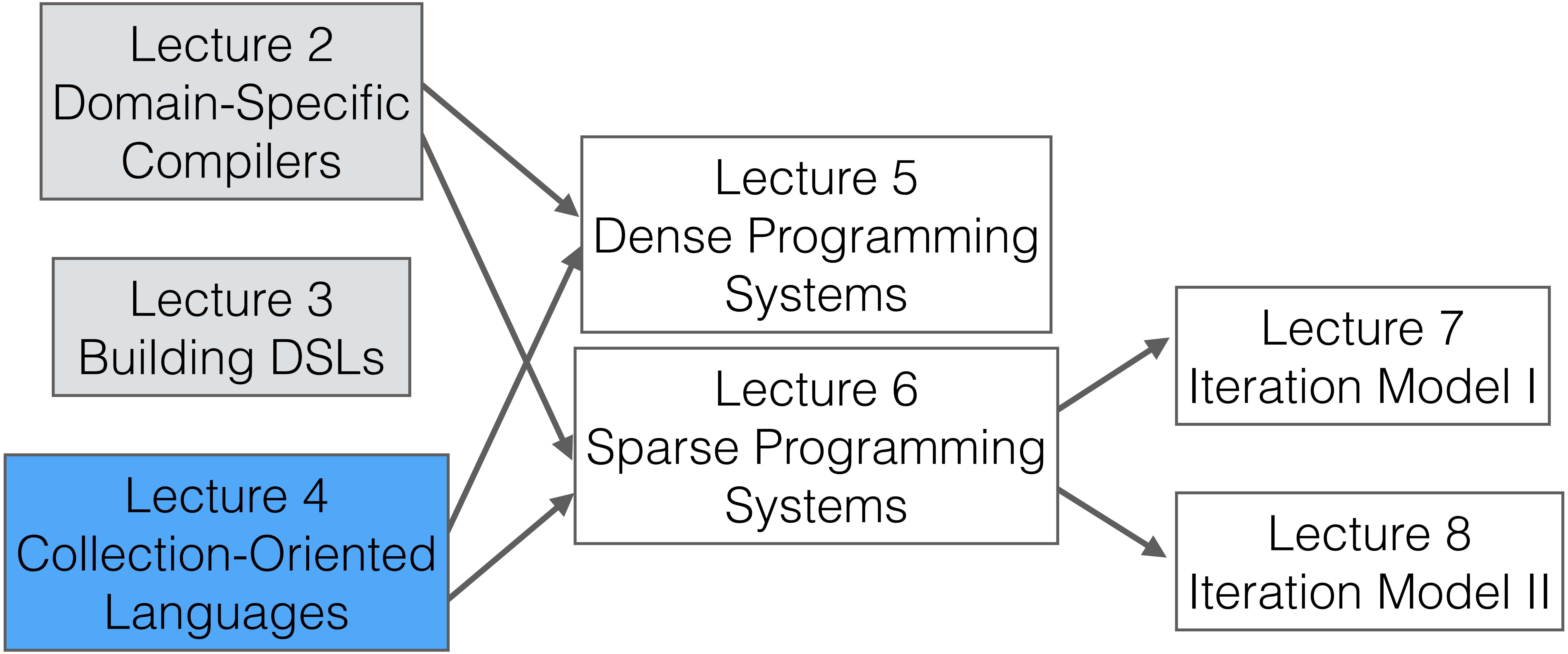
Lecture 4 — Collection-Oriented Languages

