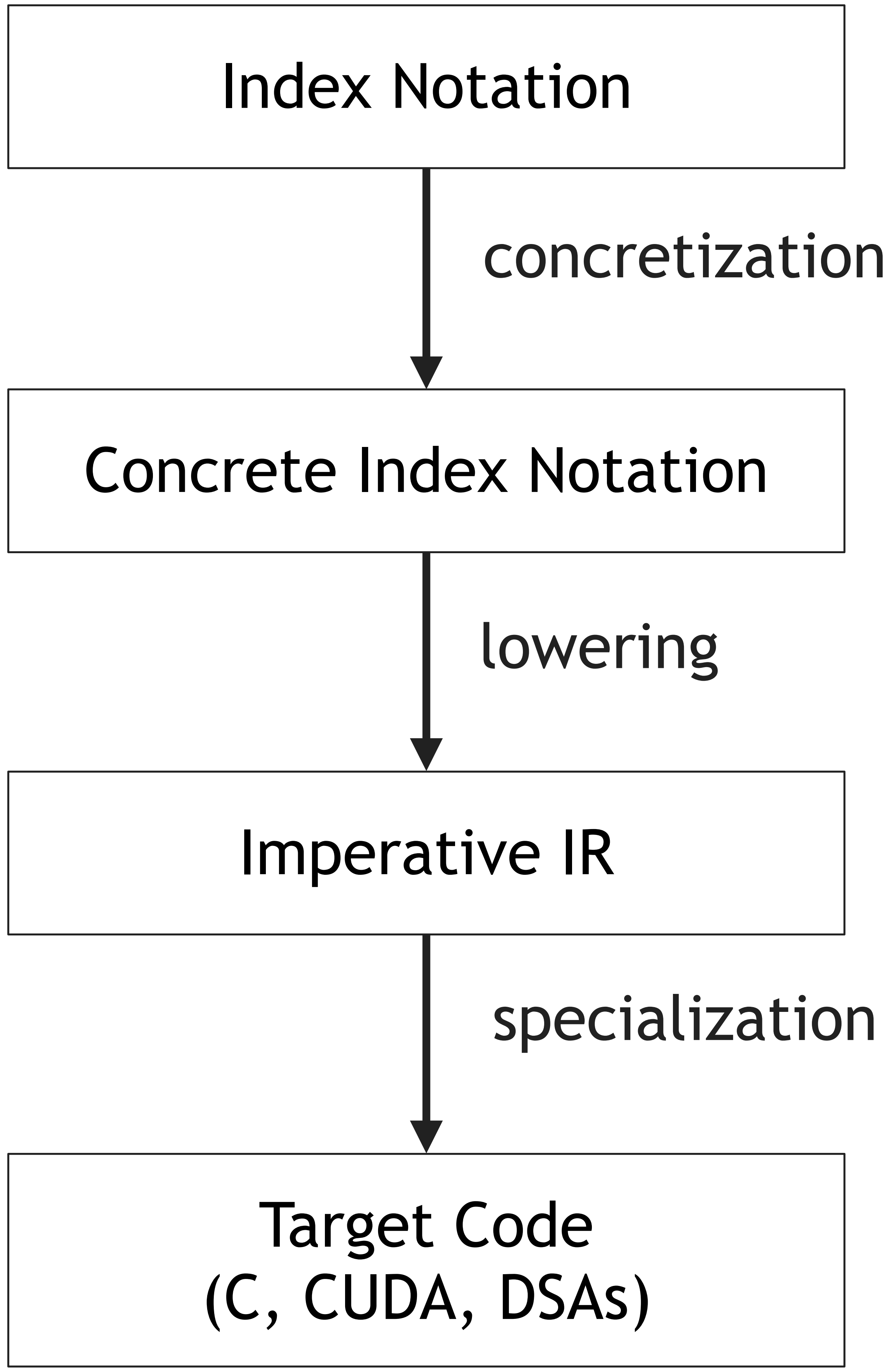


$$A_{ij} = \sum_k B_{ijk} c_k$$

Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$

Concrete index notation example



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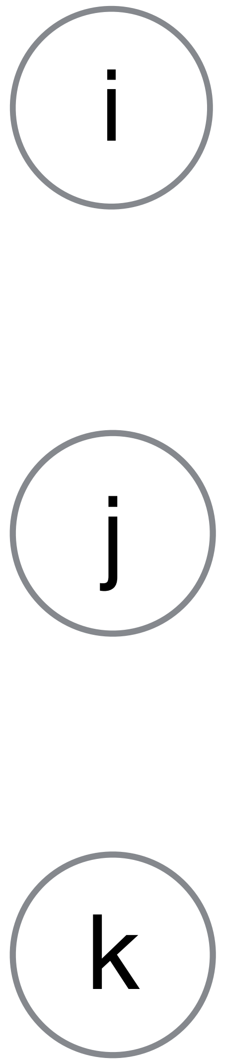
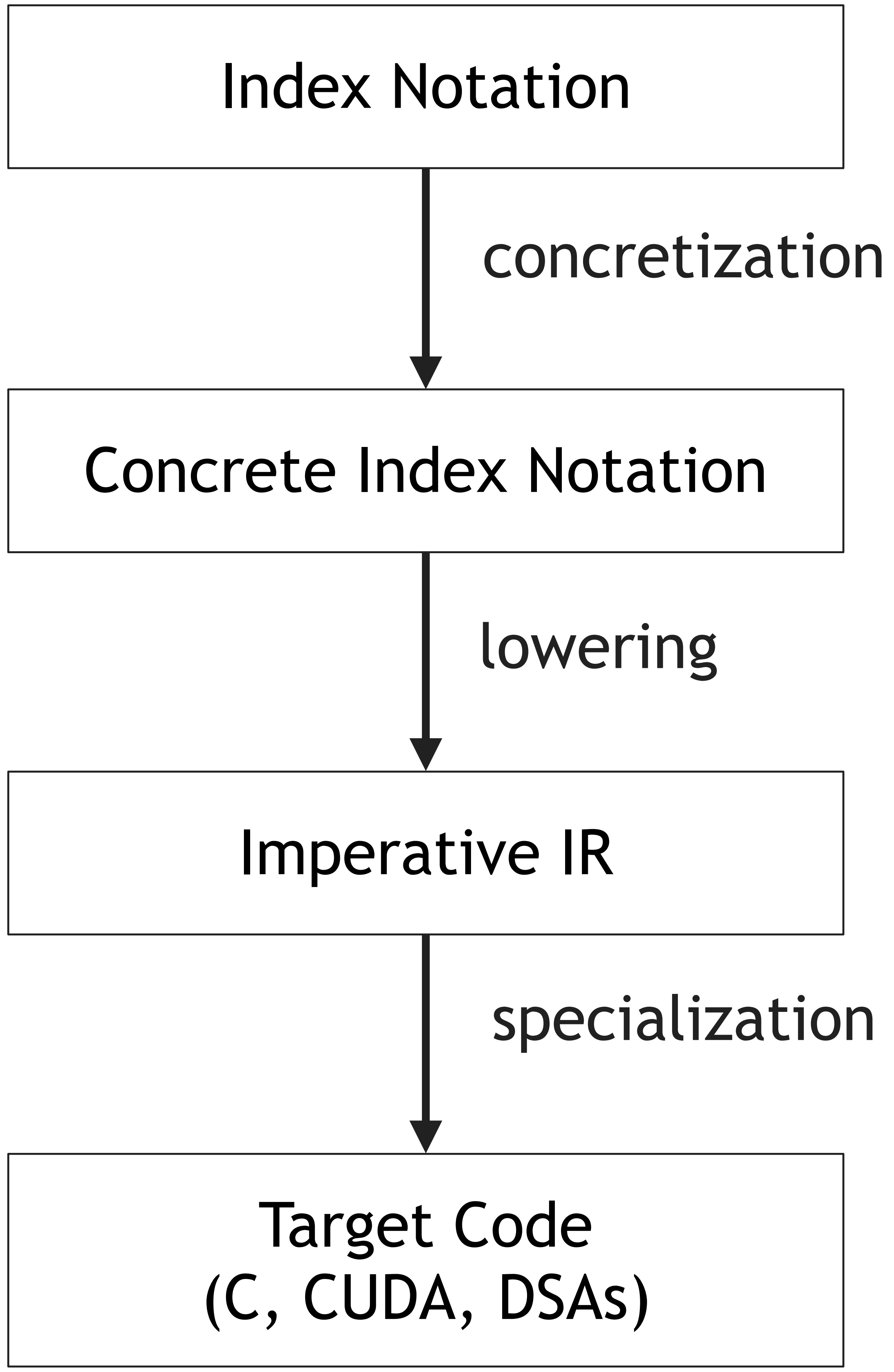


Concrete index notation example



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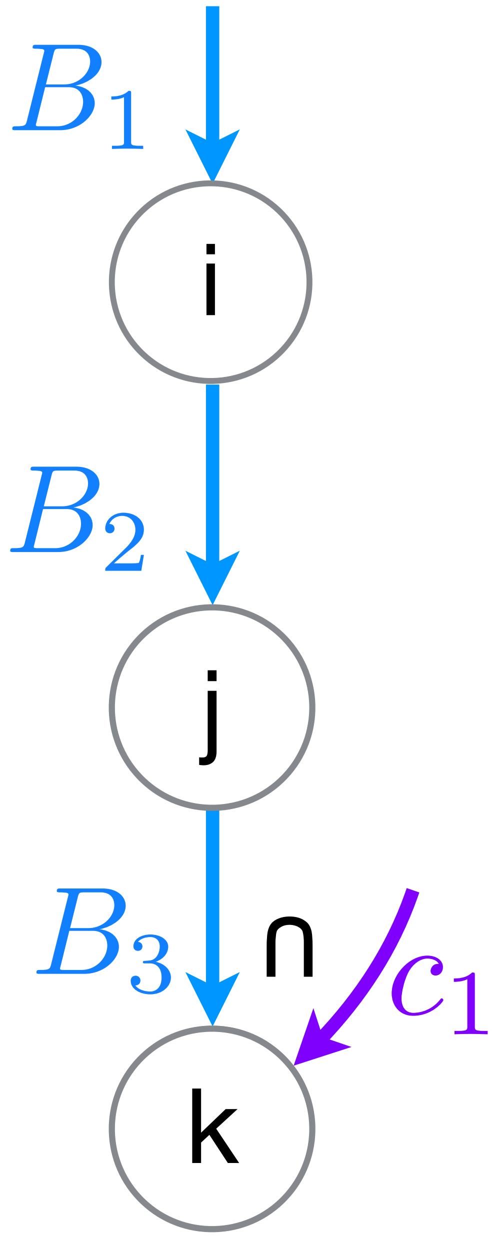
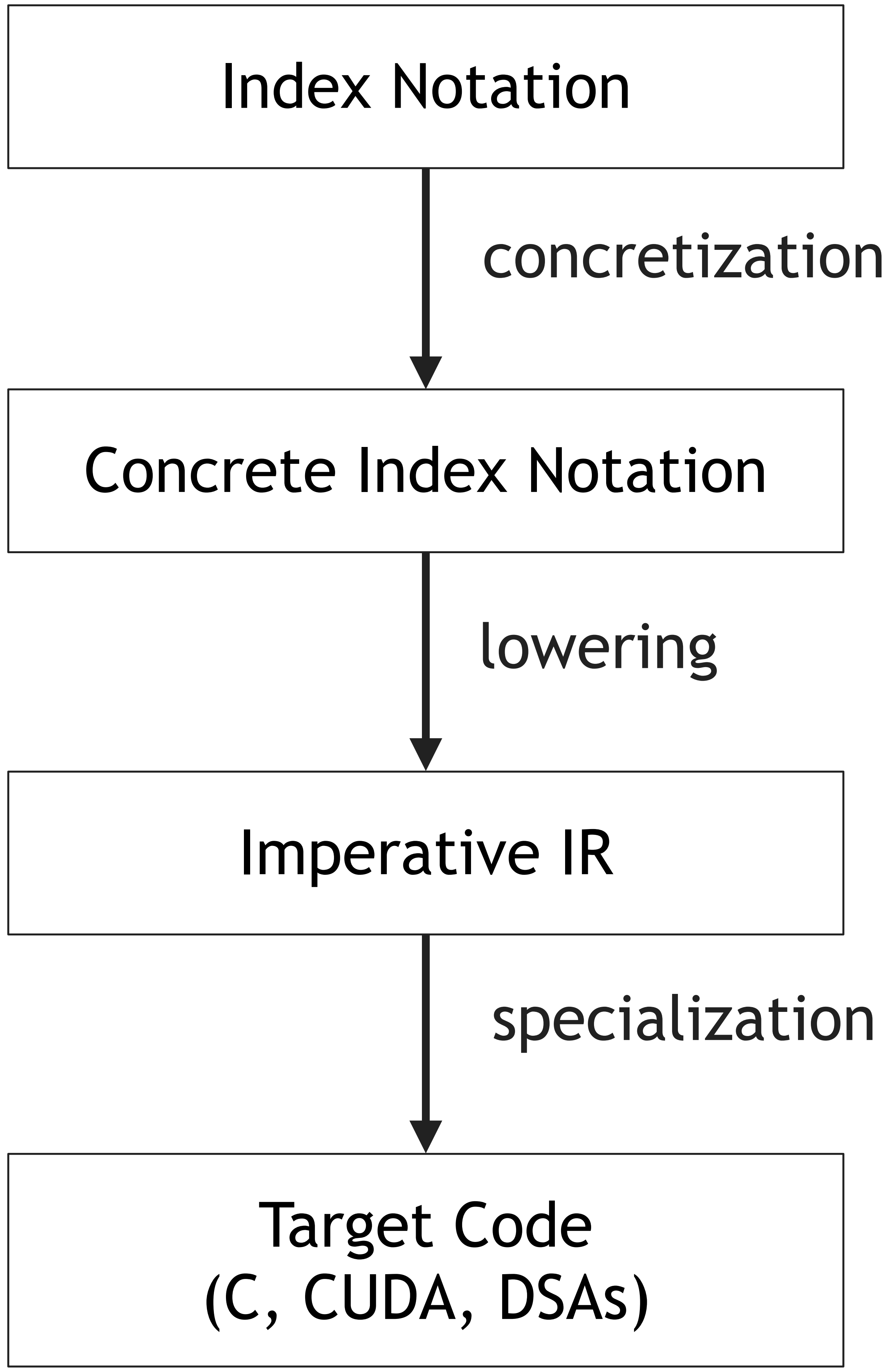
Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$



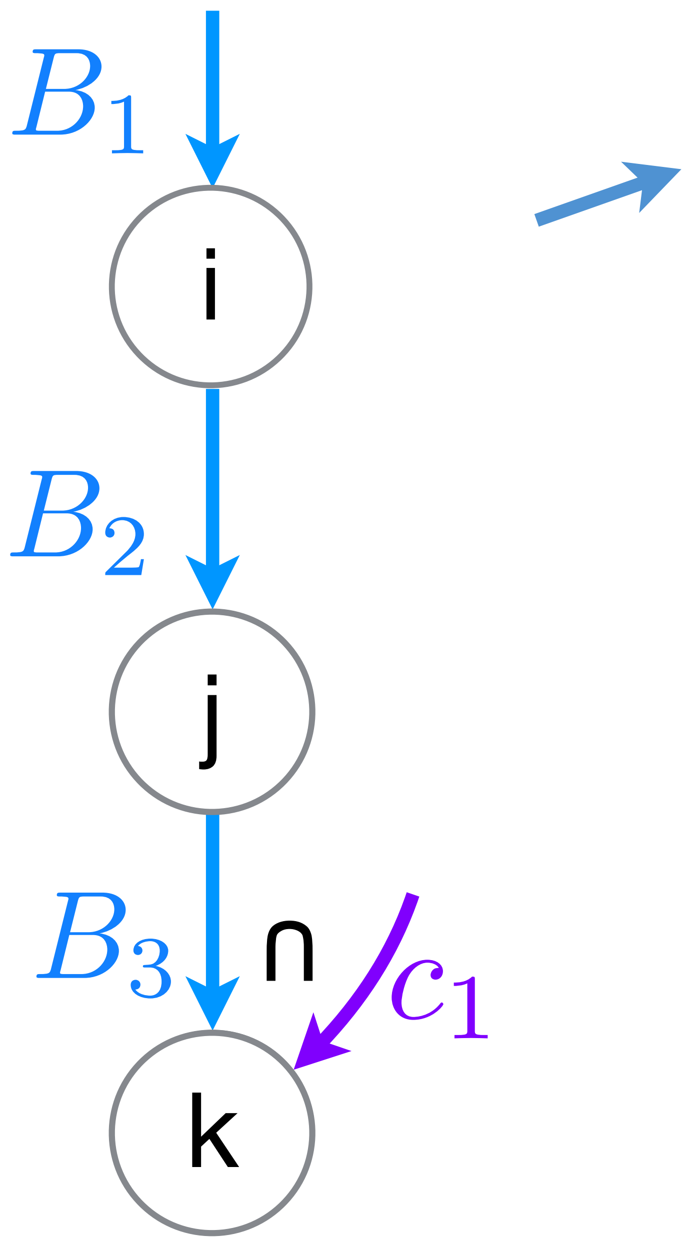
Concrete index notation example



$$A_{ij} = \sum_k B_{ijk} c_k$$

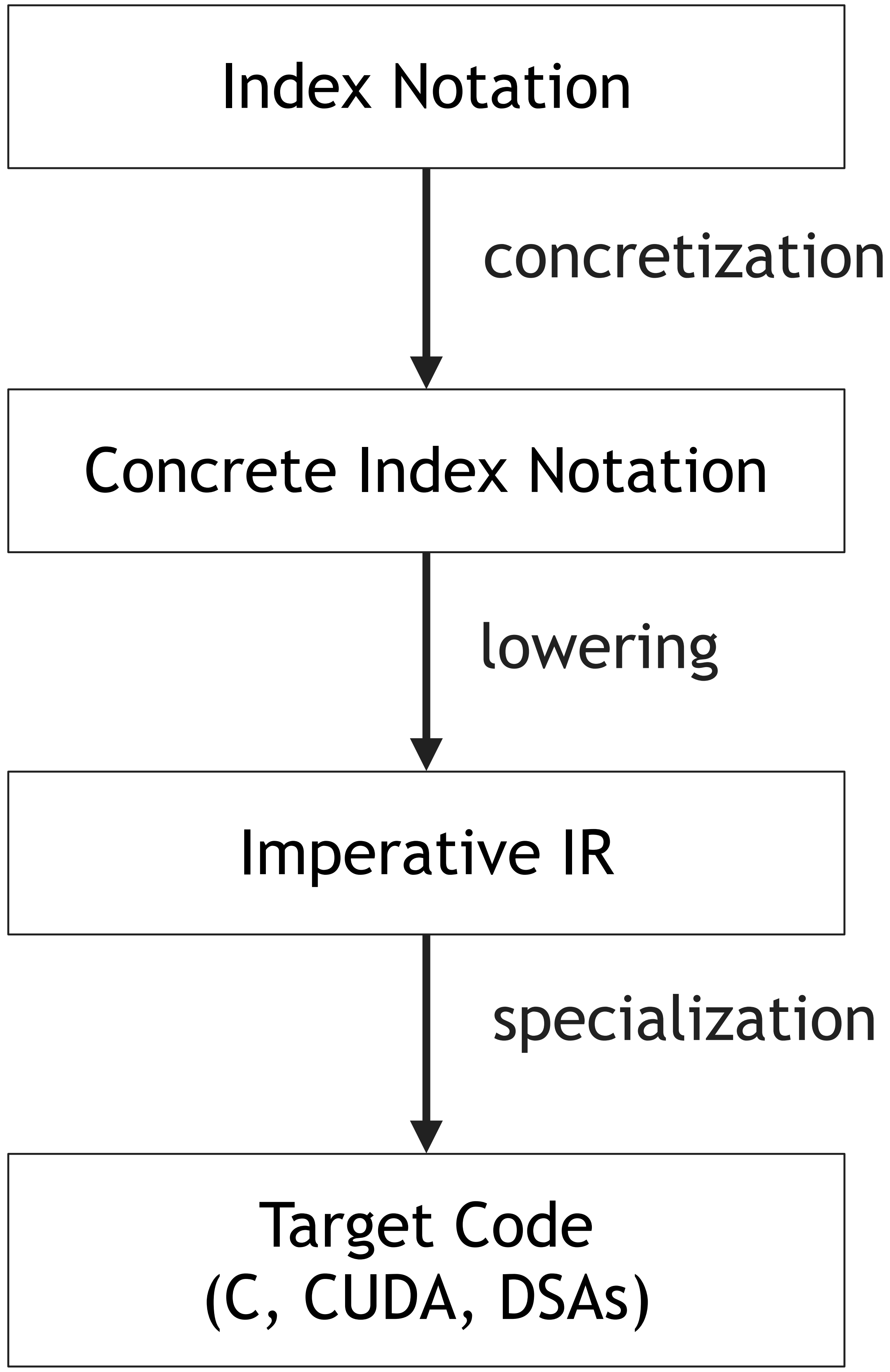
Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$




```
for (int i = 0; i < m; i++) {
```