Stanford CS343D (Winter 2025)
Fred Kjolstad

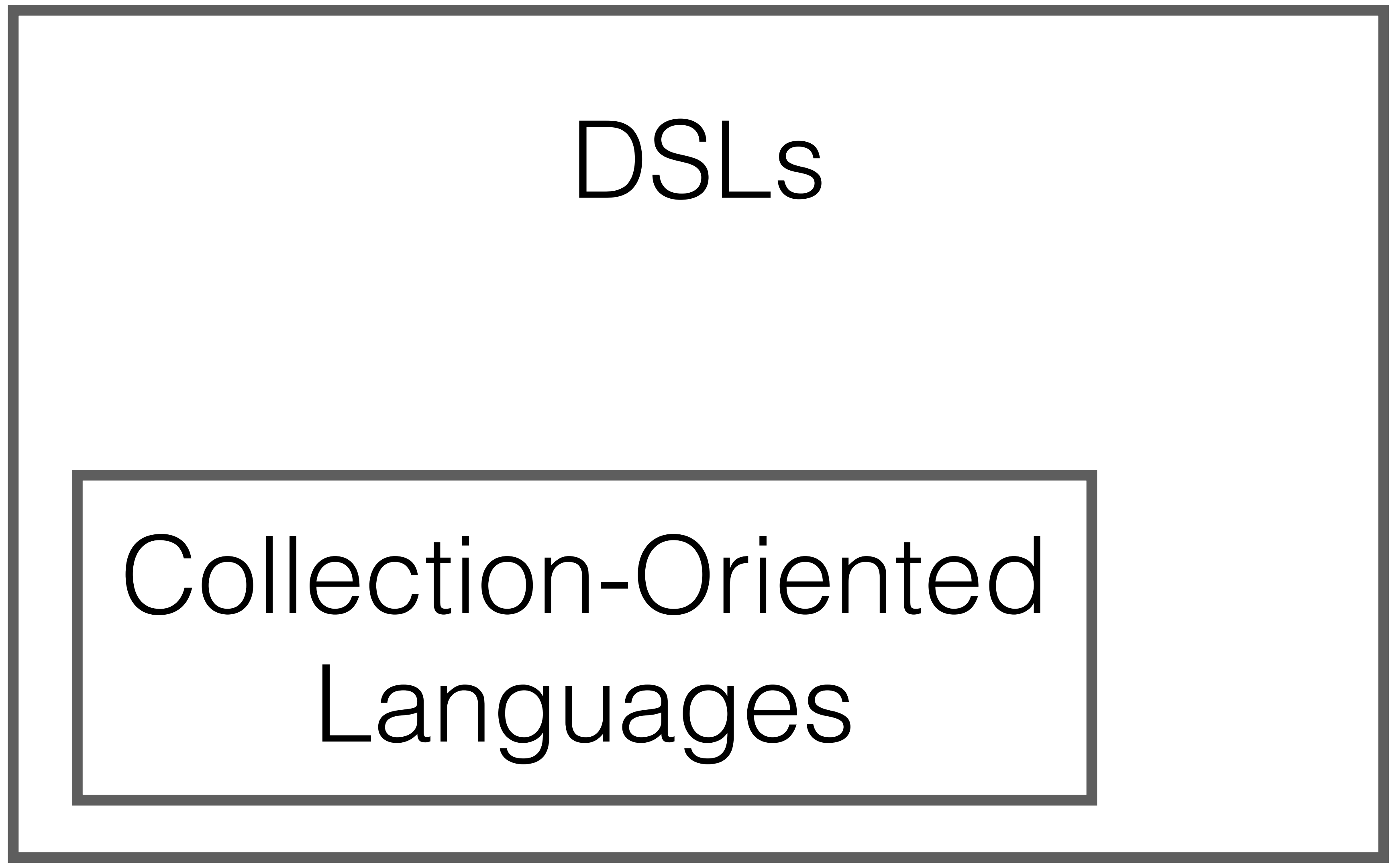

Lecture 2
Domain-Specific
Compilers
Lecture 3
Building DSLs
Lecture 4
Collection-Oriented
Languages
Lecture 5
Dense Programming
Systems
Lecture 6
Sparse Programming
Systems
Lecture 7
Iteration Model I
Lecture 8
Iteration Model II

Languages are tools for thought

“By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on the more advanced problems”

— Alfred N. Whitehead

Collection-Oriented languages are an important subclass of DSLs as discussed in this course