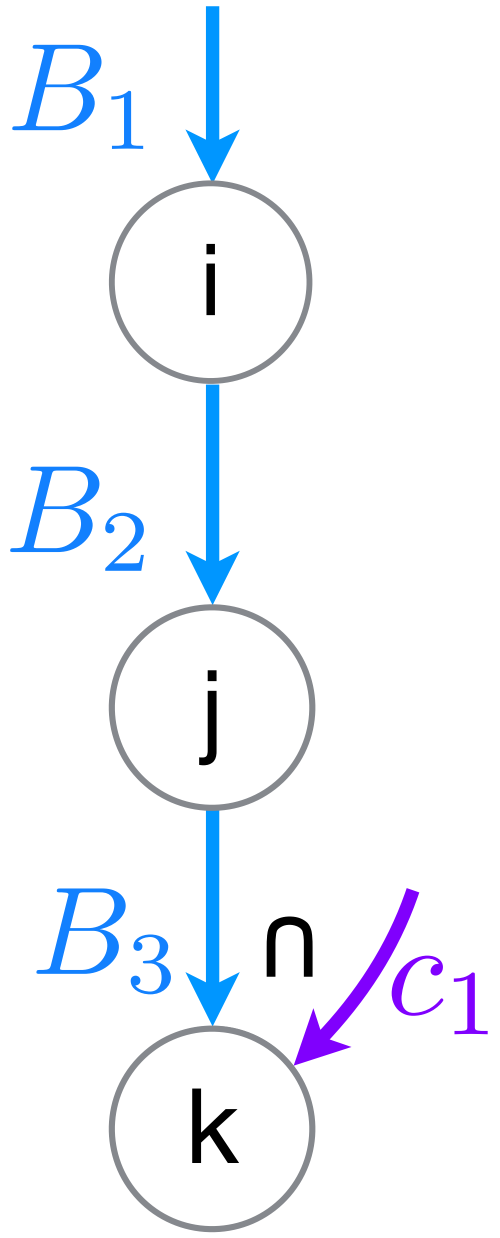
Concrete index notation example



$$A_{ij} = \sum_k B_{ijk} c_k$$

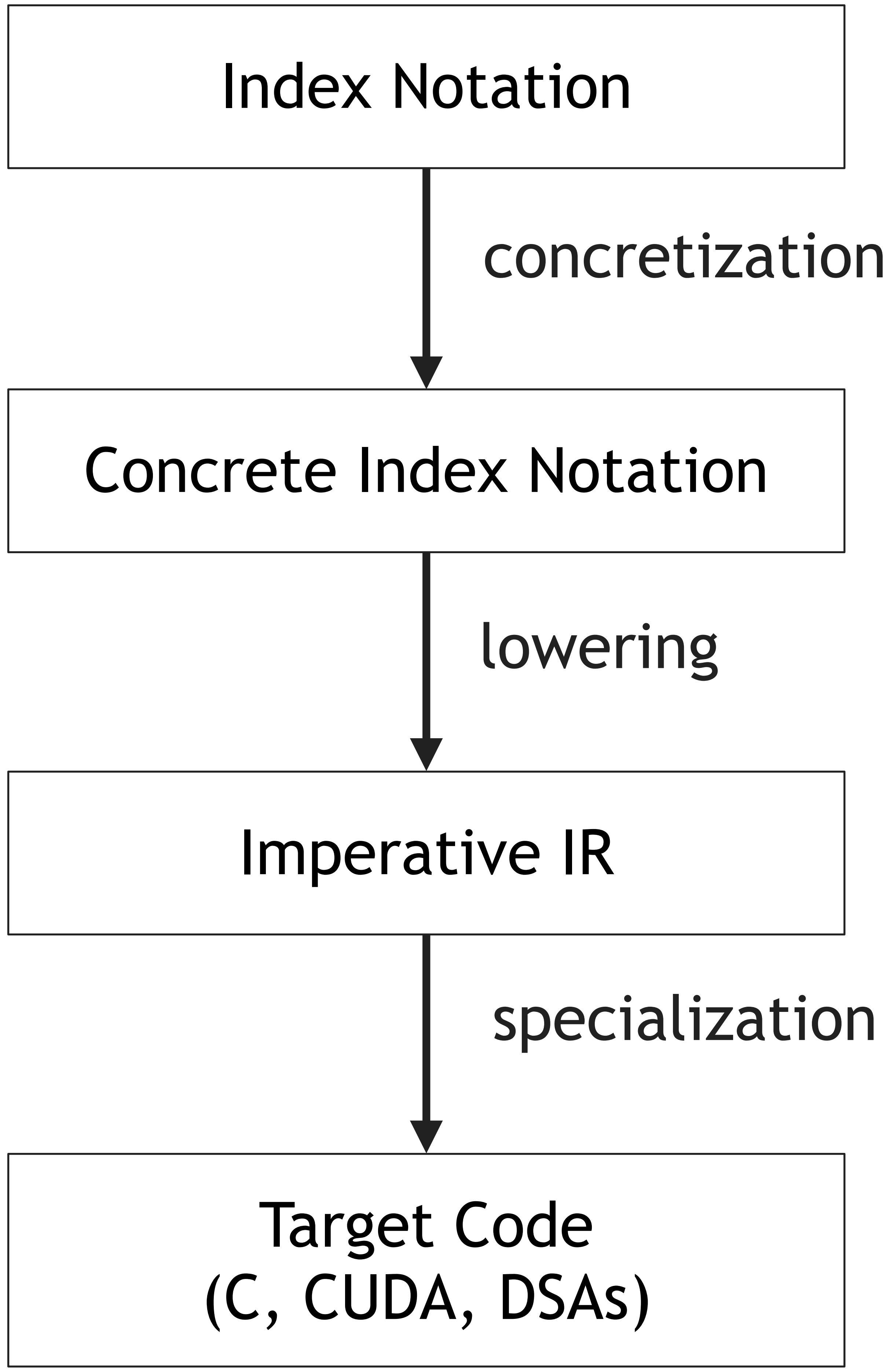
Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$




```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int j = B2_crd[pB2];
```

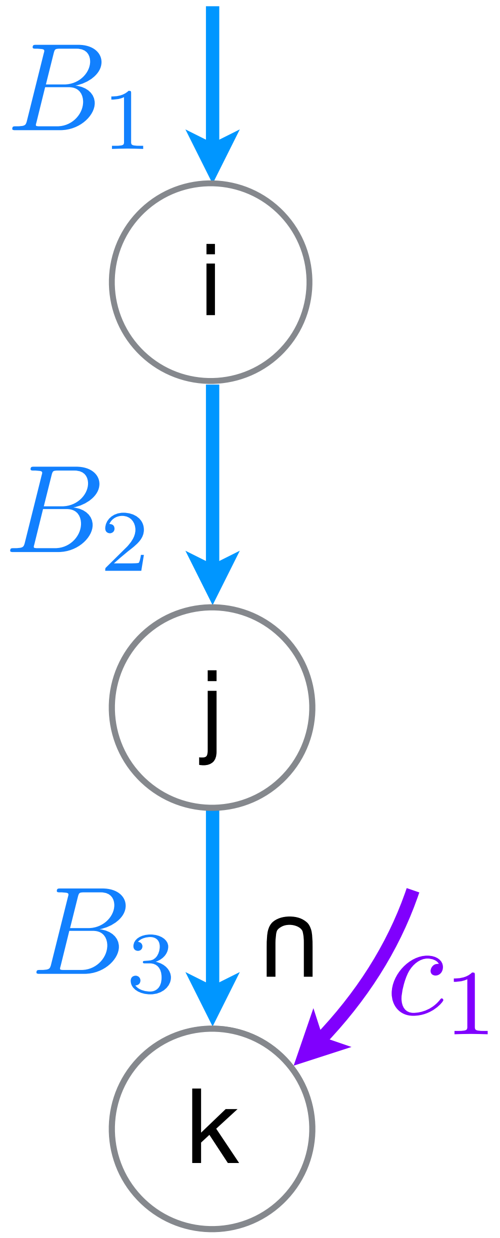
Concrete index notation example



$$A_{ij} = \sum_k B_{ijk} c_k$$

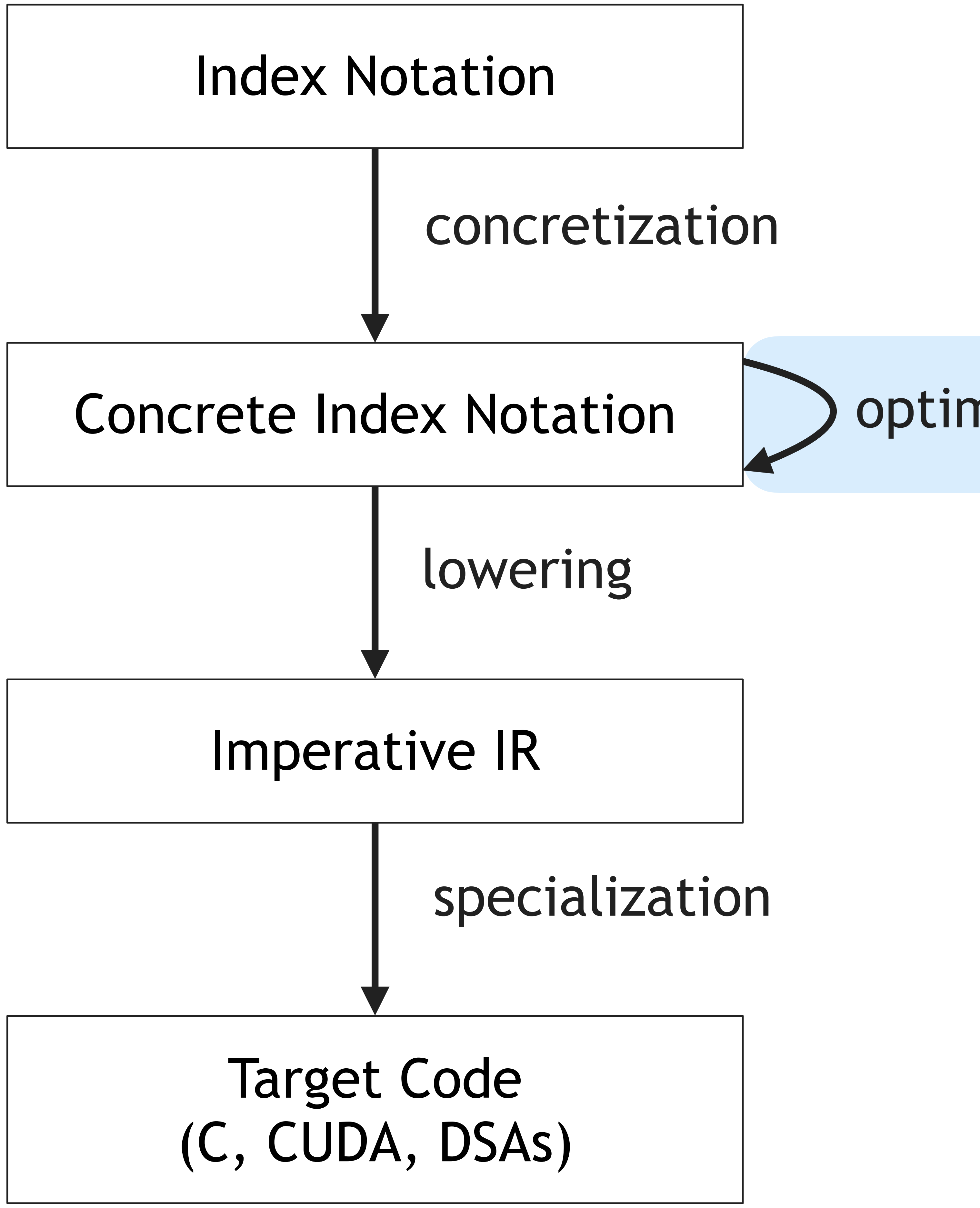
Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$




```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int j = B2_crd[pB2];


    int pA2 = i*n + j;
    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2+1] && pc1 < c1_pos[1]) {
      int kB = B3_crd[pB3];
      int kc = c1_crd[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        A[pA2] += B[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;

    }
  }
}
```

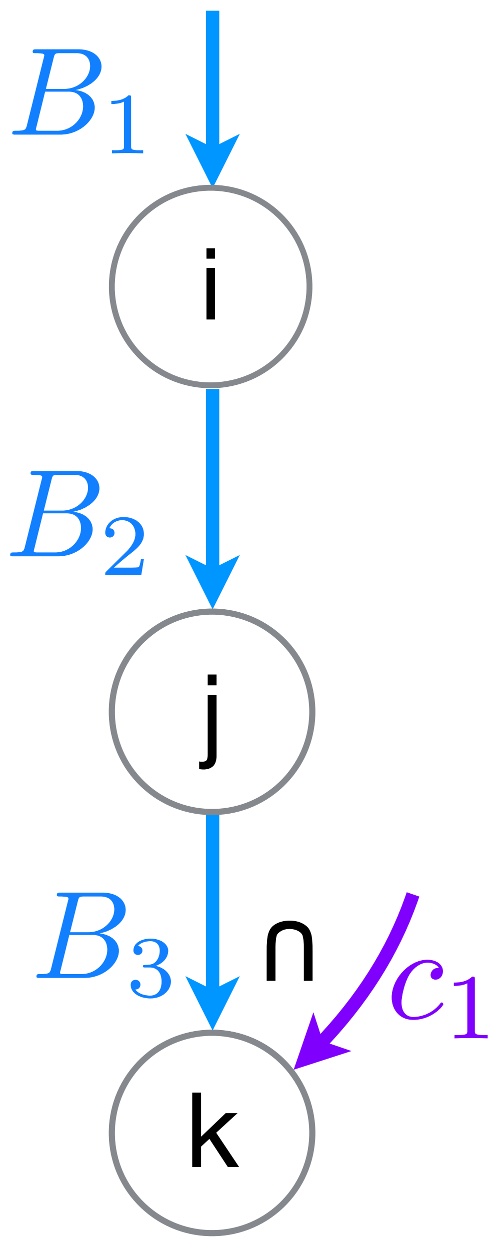
Concrete index notation example



$$A_{ij} = \sum_k B_{ijk} c_k$$

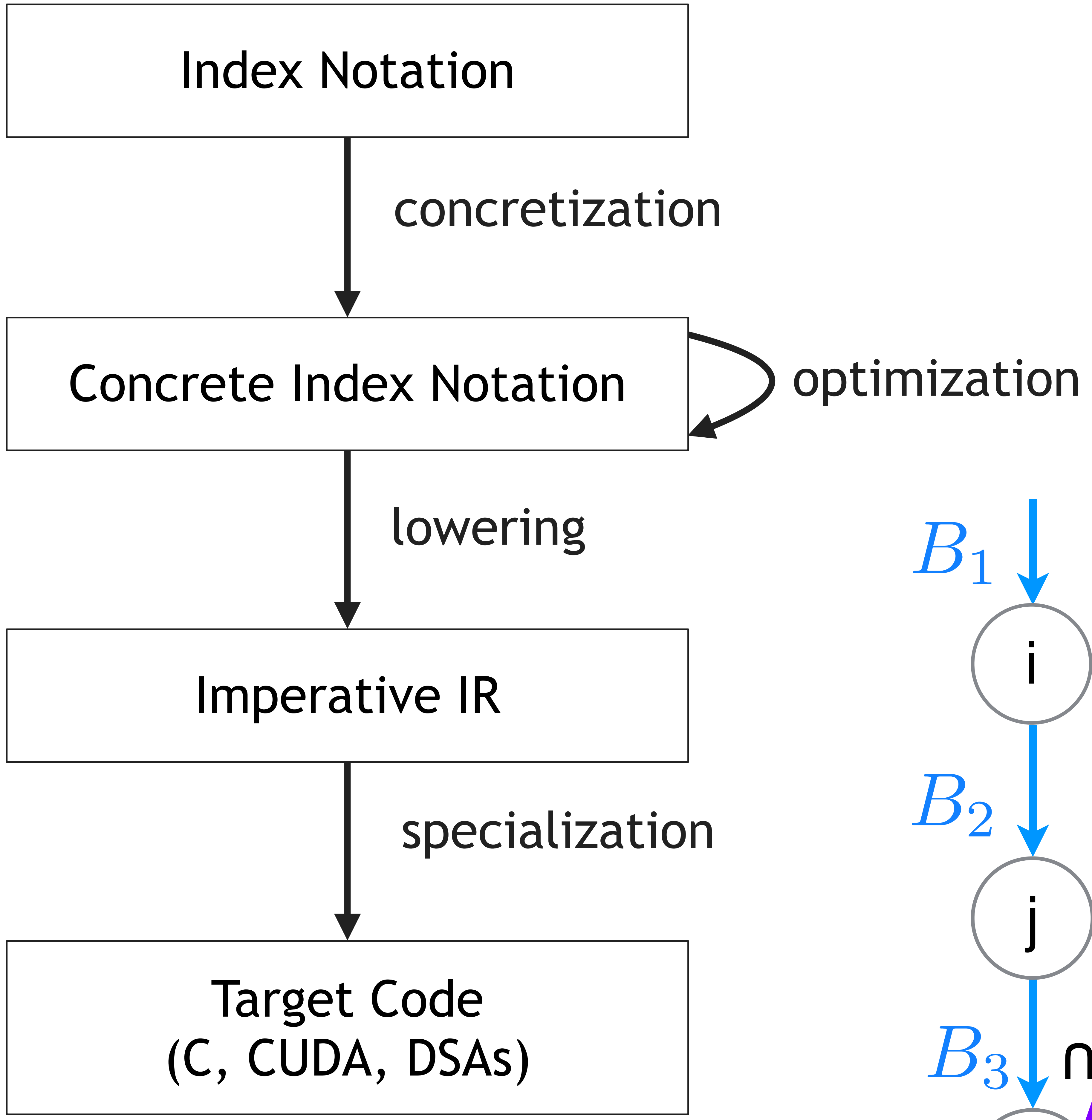
Order of computation

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ijk} c_k$$




```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int j = B2_crd[pB2];


    int pA2 = i*n + j;
    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2+1] && pc1 < c1_pos[1]) {
      int kB = B3_crd[pB3];
      int kc = c1_crd[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        A[pA2] += B[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;

    }
  }
}
```

Concrete index notation example

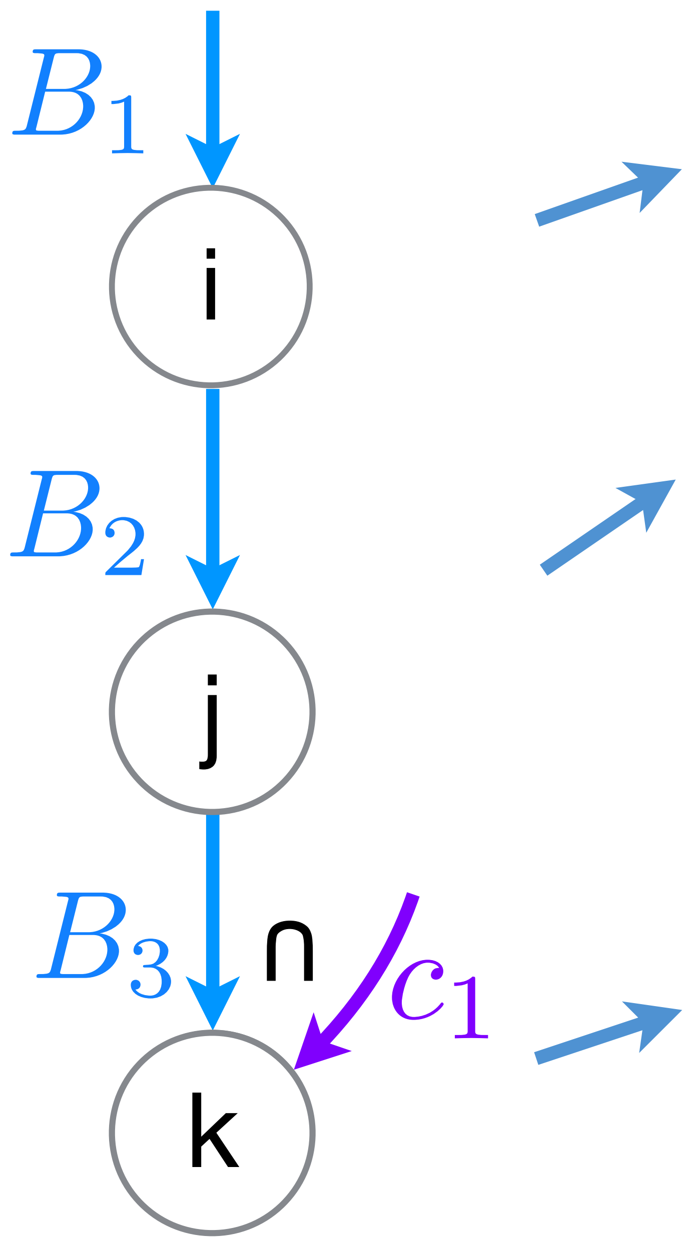


$$A_{ij} = \sum_k B_{ijk} c_k$$

Order of computation

Temporary storage

$$\forall_i \forall_j \left(A_{ij} = t\right) \textbf{ where } \left(\forall_k\, t \mathrel{+}= B_{ijk} c_k\right)$$




```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int j = B2_crd[pB2];

    float t = 0.0;
    int pA2 = i*n + j;
    int pB3 = B3_pos[pB2];
    int pc1 = c1_pos[0];
    while (pB3 < B3_pos[pB2+1] && pc1 < c1_pos[1]) {
      int kB = B3_crd[pB3];
      int kc = c1_crd[pc1];
      int k = min(kB, kc);
      if (kB == k && kc == k) {
        t += B[pB3] * c[pc1];
      }
      if (kB == k) pB3++;
      if (kc == k) pc1++;
    A[pA2] = t;
    }
  }
}
```

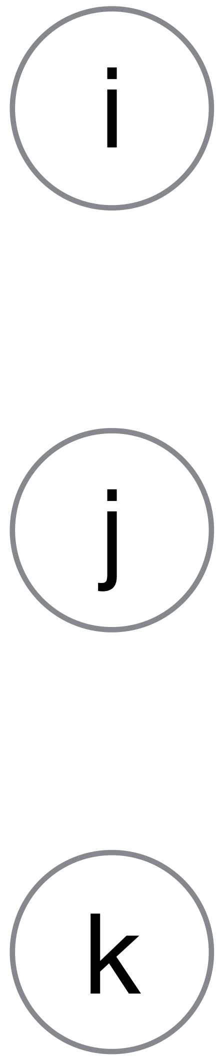
Workspace to scatter into results in sparse matrix multiplication

$$A_{ij} = \sum_k B_{ik} C_{kj}$$

Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

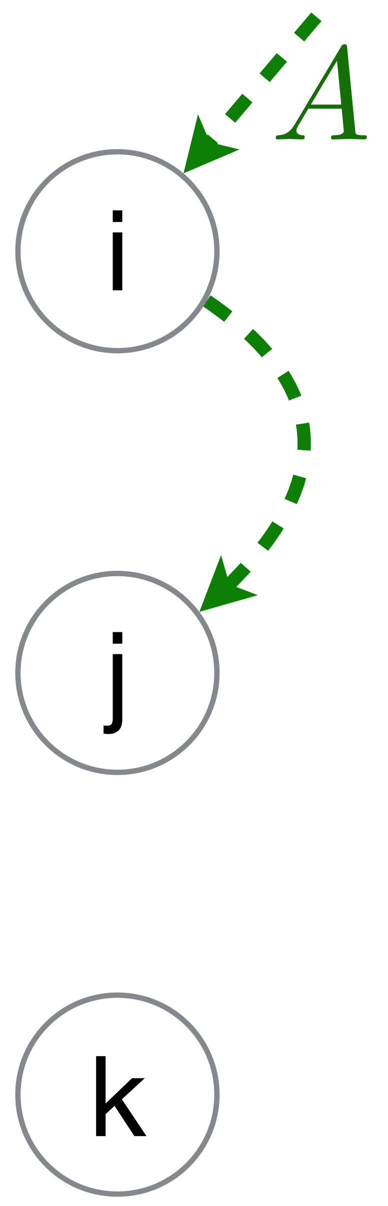


Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

		CSR
A	rows	Dense
	cols	Compressed



Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

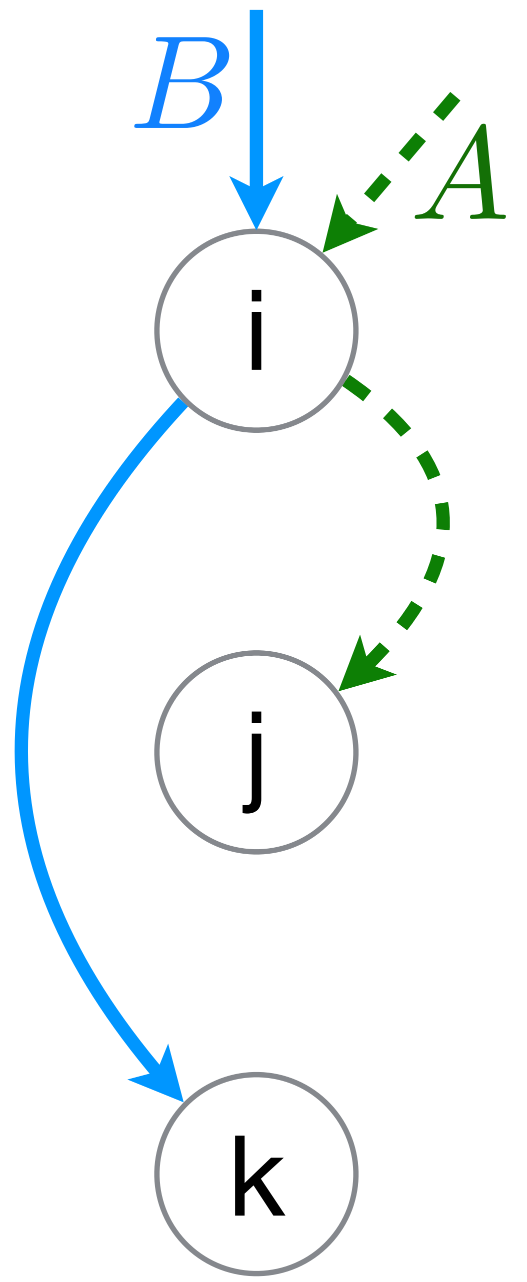
$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

CSR

A		
	rows	Dense
	cols	Compressed

CSR

B		
	rows	Dense
	cols	Compressed



Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A

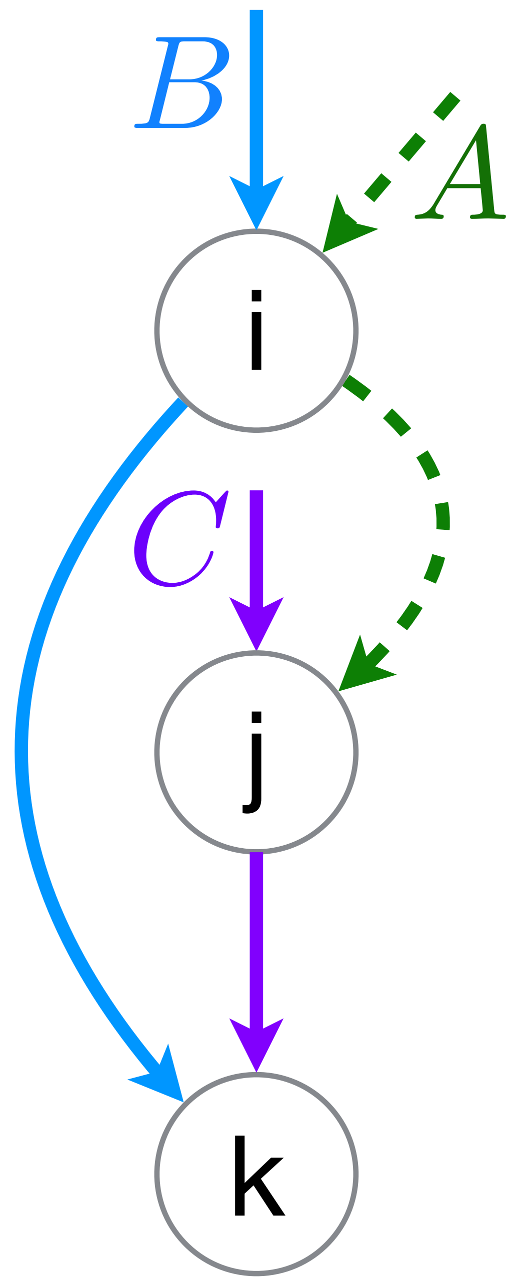
	CSR
rows	Dense
cols	Compressed

B

	CSR
rows	Dense
cols	Compressed

C

	CSC
cols	Dense
rows	Compressed



Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A

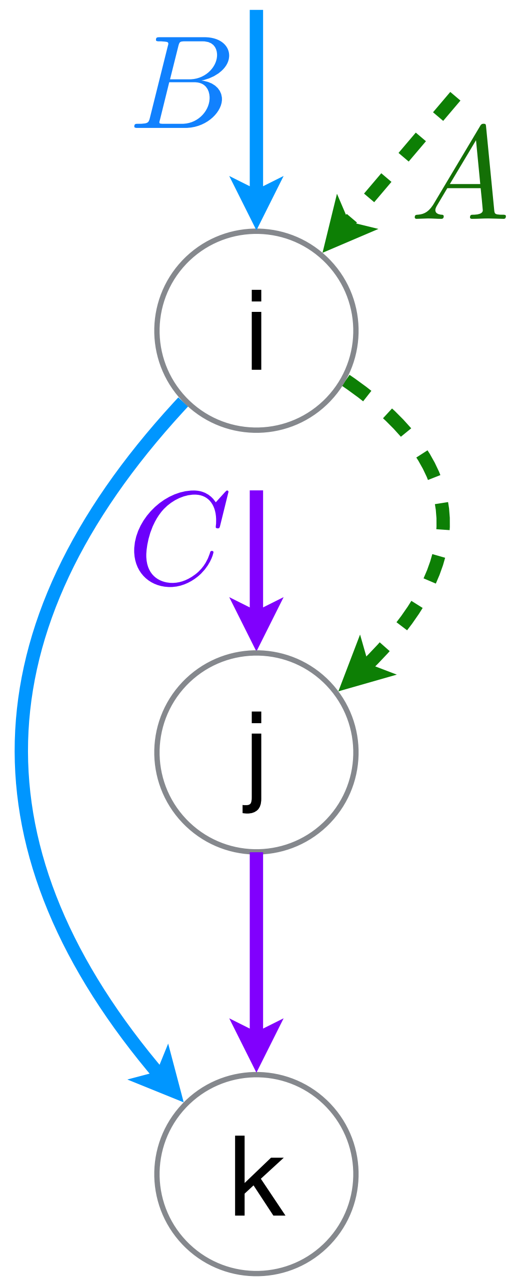
	CSR
rows	Dense
cols	Compressed

B

	CSR
rows	Dense
cols	Compressed

C

	CSC
cols	Dense
rows	Compressed



Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A — CSR

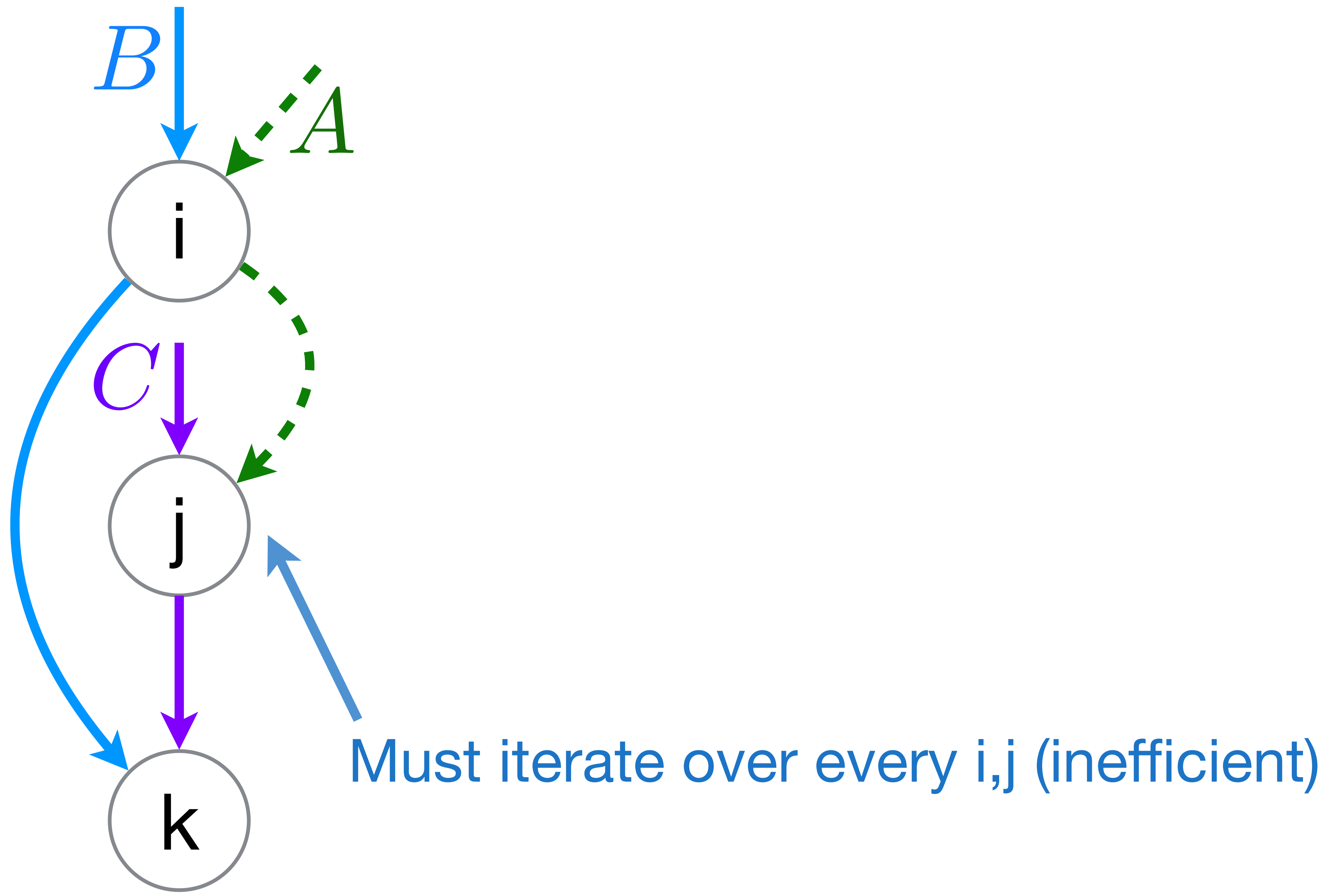
rows	Dense
cols	Compressed

B — CSR

rows	Dense
cols	Compressed

C — CSC

cols	Dense
rows	Compressed



Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

CSR

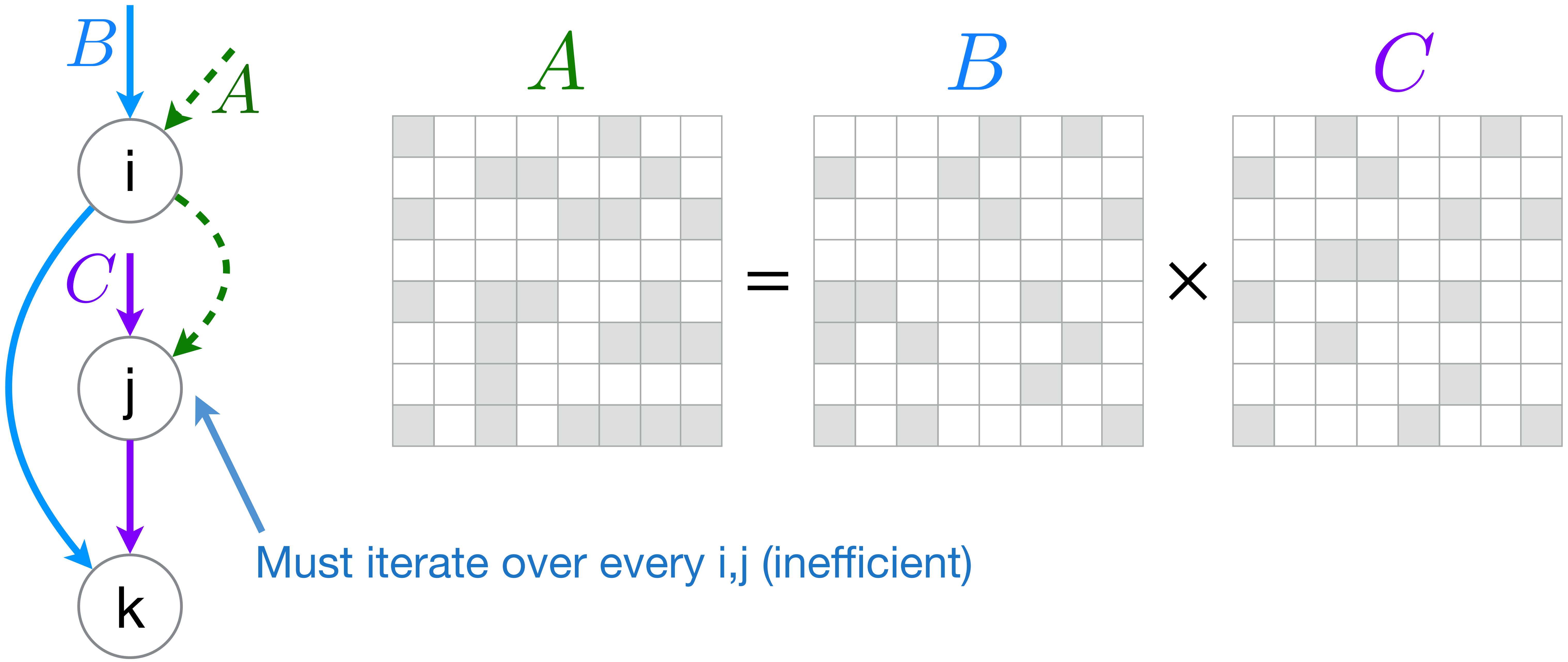
A		
	rows	Dense
	cols	Compressed

CSR

B		
	rows	Dense
	cols	Compressed

CSC

C		
	cols	Dense
	rows	Compressed

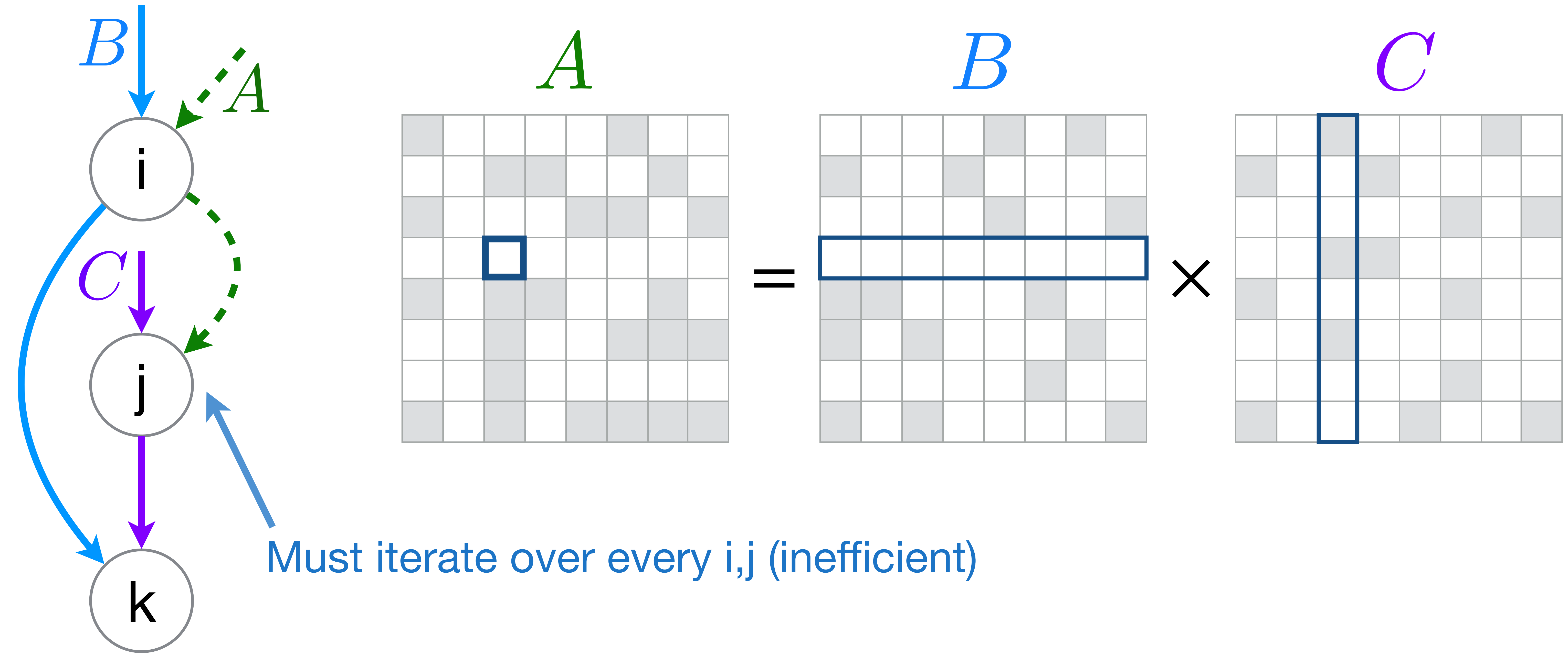


Workspace to scatter into results in sparse matrix multiplication

Inner Product
Matrix Multiplication

$$\forall_i \forall_j \forall_k \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