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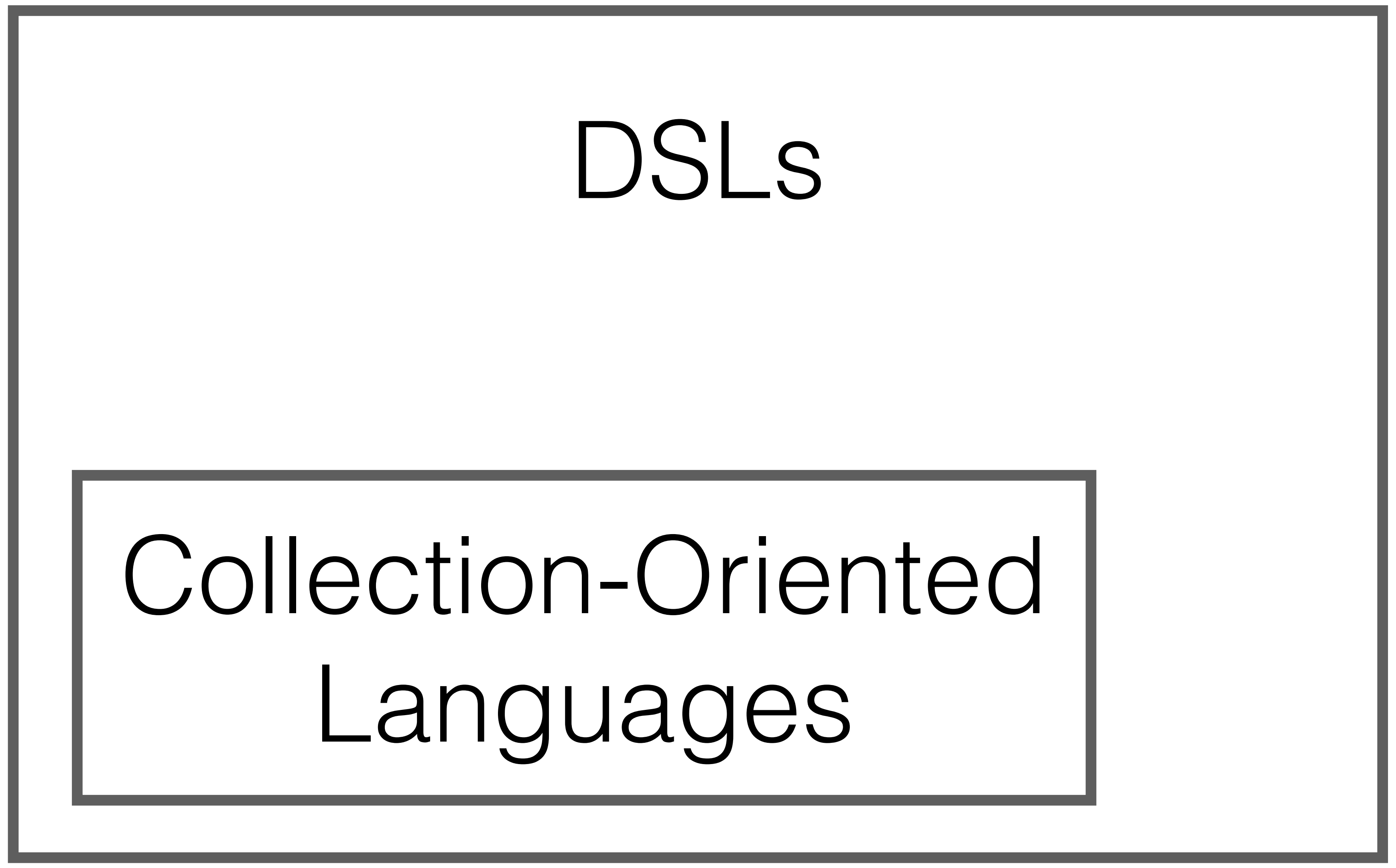


Economy of scale

in notation and execution

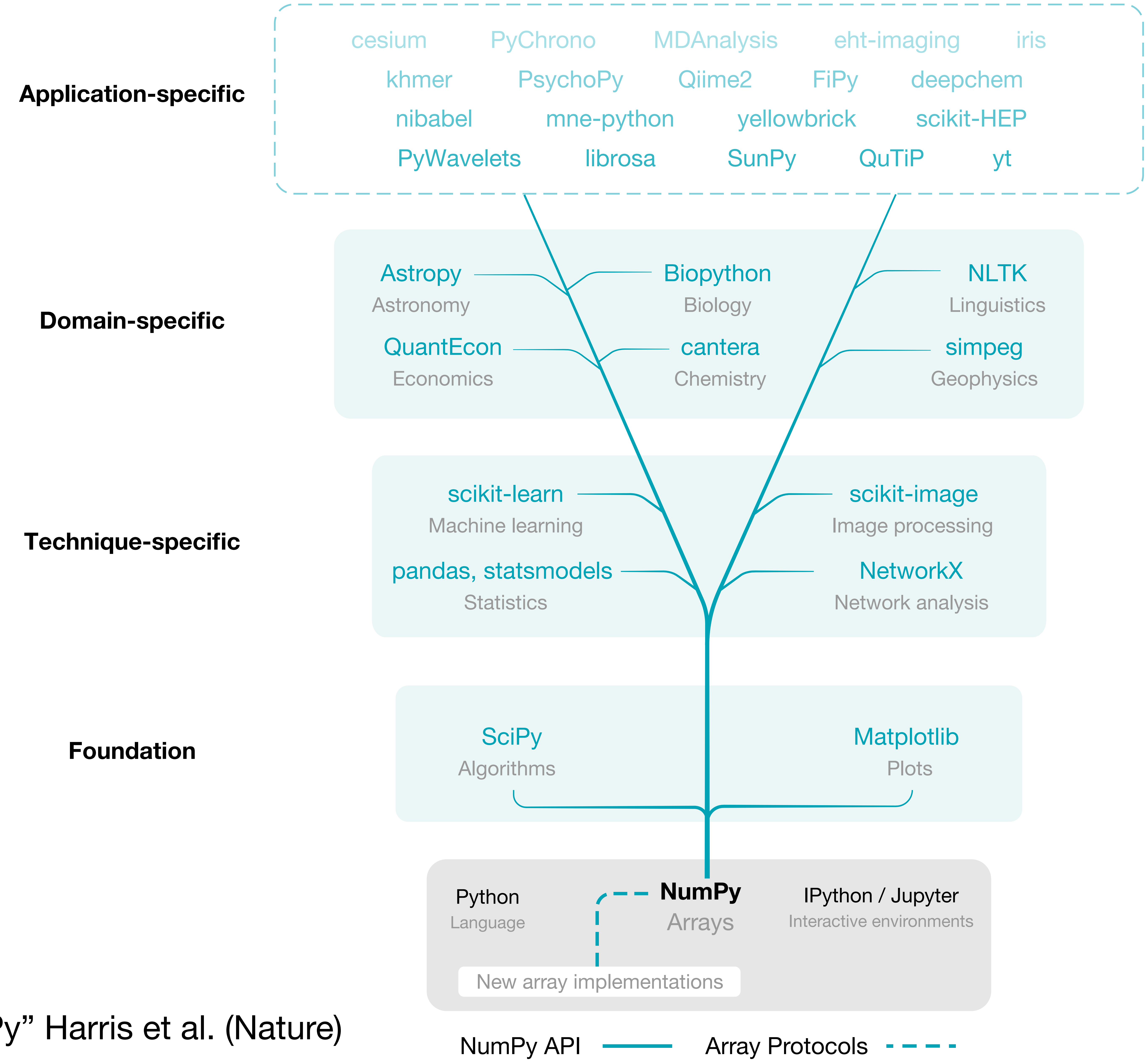
$$C = A \bowtie B$$

$$c = Ab$$

```
[x * 2 for x in my_list]
```

How many operations?