Tensor	Format	Level	Storage
A	CSR	rows	Dense
		cols	Compressed
B	CSR	rows	Dense
		cols	Compressed
C	CSC	cols	Dense
		rows	Compressed



Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \;\; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A — CSR

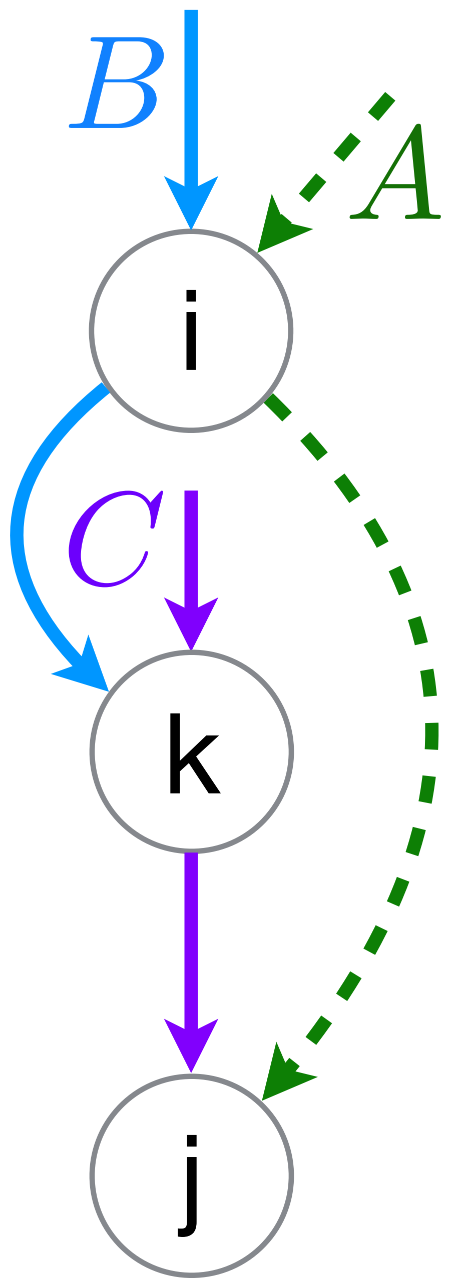
rows	Dense
cols	Compressed

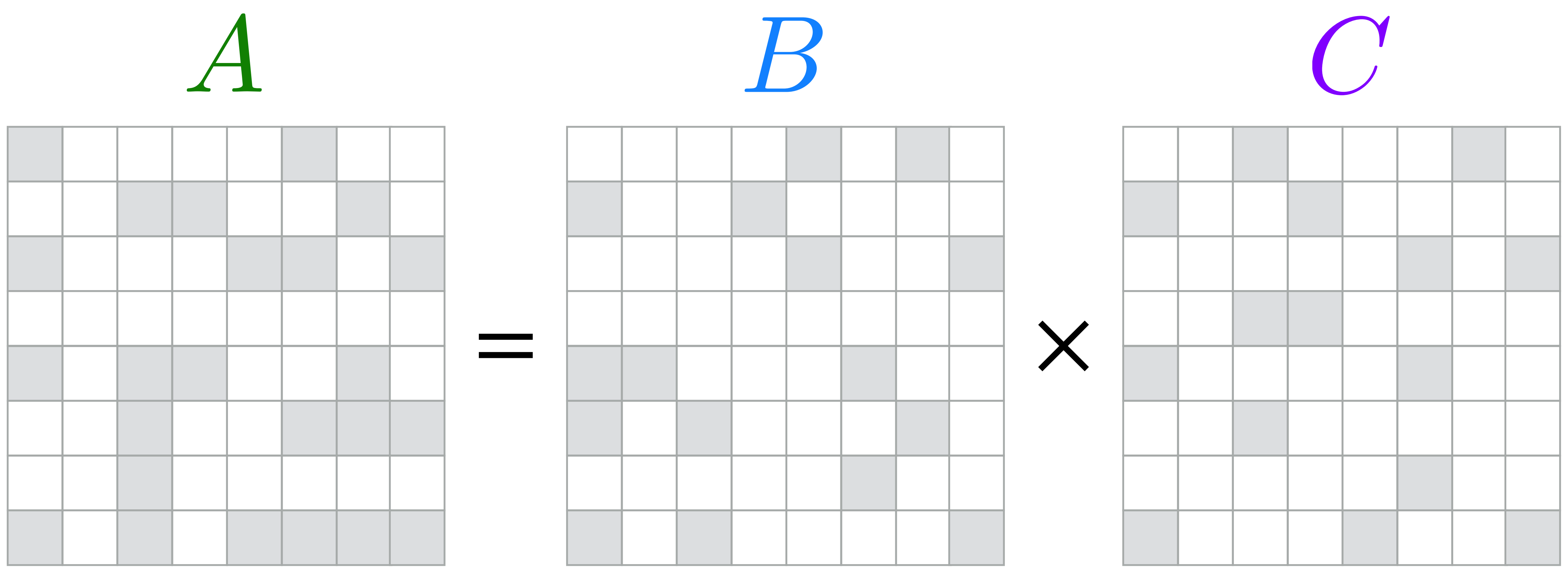
B — CSR

rows	Dense
cols	Compressed

C — CSR

rows	Dense
cols	Compressed





Workspace to scatter into results in sparse matrix multiplication

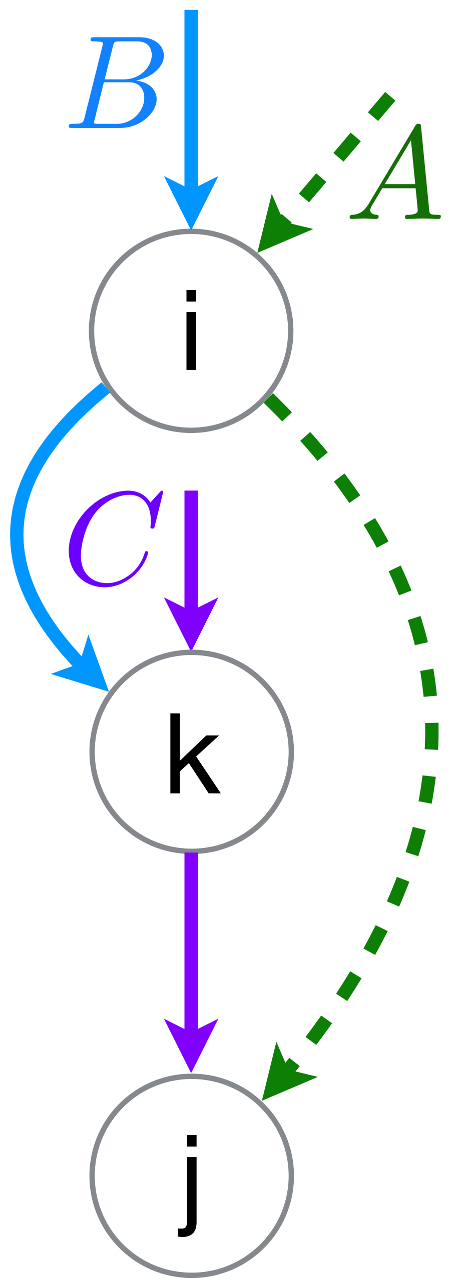
Linear Combination of Rows
Matrix Multiplication

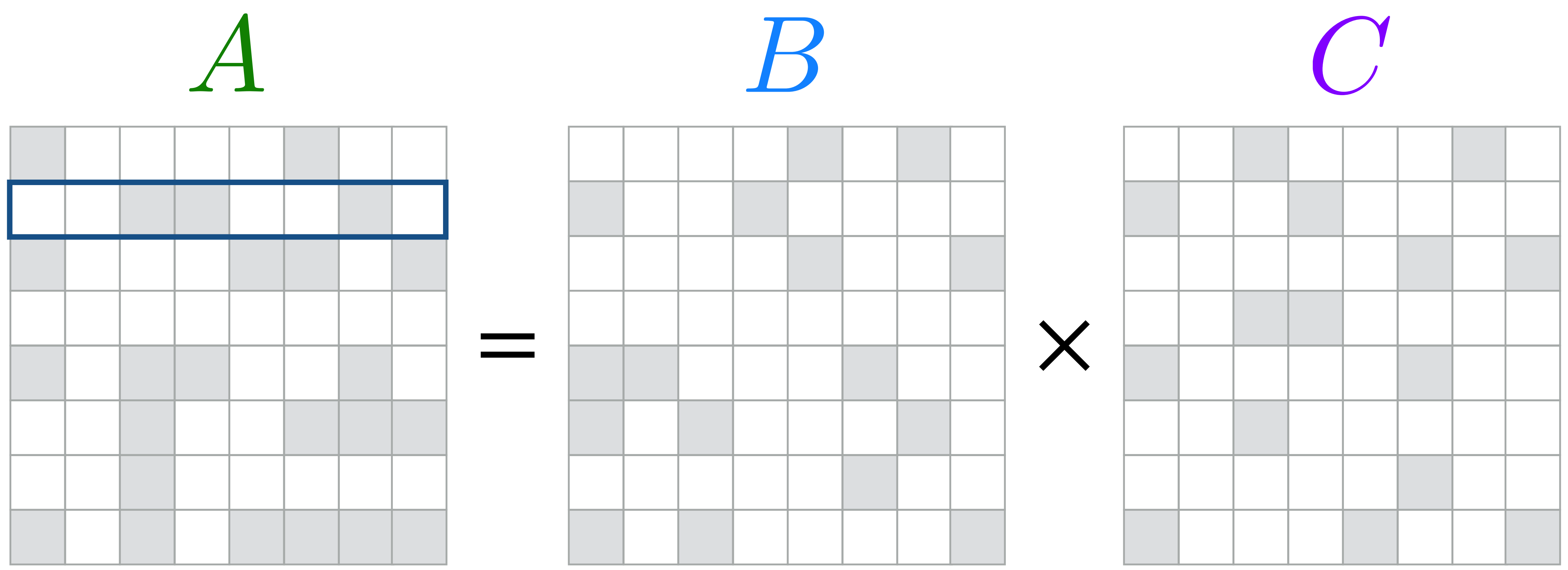
$$\forall_i \forall_k \forall_j \;\; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

		CSR
A	rows	Dense
	cols	Compressed

		CSR
B	rows	Dense
	cols	Compressed

		CSR
C	rows	Dense
	cols	Compressed





Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \;\; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A — CSR