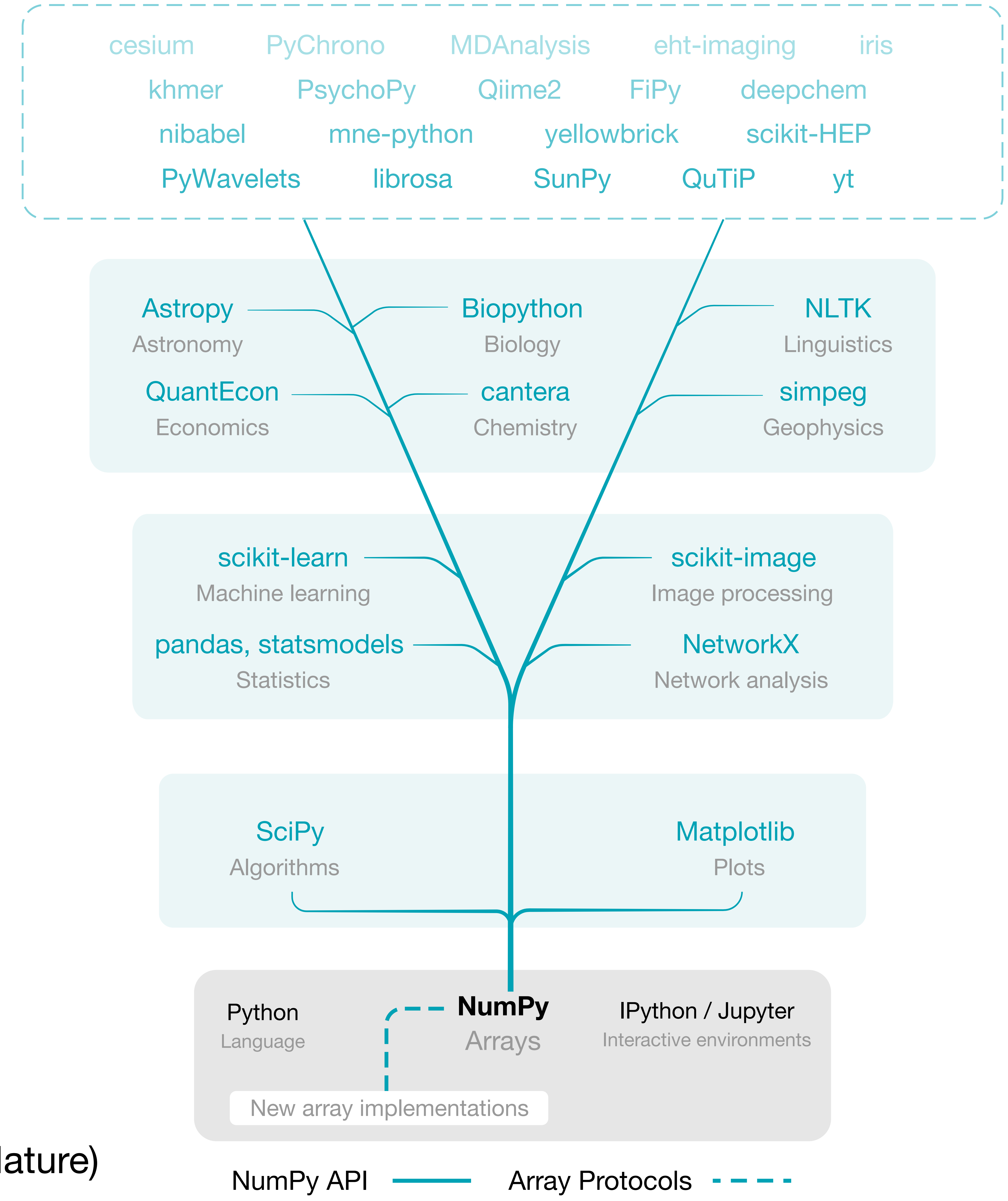
Collection-oriented languages are relatively general



“Array Programming with NumPy” Harris et al. (Nature)