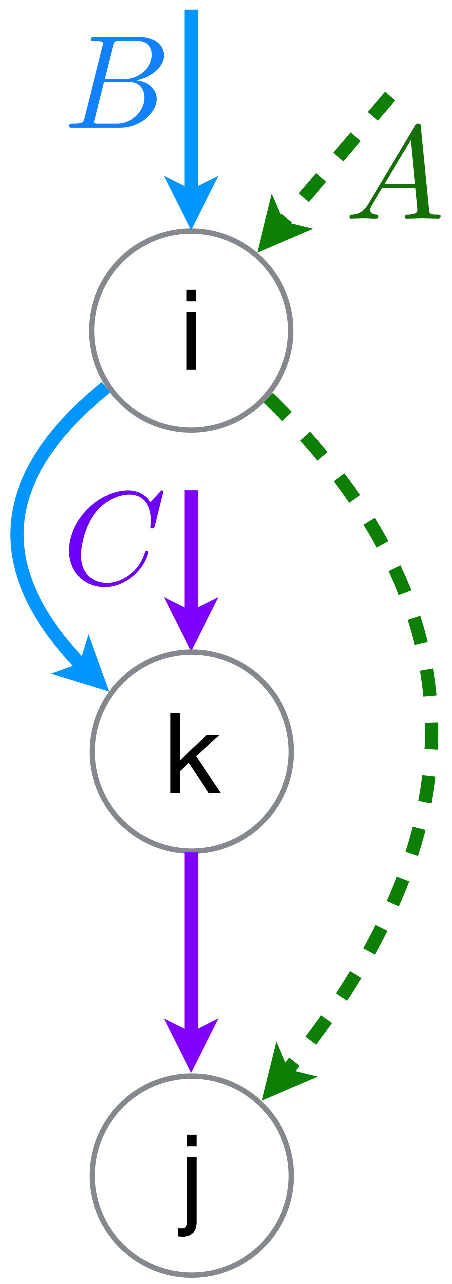
rows	Dense
cols	Compressed

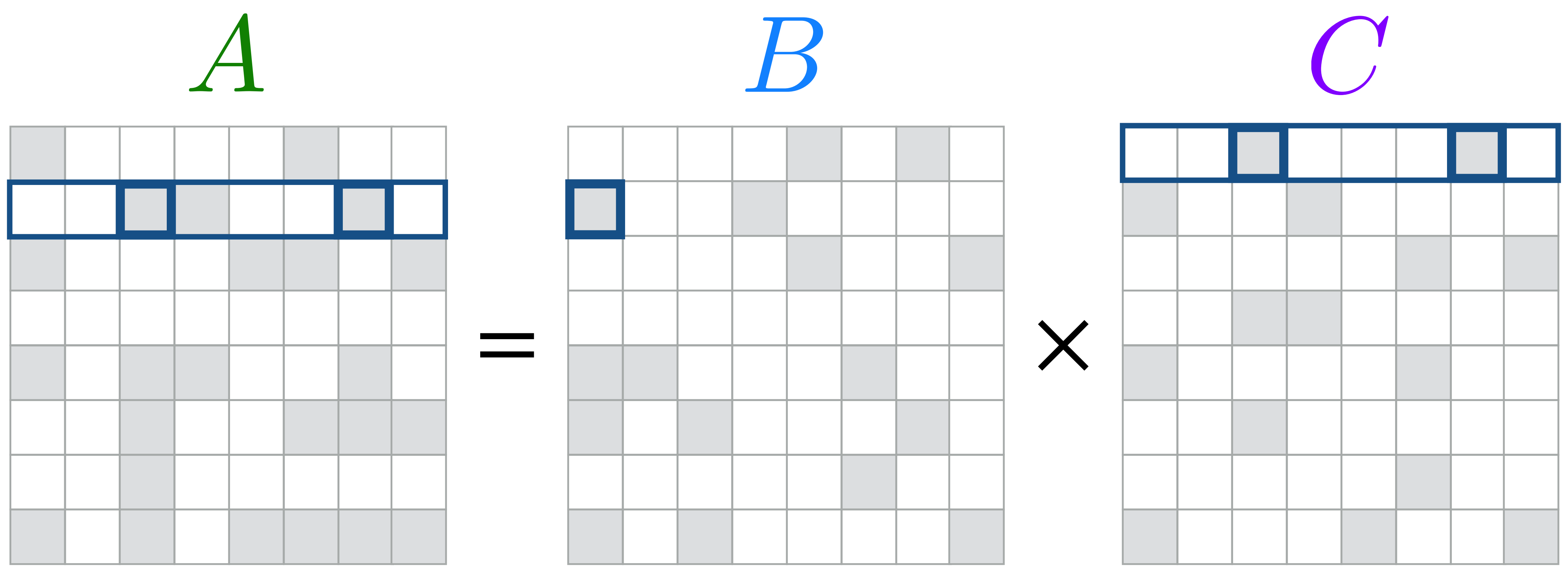
B — CSR

rows	Dense
cols	Compressed

C — CSR

rows	Dense
cols	Compressed





Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A — CSR

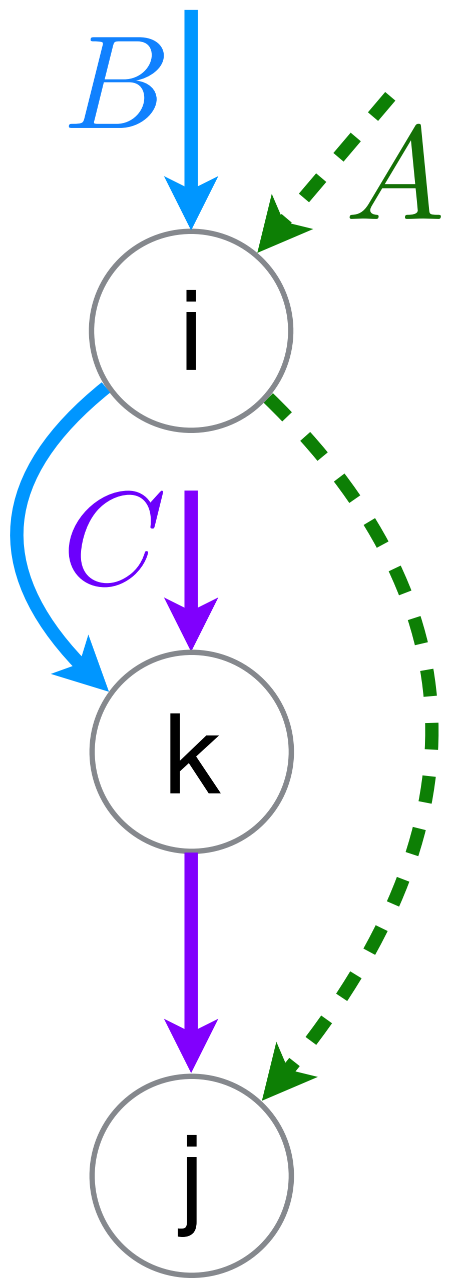
rows	Dense
cols	Compressed

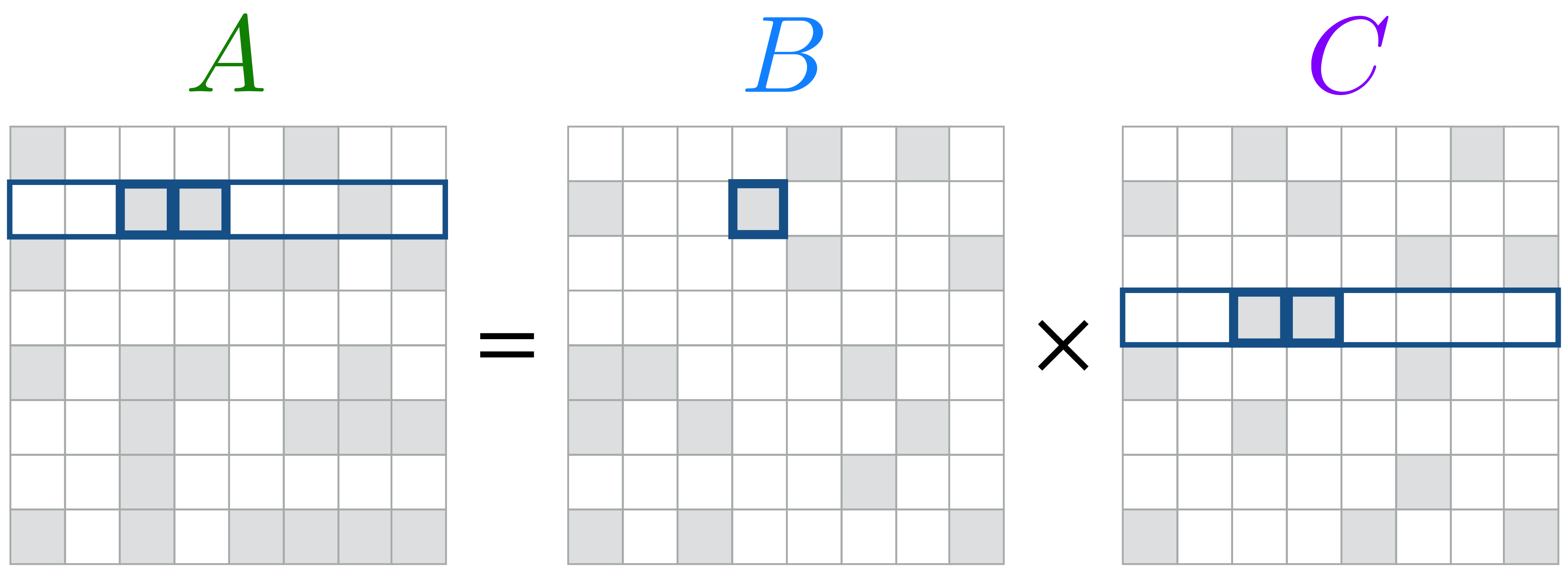
B — CSR

rows	Dense
cols	Compressed

C — CSR

rows	Dense
cols	Compressed





Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

A — CSR

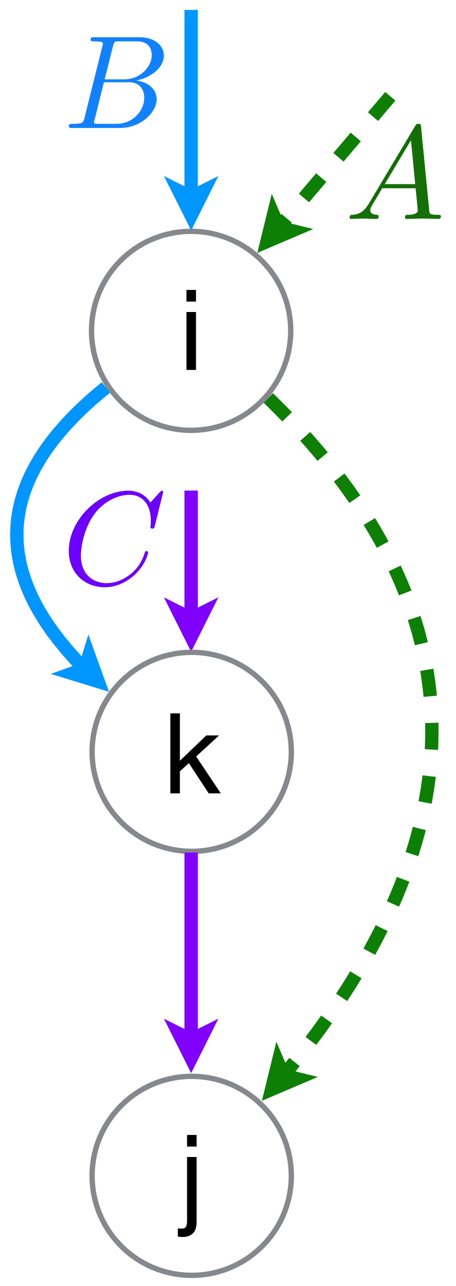
rows	Dense
cols	Compressed

B — CSR

rows	Dense
cols	Compressed

C — CSR

rows	Dense
cols	Compressed



Workspace to scatter into results in sparse matrix multiplication

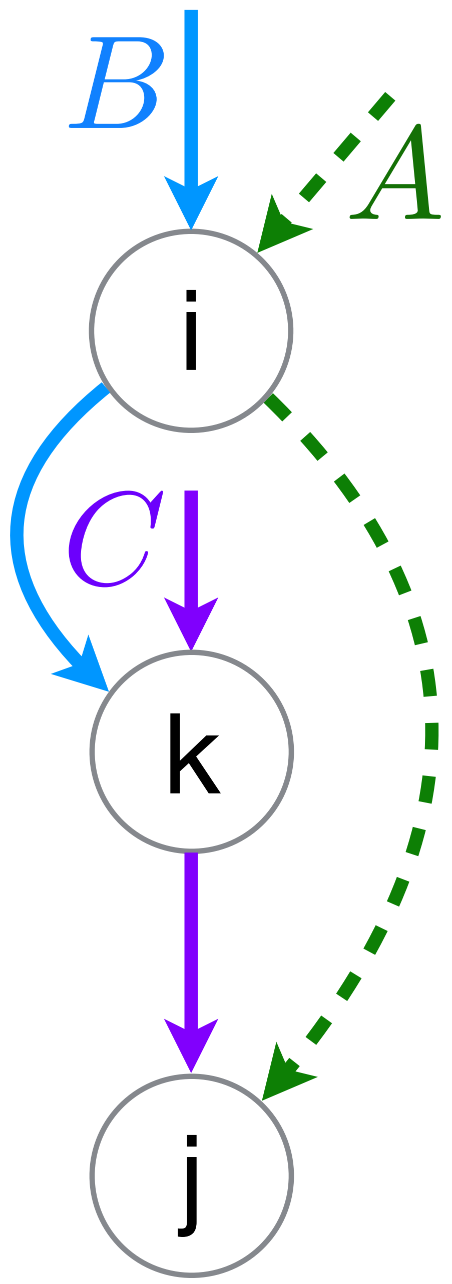
Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

		CSR
A	rows	Dense
	cols	Compressed

		CSR
B	rows	Dense
	cols	Compressed

		CSR
C	rows	Dense
	cols	Compressed



dense B_1 →

```
for (int i = 0; i < m; i++) {

}
```

Workspace to scatter into results in sparse matrix multiplication

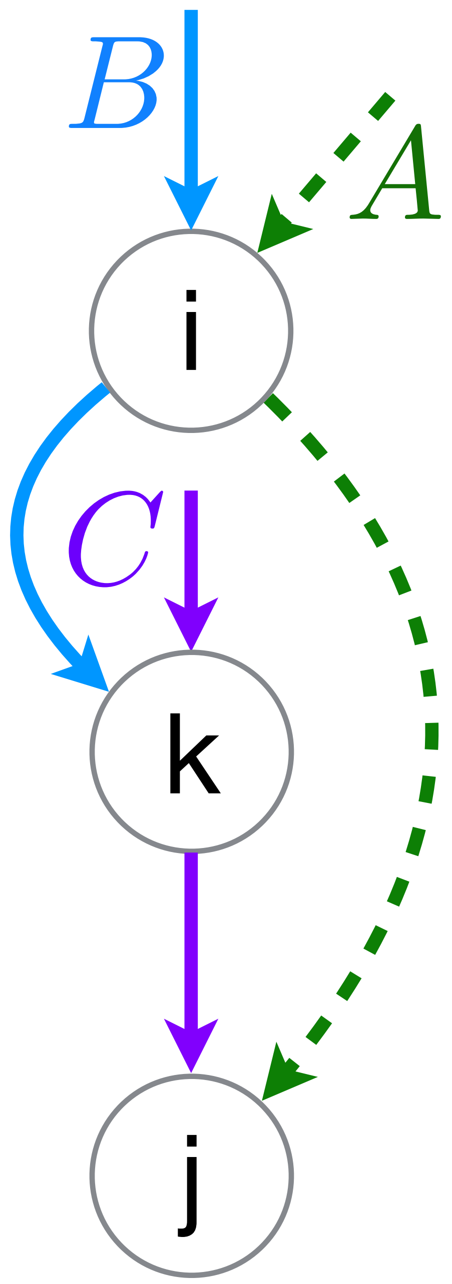
Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

		CSR
A	rows	Dense
	cols	Compressed

		CSR
B	rows	Dense
	cols	Compressed

		CSR
C	rows	Dense
	cols	Compressed



compressed B_2
dense C_1

```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];




  }



}
```

Workspace to scatter into results in sparse matrix multiplication

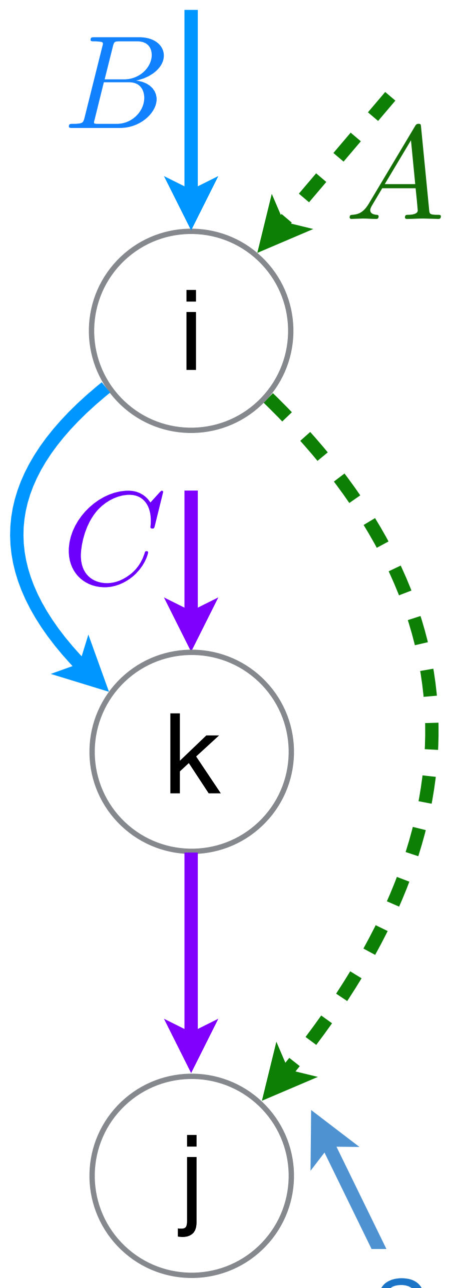
Linear Combination of Rows
Matrix Multiplication

$$\forall_i \forall_k \forall_j \;\; A_{ij} \mathrel{+}= B_{ik} C_{kj}$$

		CSR
A	rows	Dense
	cols	Compressed

		CSR
B	rows	Dense
	cols	Compressed

		CSR
C	rows	Dense
	cols	Compressed



Scatter results into compressed A (inefficient)

```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];



  }



}
```

Workspace to scatter into results in sparse matrix multiplication

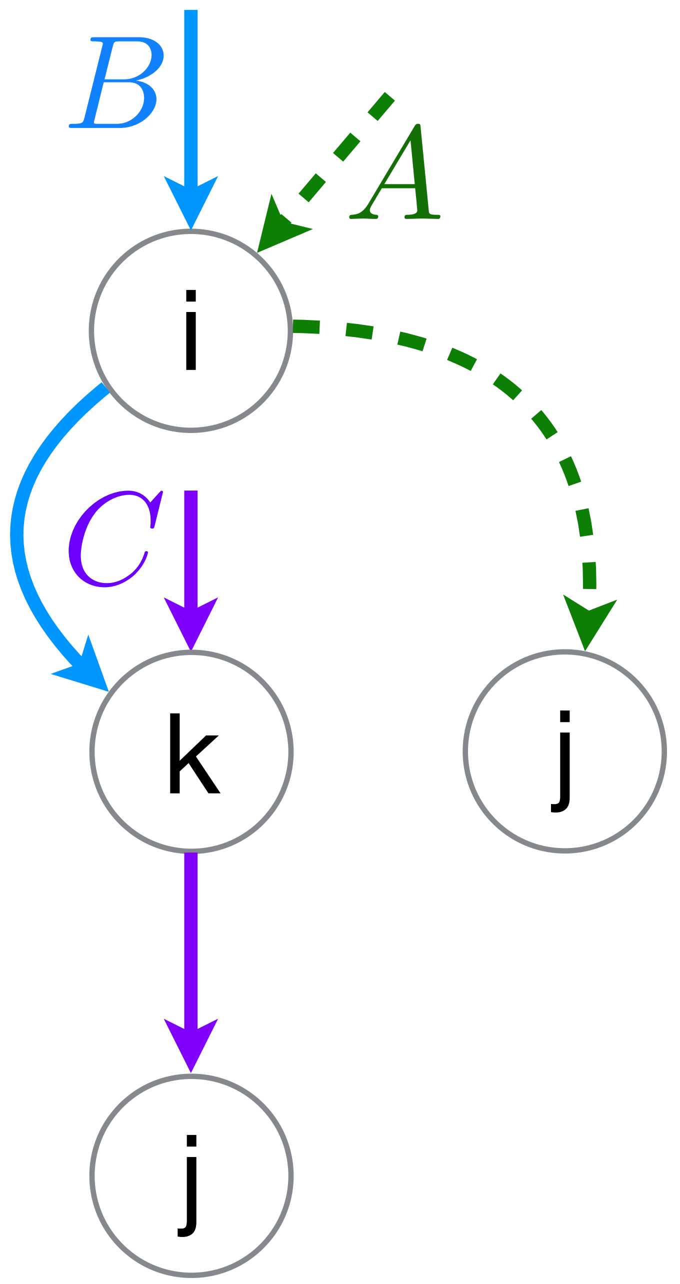
Linear Combination of Rows
Matrix Multiplication

$$\forall_i \left(\forall_j \ A_{ij} \qquad \right) \textbf{ where } \left(\forall_k \forall_j \qquad B_{ik} C_{kj} \right)$$

A	CSR
rows	Dense
cols	Compressed

B	CSR
rows	Dense
cols	Compressed

C	CSR
rows	Dense
cols	Compressed




```
for (int i = 0; i < m; i++) {

  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];

  }

}
```

Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i \big(\forall_j \; A_{ij} \qquad \big) \;\textbf{where}\; \big(\forall_k \forall_j \; \mathrm{row}_j \mathrel{+}= B_{ik} C_{kj}\big)$$

A

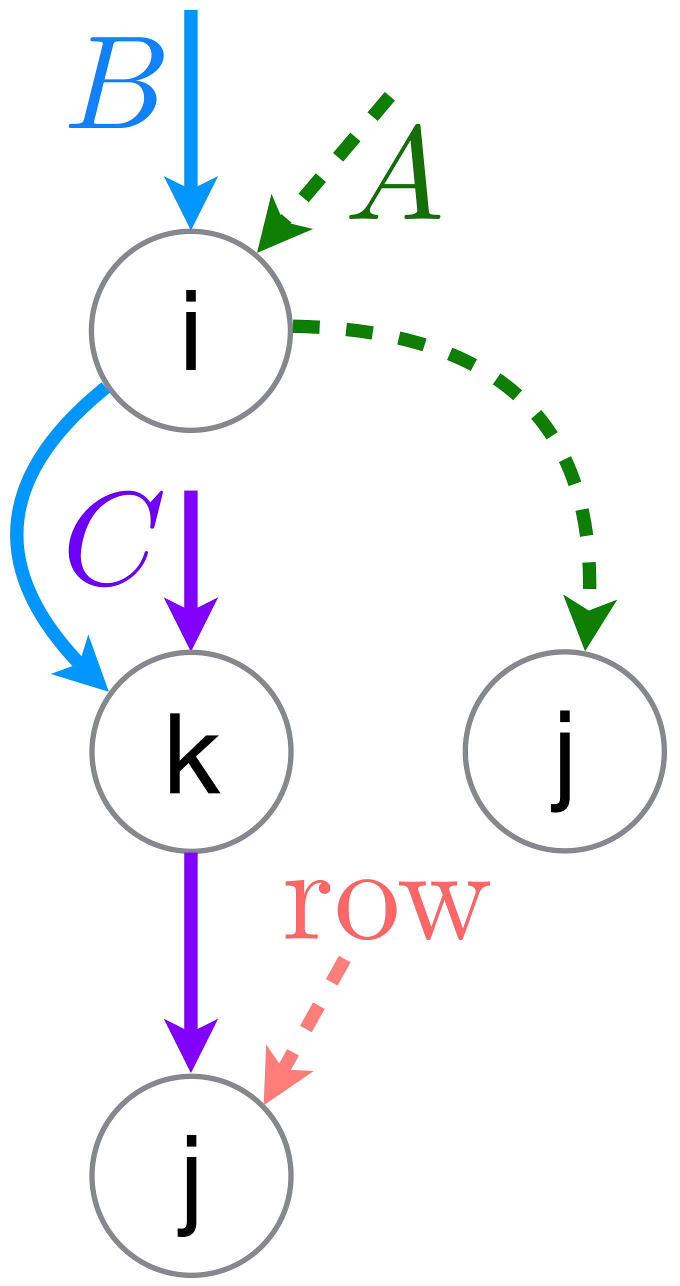
	CSR
rows	Dense
cols	Compressed

B

	CSR
rows	Dense
cols	Compressed

C

	CSR
rows	Dense
cols	Compressed




```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];



  }



}
```

Workspace to scatter into results in sparse matrix multiplication

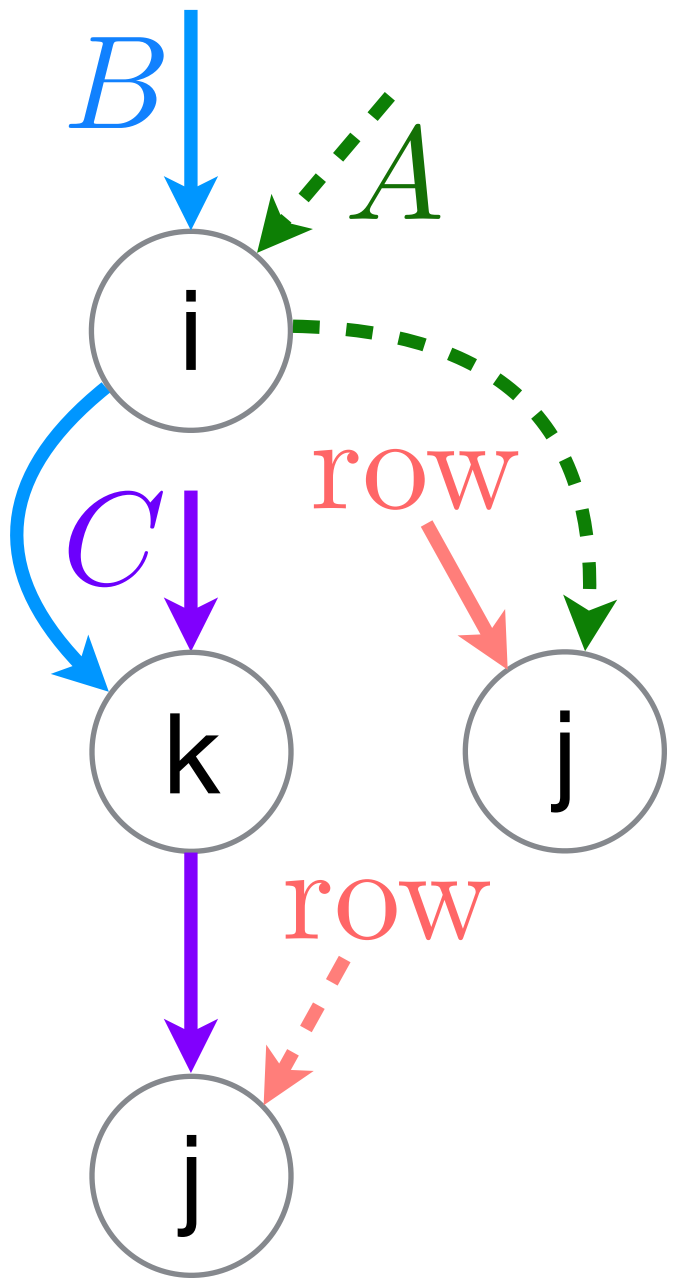
Linear Combination of Rows
Matrix Multiplication

$$\forall_i (\forall_j \; A_{ij} = \text{row}_j) \;\textbf{where}\; (\forall_k \forall_j \; \text{row}_j \mathrel{+}= B_{ik} C_{kj})$$