Collection-oriented languages are relatively general

DSLs



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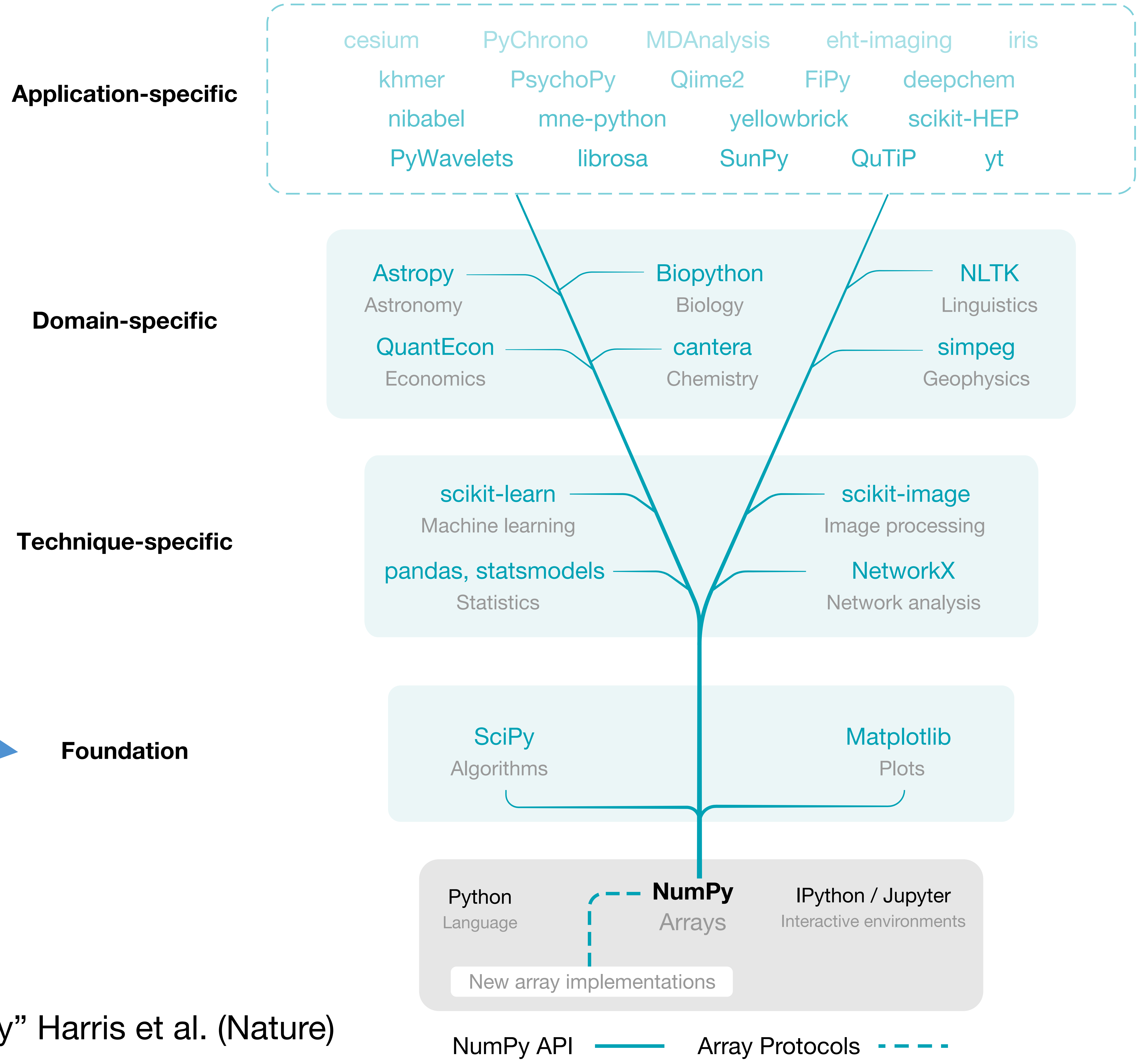
Collection-Oriented
(also DSLs?)



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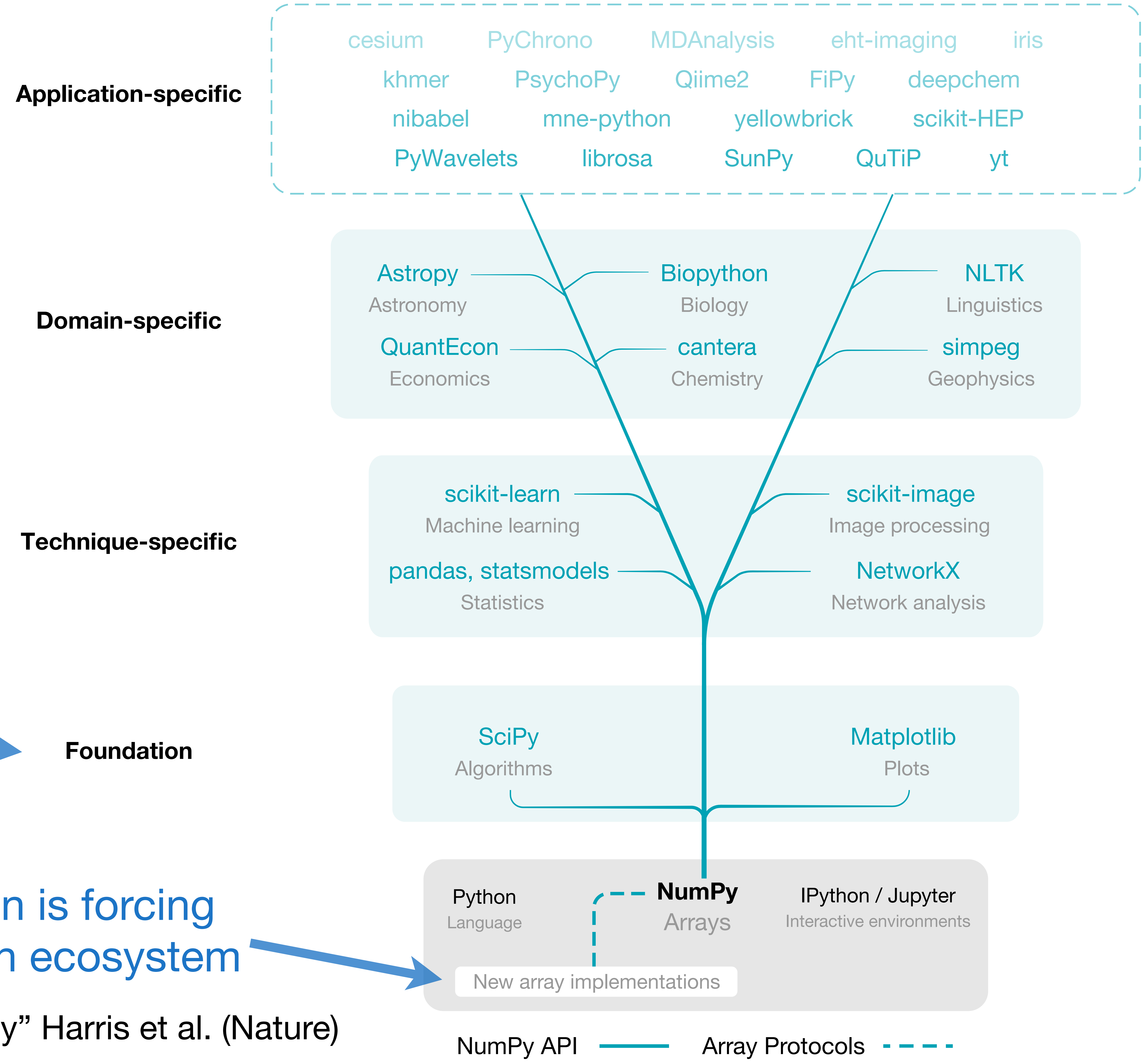
Collection-oriented languages are relatively general

DSLs

Collection-Oriented
(also DSLs?)