		CSR
A	rows	Dense
	cols	Compressed

		CSR
B	rows	Dense
	cols	Compressed

		CSR
C	rows	Dense
	cols	Compressed

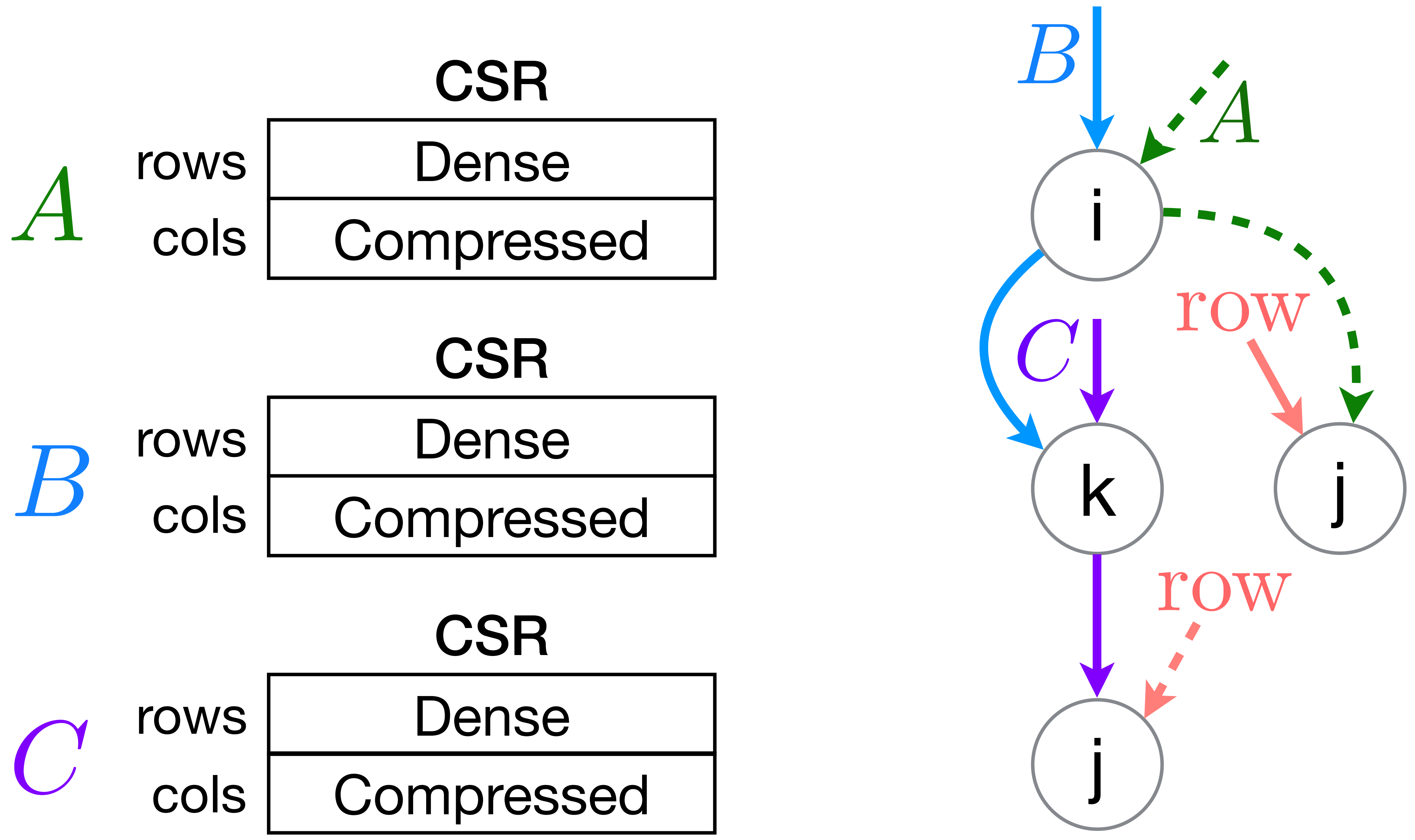



```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];



  }


}
```

Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i (\forall_j \; A_{ij} = \text{row}_j) \;\textbf{where}\; (\forall_k \forall_j \; \text{row}_j \mathrel{+}= B_{ik} C_{kj})$$



compressed C_2
dense row

```
for (int i = 0; i < m; i++) {


  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];


    for (int pC2 = C2_pos[k]; pC2 < C2_pos[k+1]; pC2++) {
      int j = C2_crd[pC2];
      row[j] += B[pB2] * C[pC2];
    }


  }


}
```

Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i (\forall_j \; A_{ij} = \mathrm{row}_j) \; \textbf{where} \; (\forall_k \forall_j \; \mathrm{row}_j \mathrel{+}= B_{ik} C_{kj})$$

CSR

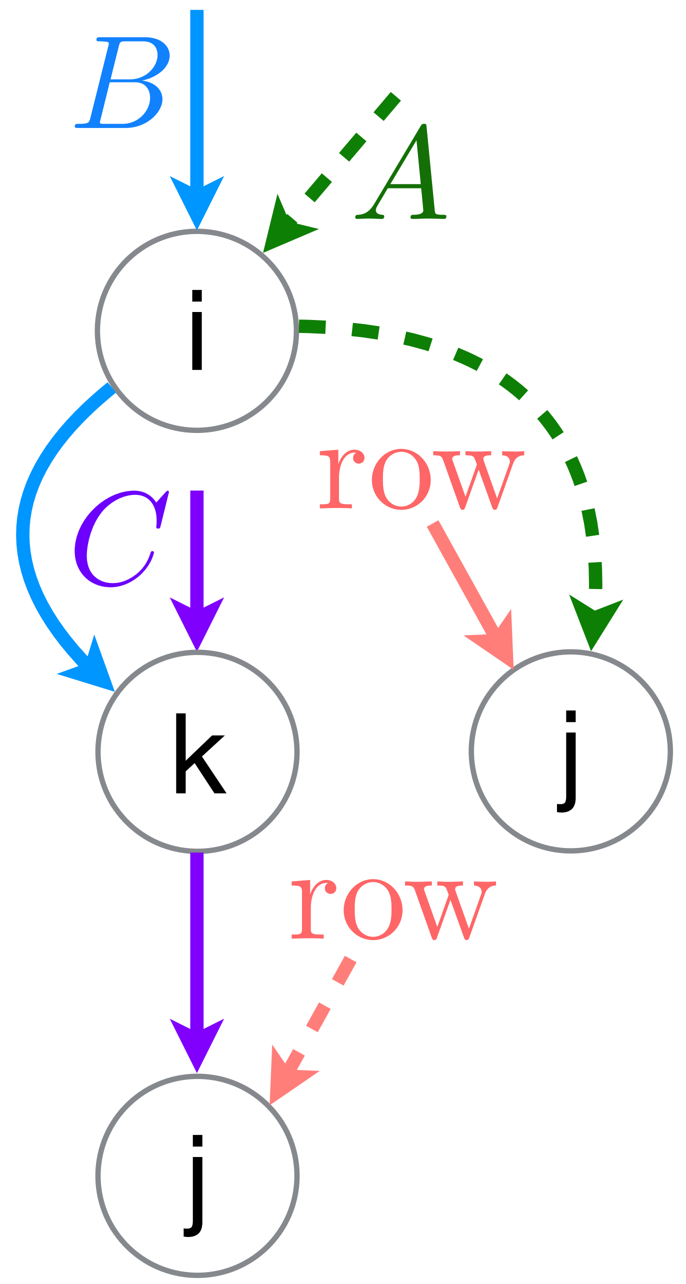
A		
rows	Dense	
cols	Compressed	

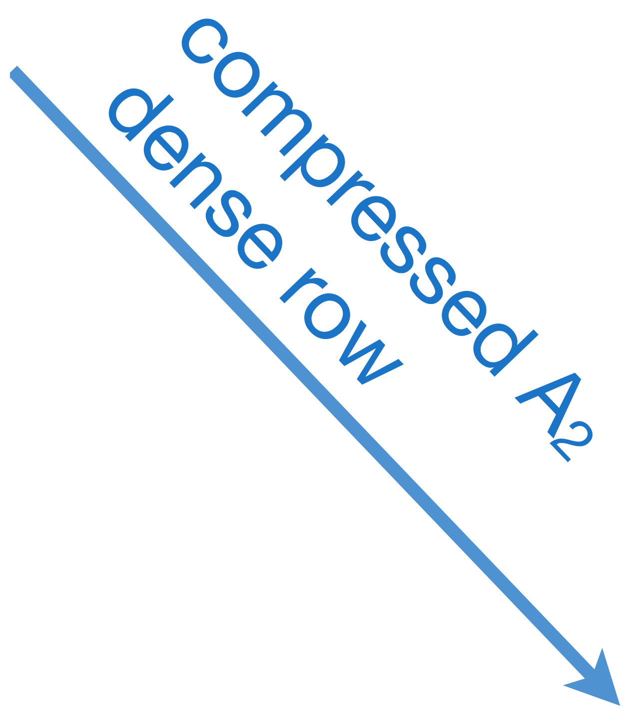
CSR

B		
rows	Dense	
cols	Compressed	

CSR

C		
rows	Dense	
cols	Compressed	






```
for (int i = 0; i < m; i++) {



  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];


    for (int pC2 = C2_pos[k]; pC2 < C2_pos[k+1]; pC2++) {
      int j = C2_crd[pC2];
      row[j] += B[pB2] * C[pC2];
    }


  }

  for (int pA2 = A2_pos[i]; pA2 < A2_pos[i+1]; pA2++) {
    int j = A2_crd[pA2];
    A[pA2] = row[j];
    row[j] = 0.0;
  }

}
```

Workspace to scatter into results in sparse matrix multiplication

Linear Combination of Rows
Matrix Multiplication

$$\forall_i (\forall_j \ A_{ij} = \mathrm{row}_j) \ \textbf{where} \ (\forall_k \forall_j \ \mathrm{row}_j \mathrel{+}= B_{ik} C_{kj})$$

A — CSR

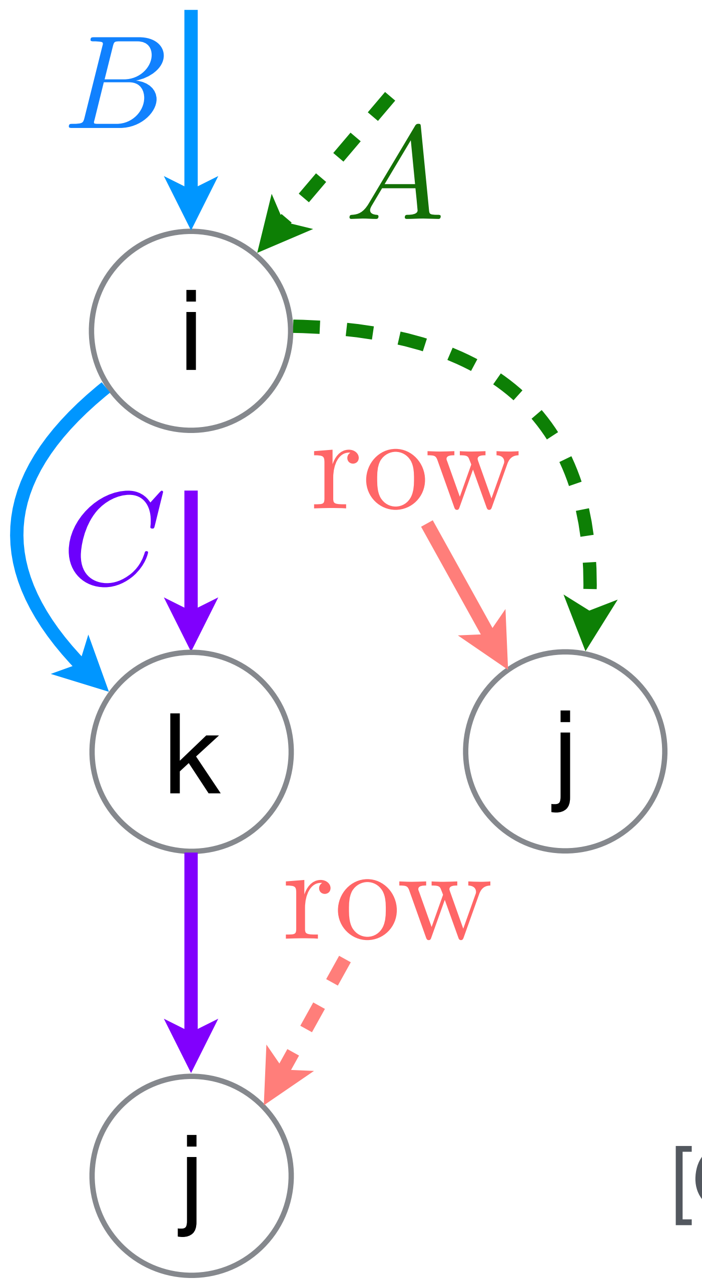
rows	Dense
cols	Compressed

B — CSR

rows	Dense
cols	Compressed

C — CSR

rows	Dense
cols	Compressed

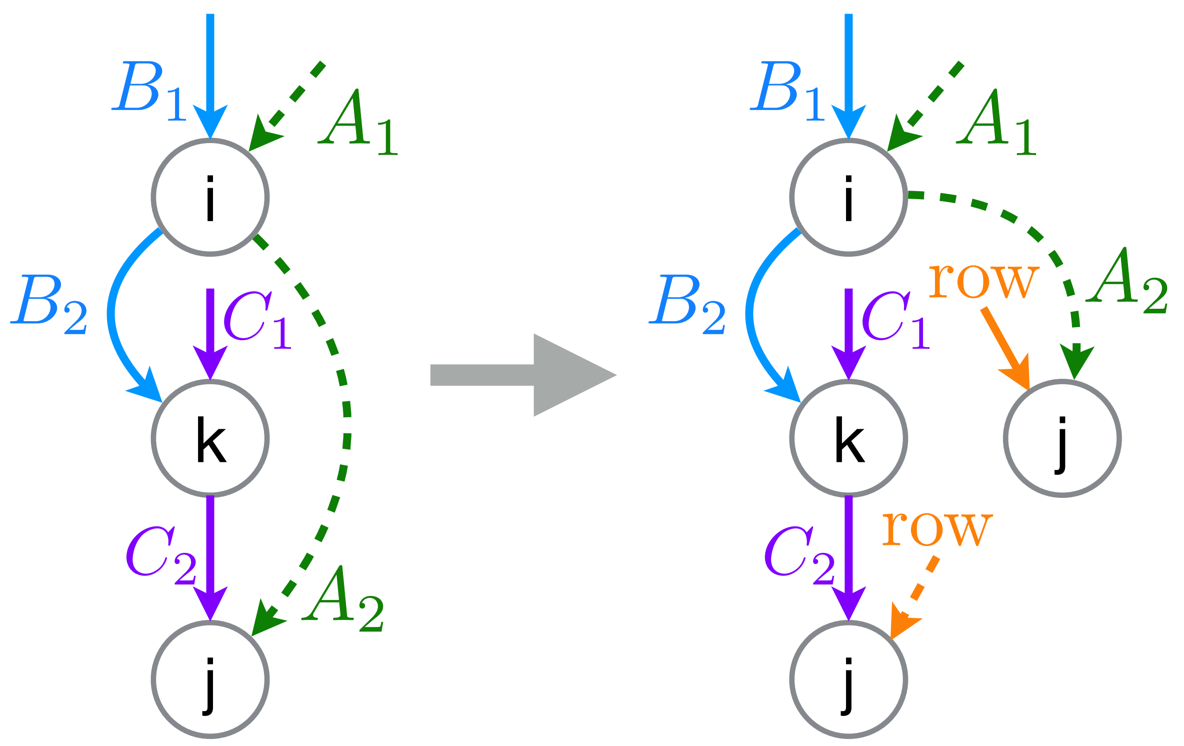


[Gustavson 1978]

```
for (int i = 0; i < m; i++) {



  for (int pB2 = B2_pos[i]; pB2 < B2_pos[i+1]; pB2++) {
    int k = B2_crd[pB2];



    for (int pC2 = C2_pos[k]; pC2 < C2_pos[k+1]; pC2++) {
      int j = C2_crd[pC2];
      row[j] += B[pB2] * C[pC2];
    }


  }

  for (int pA2 = A2_pos[i]; pA2 < A2_pos[i+1]; pA2++) {
    int j = A2_crd[pA2];
    A[pA2] = row[j];
    row[j] = 0.0;
  }

}
```

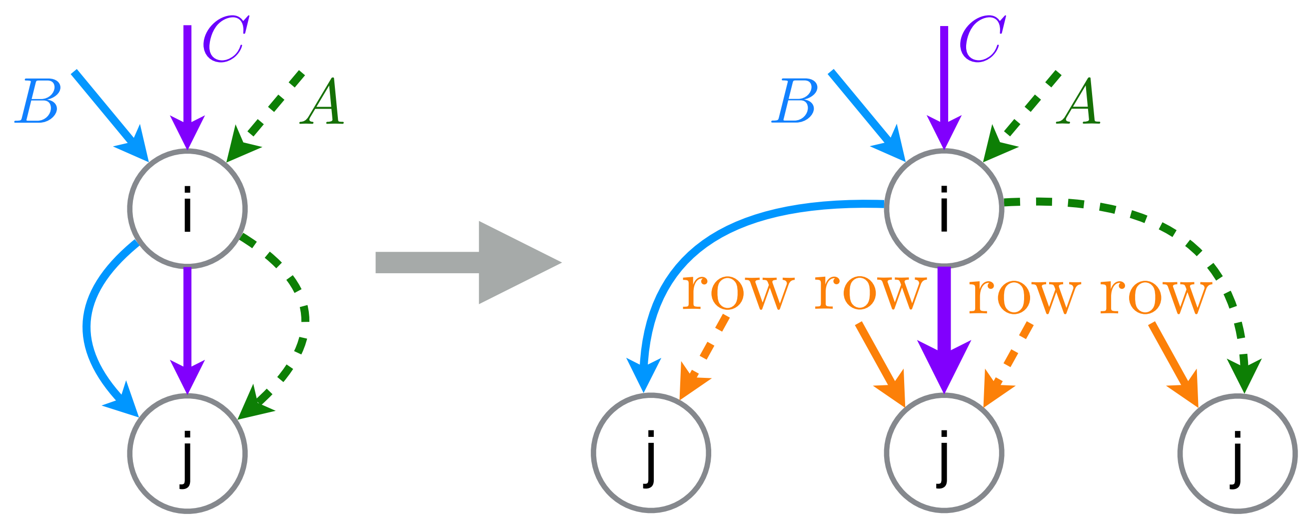
Other uses of where clauses

Scatter into results



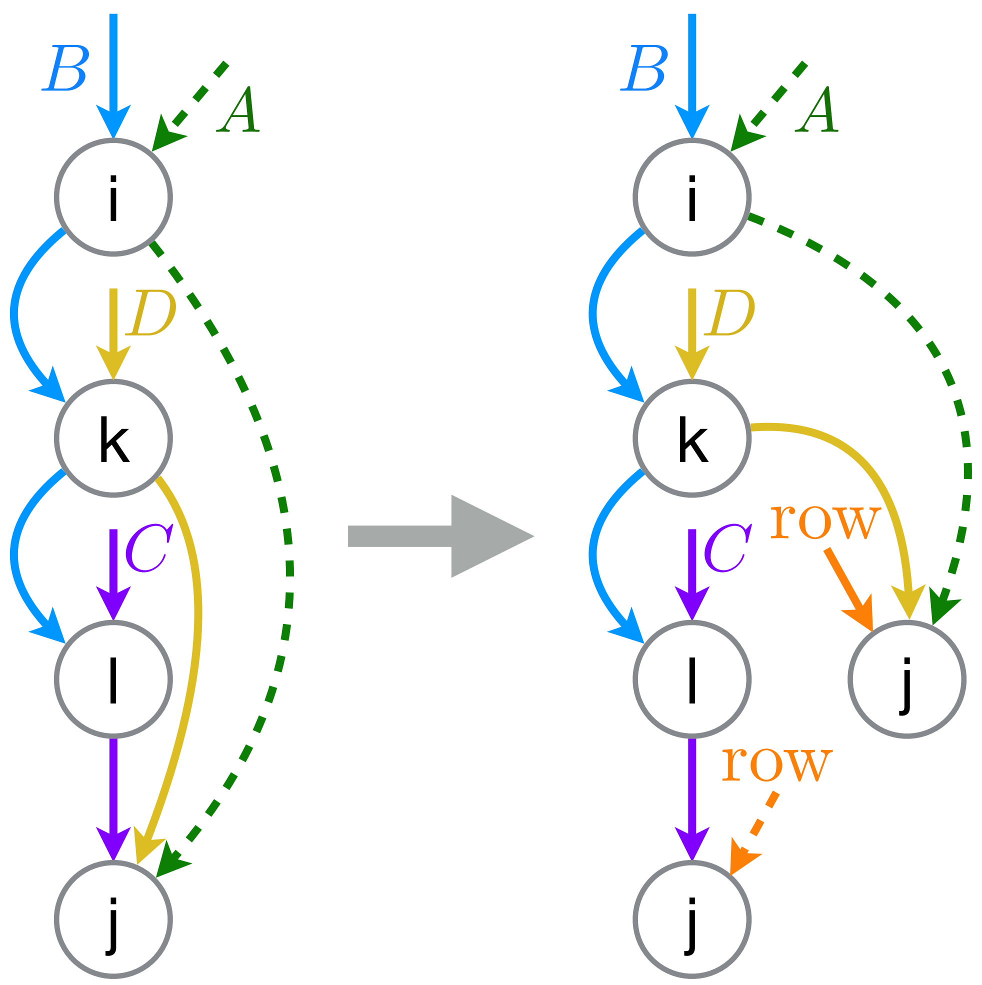
SpGEMM

Simplify merge code



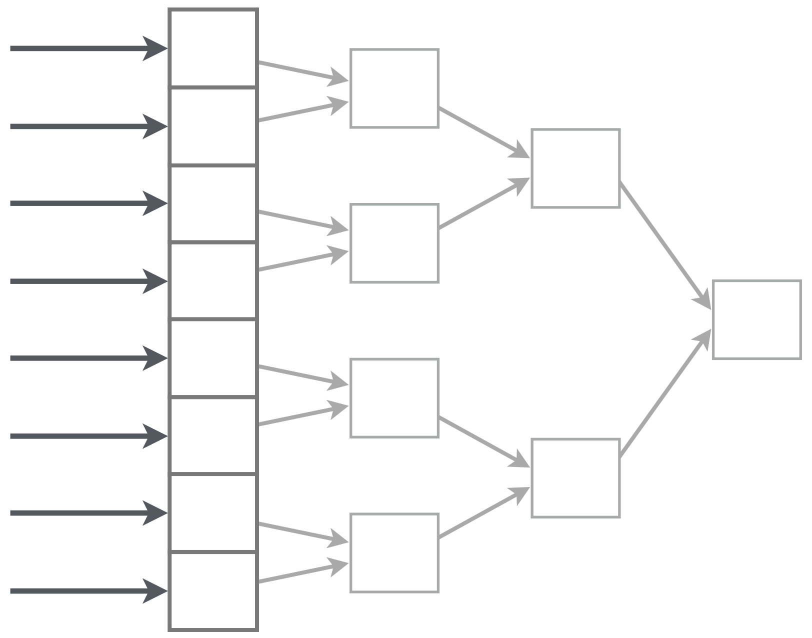
SpAdd

Loop-invariant code motion



MTTKRP

Prepare reductions



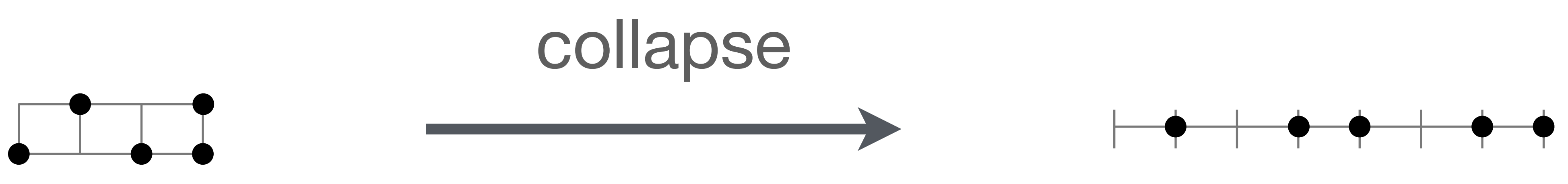
GPU shared memories

Mixed precision

```
for (int i = 0; i < m; i++) {
  double tj = 0.0;
  for (int pB2 = B2_pos[i];
           pB2 < B2_pos[i+1];
           pB2++) {
    int j = B2_crd[pB2];
    tj += B[pB2] * c[j];
  }
  a[i] = tj;
}
```

Single-precision floating point

A derived iteration space is one where dimensions have been split or collapsed



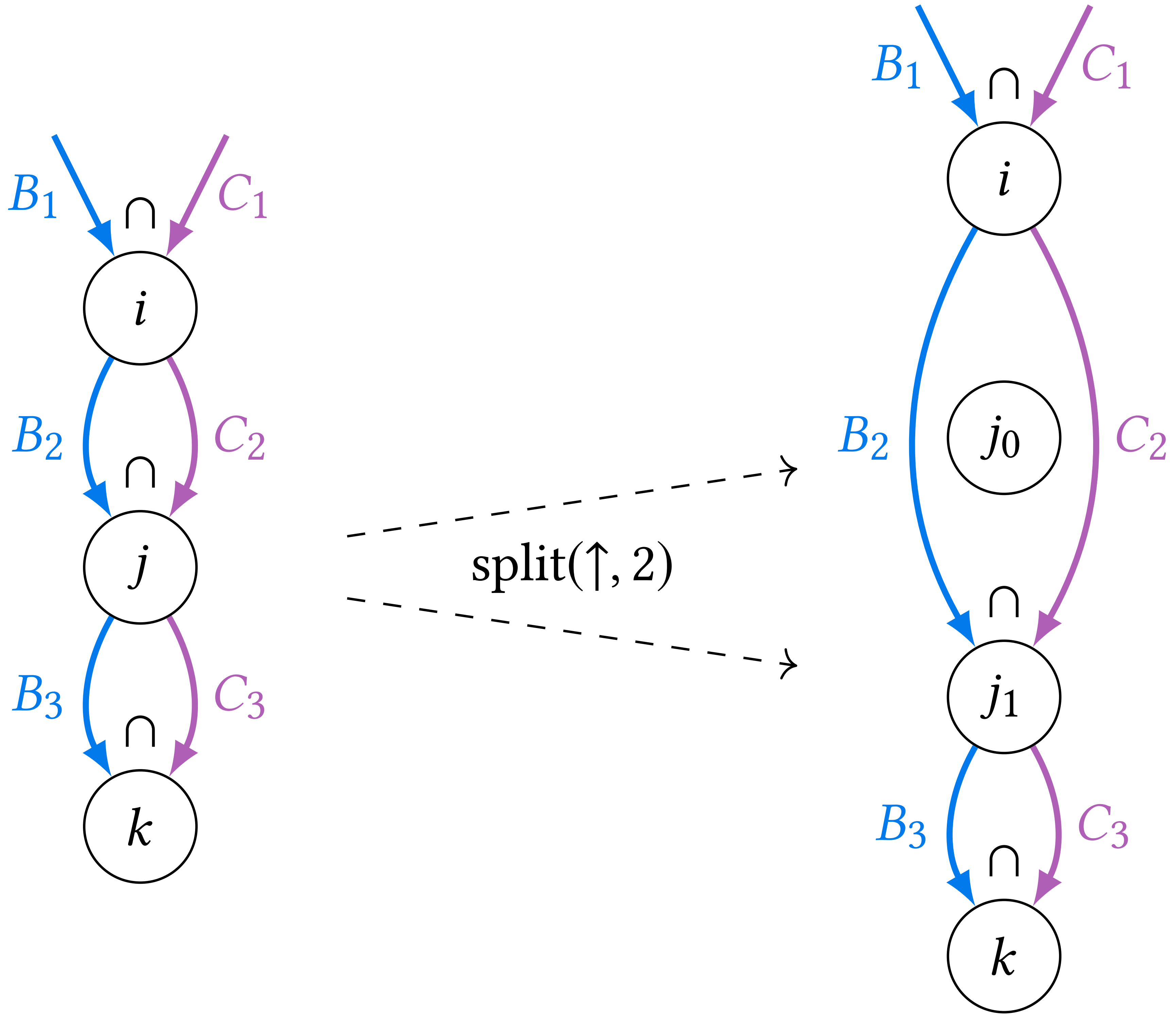
$$\forall_i \, a_i = b_i + c_i \quad \xrightarrow{\text{split}} \quad \forall_{i_0} \forall_{i_1} \, a_i = b_i + c_i \; : \; i \xrightarrow{\text{split}(\uparrow,4)} i_0 i_1$$

derived index variables

original index variable i

Cannot simply rewrite access expressions as with dense, so the mapping functions must be stored in the IR, so we can later emit code to remap coordinates.

The split iteration space function



$$\frac{\forall_i \forall_j \forall_k \, B_{ijk} \cap C_{ijk}}{\begin{array}{c} i \in B_1 \cap C_1 \\ j \in B_2 \cap C_2 \\ k \in B_3 \cap C_3 \end{array}}$$

$$\frac{\forall_i \forall_{j_0} \forall_{j_1} \forall_k \, B_{ijk} \cap C_{ijk} \; : \; j \xrightarrow{\text{split}(\uparrow,2)} j_0 j_1}{\begin{array}{l} i \in B_1 \cap C_1 \\ j_0 \in [0, 2) \\ j_1 \in B_2 \cap C_2 \cap [j_0 \cdot n/2, (j_0 + 1) \cdot n/2] \\ k \in B_3 \cap C_3 \end{array}}$$

The split iteration space function comes in several variants

$$\text{split}(\underline{\text{direction}, \text{stride}}, \underline{[\text{tensor operand}]})$$

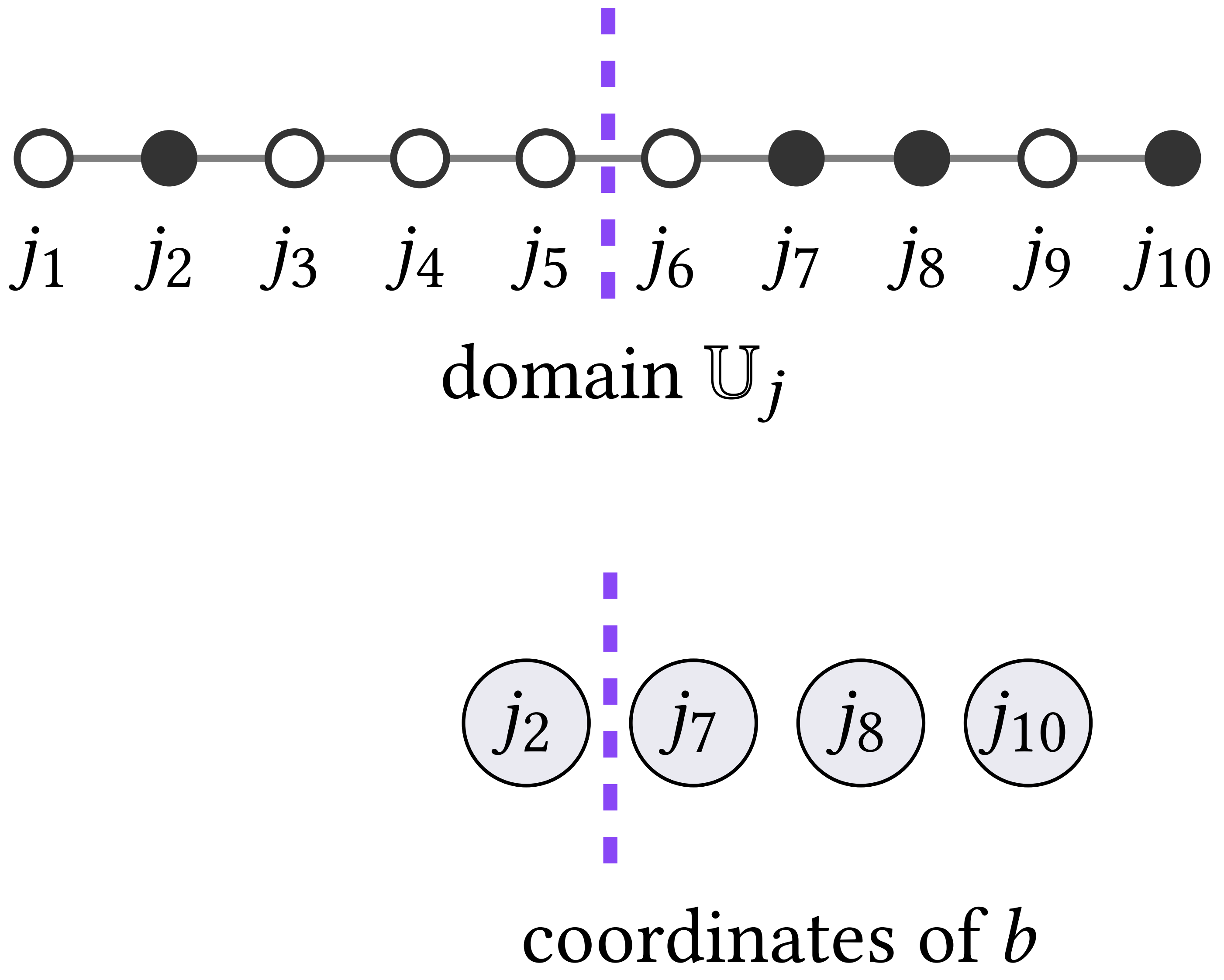
Split off a loop with stride iterations as the:

- outer loop (direction $= \uparrow$), or
- inner loop (direction $= \downarrow$)

Optional tensor operand whose nonzero coordinates the split applies to. If not given, the split applies to the universe of coordinates of the split index variable.

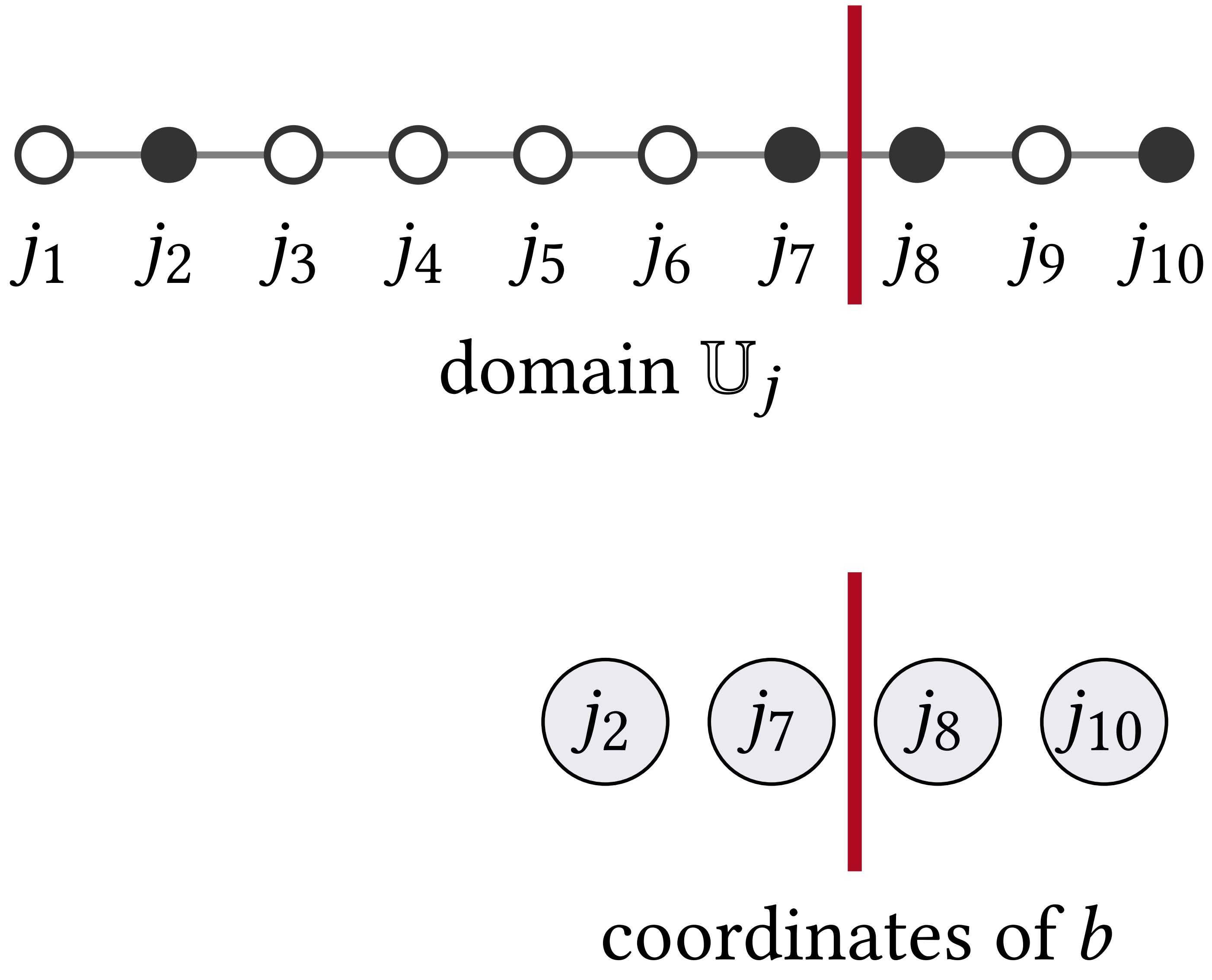
The split iteration space function can split with respect to the universe of coordinates or the coordinates of one operand

Universe split



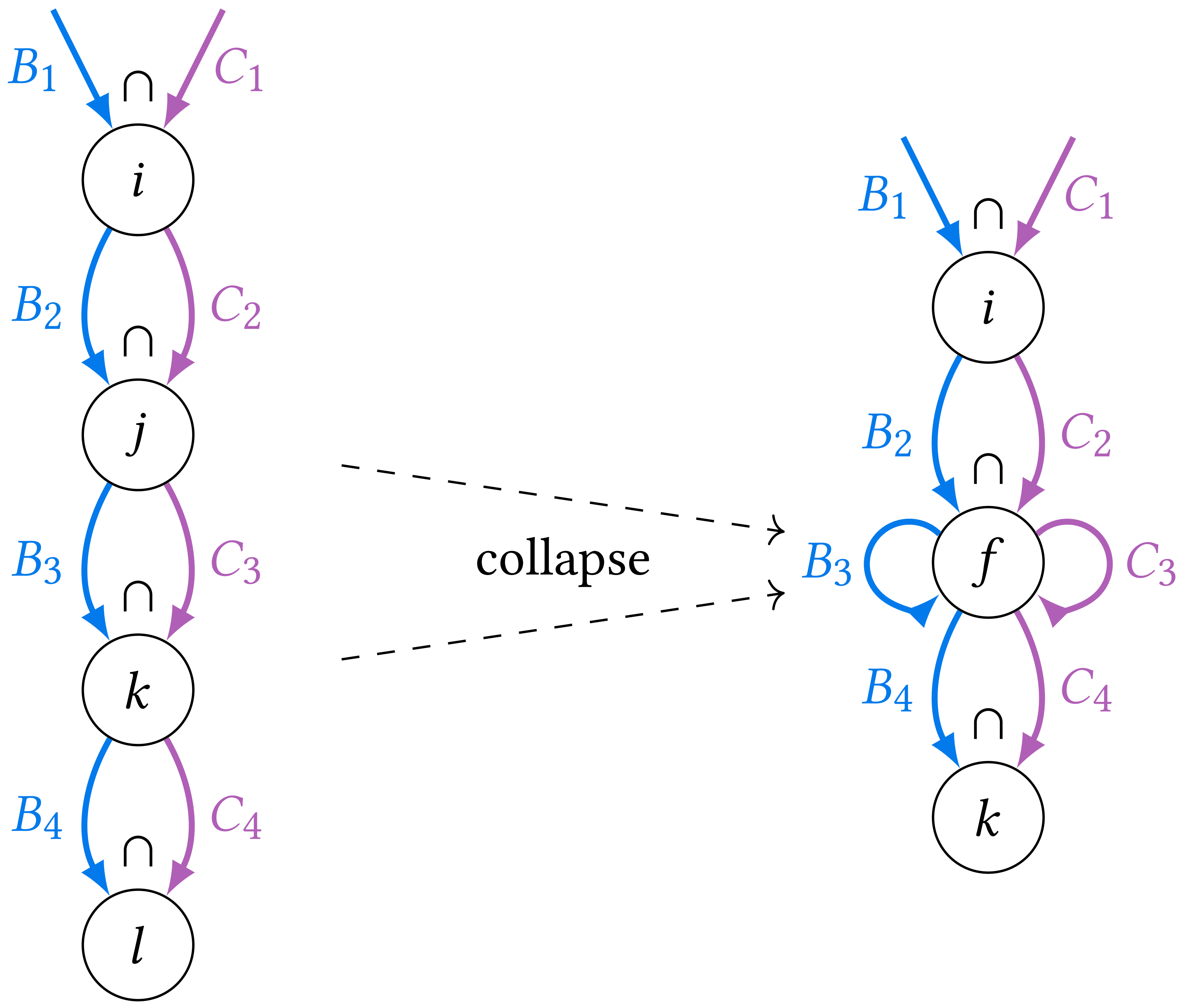
even split in coordinate universe, but
uneven split in b’s nonzero coordinates

Coordinate tree split



uneven split in coordinate universe, but
even split in b’s nonzero coordinates

The collapse iteration space function



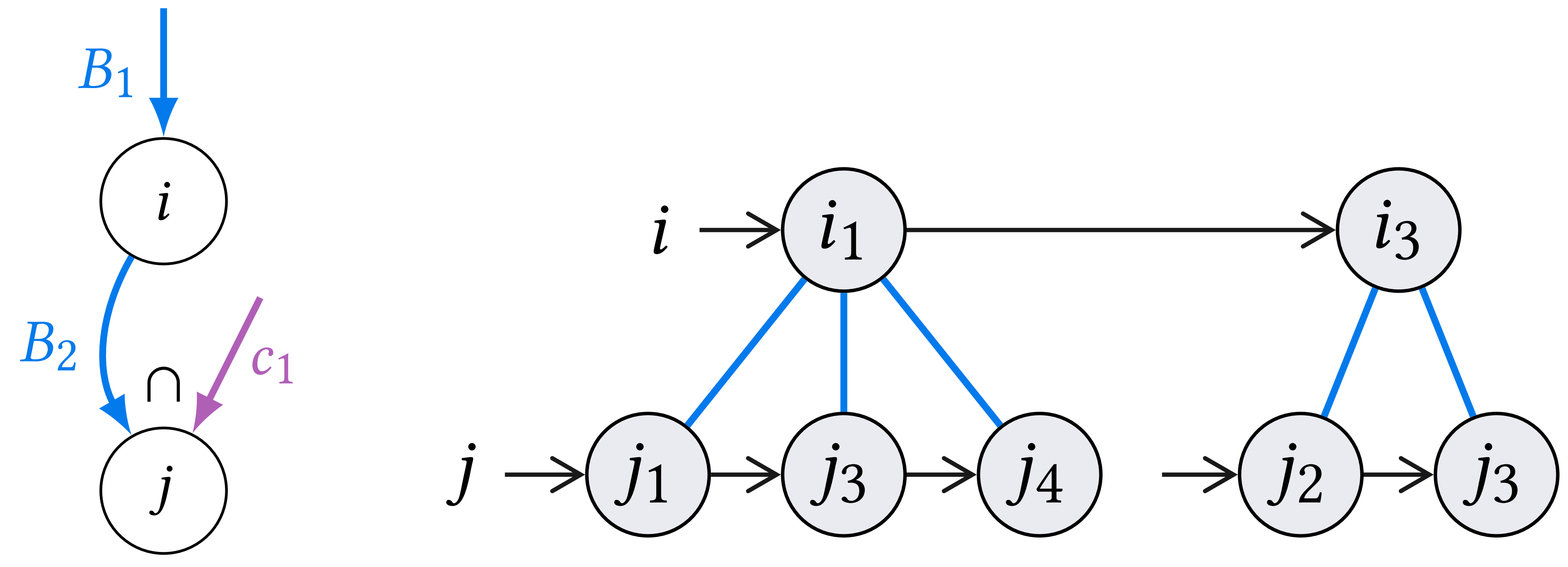
$$\frac{\forall_i \forall_j \forall_k \forall_l\, B_{ijkl} \cap C_{ijkl}}{\begin{array}{c} i \in B_1 \cap C_1 \\ j \in B_2 \cap C_2 \\ k \in B_3 \cap C_3 \\ l \in B_4 \cap C_4 \end{array}}$$

$$\frac{\forall_i \forall_f \forall_l\, B_{ijkl} \cap C_{ijkl} : jk \xrightarrow{\text{collapse}} f}{\begin{array}{l} i \in B_1 \cap C_1 \\ f \in (B_2 \times B_3) \cap (C_2 \times C_3) \\ k \in B_4 \cap C_4 \end{array}}$$

Iterate over Cartesian combination of coordinates in i and j.

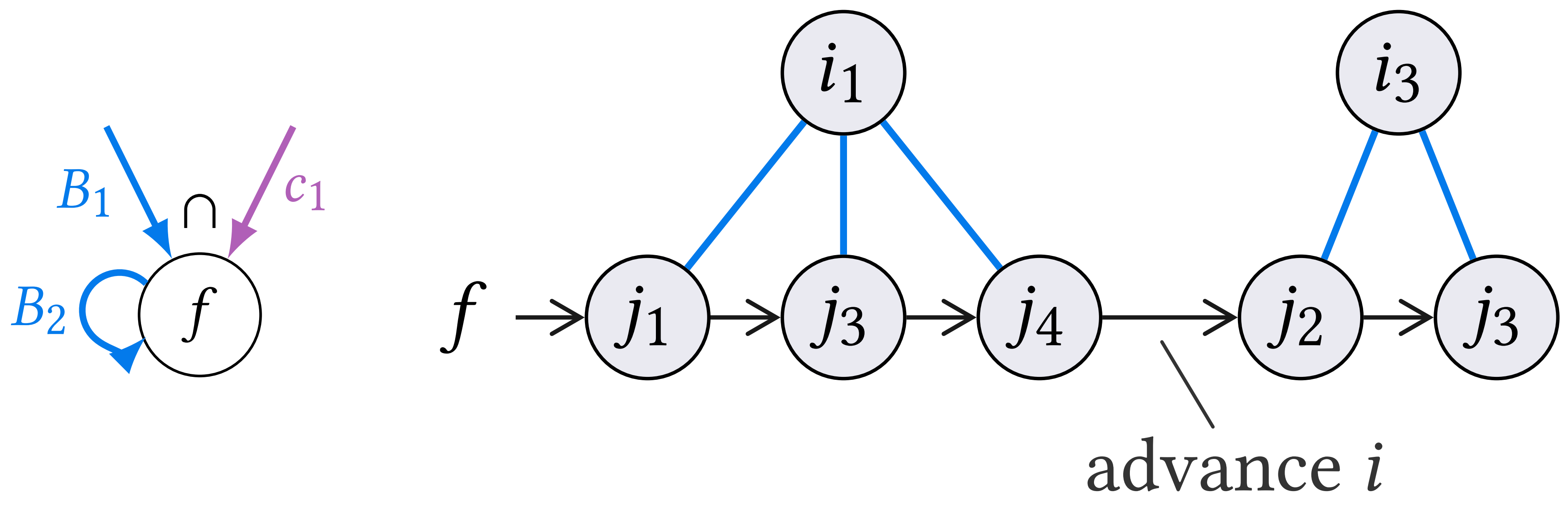
The collapse function leads to bottom-up iteration

Pre-collapse top-down iteration




```
for (int i = 0; i < m; i++) {
  for (int jB = B2_pos[i];
           jB < B2_pos[i+1]; jB++) {
    int j = B2_crd[jB];
    a[i] += B[jB] * c[j];
  }
}
```

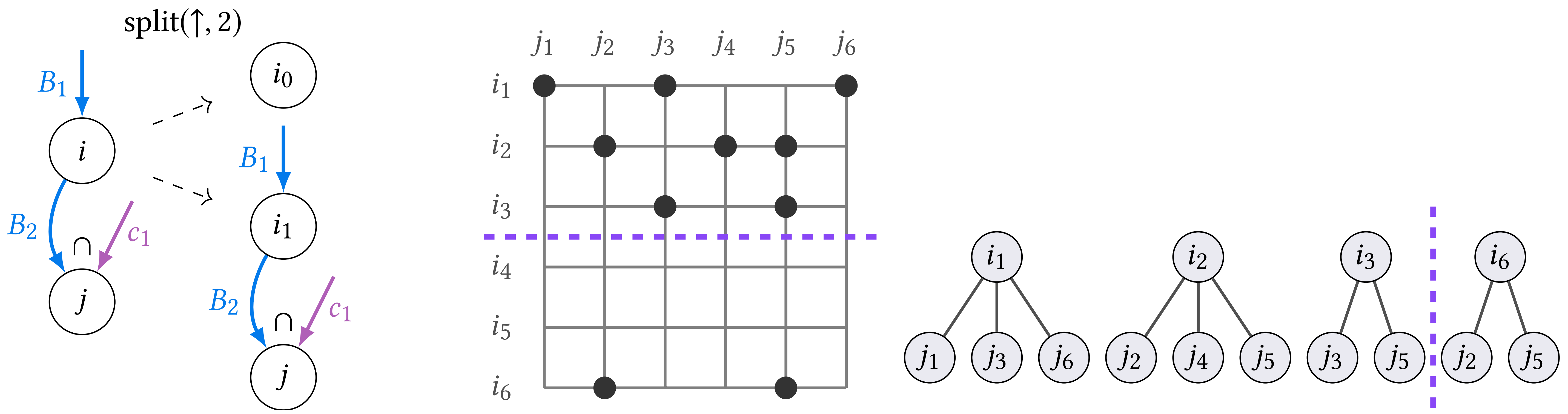
Post-collapse bottom-up iteration



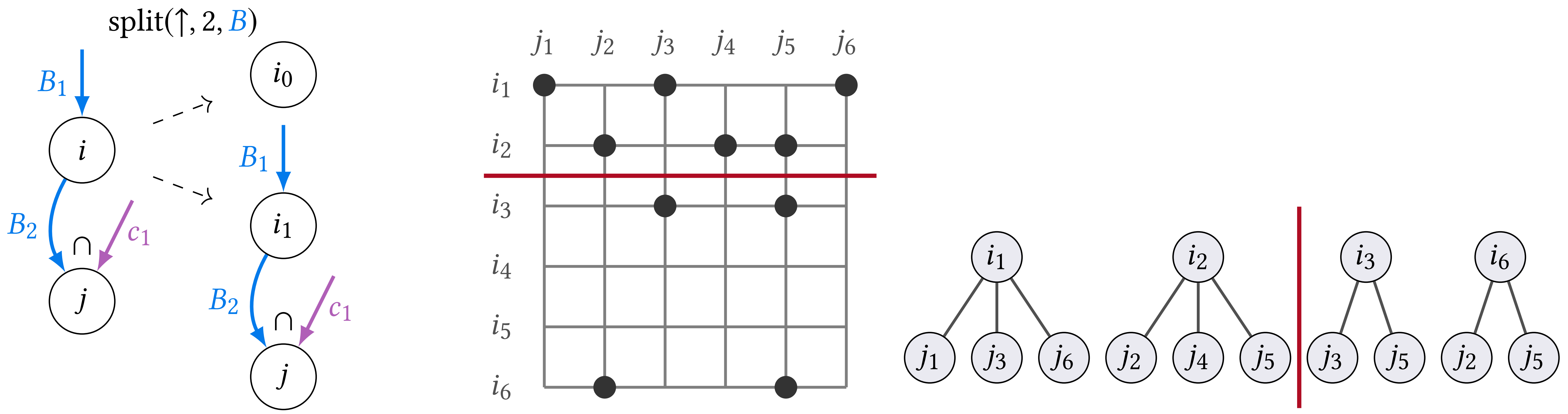

```
for (int f = 0, i = 0;
         f < B2_pos[m]; f++) {
  if (f >= B2_pos[m]) break;

  int j = B2_crd[f];
  while (f == B2_pos[i+1]) i++;

  a[i] += B[f] * c[j];
}
```

Two-dimensional tiling examples

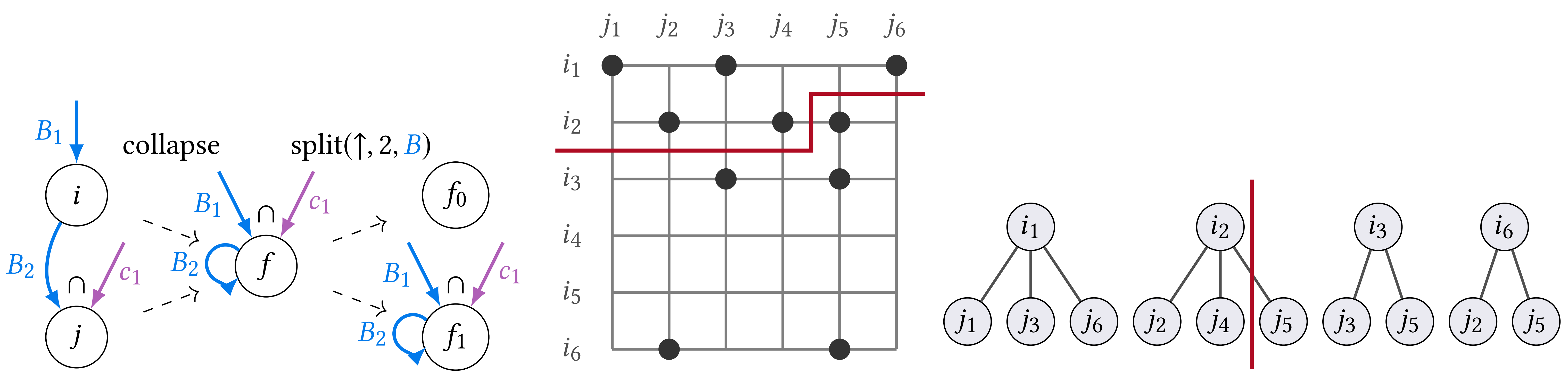
Universe split



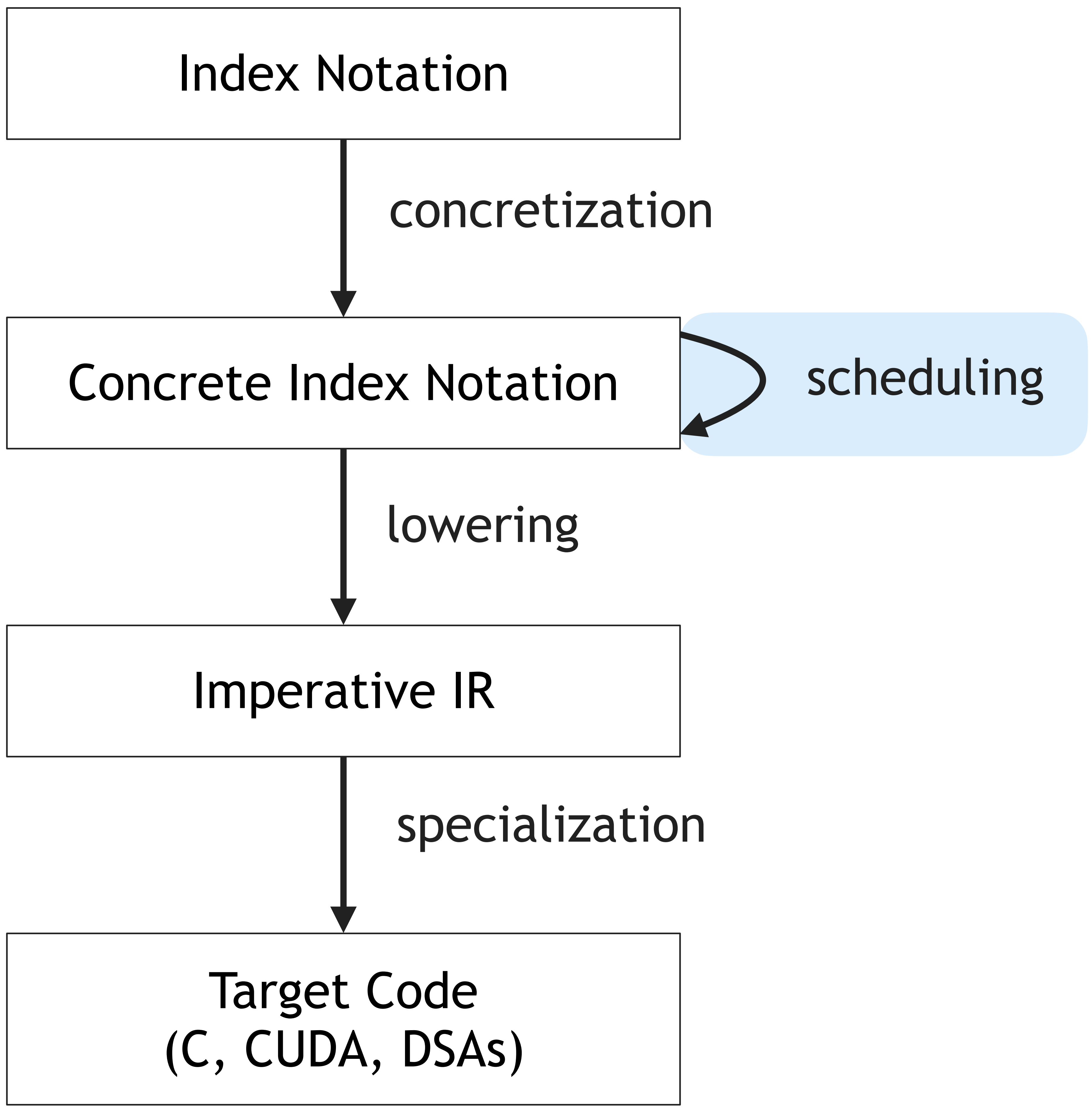
Coordinate tree split



Collapse+coordinate tree split



A sparse (tensor algebra) scheduling language



- **reorder(i, j)** interchanges loops i and j
- **split(i, i_1, i_2, d, s, t)** strip-mines i into two loops i_1 and i_2, where i_1 or i_1 is of size s depending on the direction d. The tensor t is optional and, if given, means the loop is strip-mined w.r.t. its nonzeros.
- **collapse(i, j, f)** collapses loops i and j into a new loop f, which iterates over their Cartesian combination.
- **precompute(S, e, t, I)** precomputes expression e in index statement S before the loops I and stores the results in tensor t.
- **unroll**, **parallelize**, **vectorize**, …

Lowering algorithm for code generation

function LOWER(assignment statement $S_{\text{assignment}}$)
  Emit compute code
end function

function LOWER(where statement S_{where})
  LOWER(producer statement of S_{where})
  LOWER(consumer statement of S_{where})
end function

function LOWER(sequence statement S_{sequence})
  LOWER(definition statement of S_{sequence})
  LOWER(mutation statement of S_{sequence})
end function

function LOWER(multi statement S_{multi})
  LOWER(left statement of S_{multi})
  LOWER(right statement of S_{multi})
end function

function LOWER(forall statement S_{forall} of index variable i)
  let $\mathcal{L}$ be an iteration lattice constructed from S_{forall}
  Emit initialize iterators
  for each lattice point $\mathcal{L}_p$ in $\mathcal{L}$ **do**
    Emit loop header
    Emit access iterators
    Emit map candidate coordinates to the original space
    Emit resolve the coordinate of i
    Emit map resolved coordinate to each derived space
    Emit locate from locators
    for each lattice point $\mathcal{L}_q < \mathcal{L}_p$ in $\mathcal{L}$ **do**
      Emit conditional header
      let $S_{\text{simplified}}$ be a statement constructed from the body of S_{forall} by removing operands that have run out of values in $\mathcal{L}_q$
      LOWER($S_{\text{simplified}}$)
      Emit assembly code
      Emit conditional footer
    end for
    Emit advance iterators
    Emit loop footer
  end for
end function

Course Project