Fragmentation is forcing
implementation ecosystem



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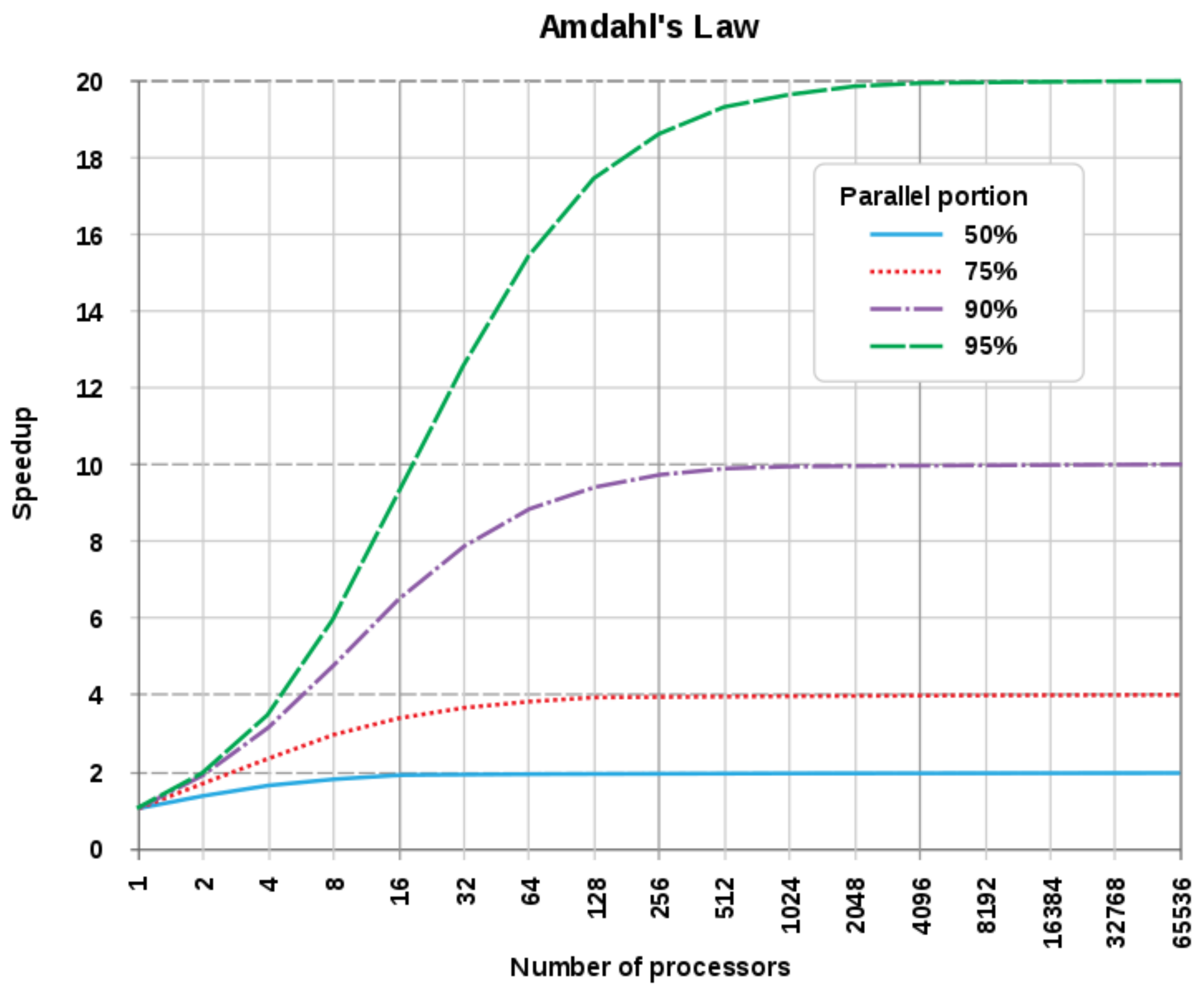
We need collections for performance due to Amdahl’s law



Plot from Wikipedia

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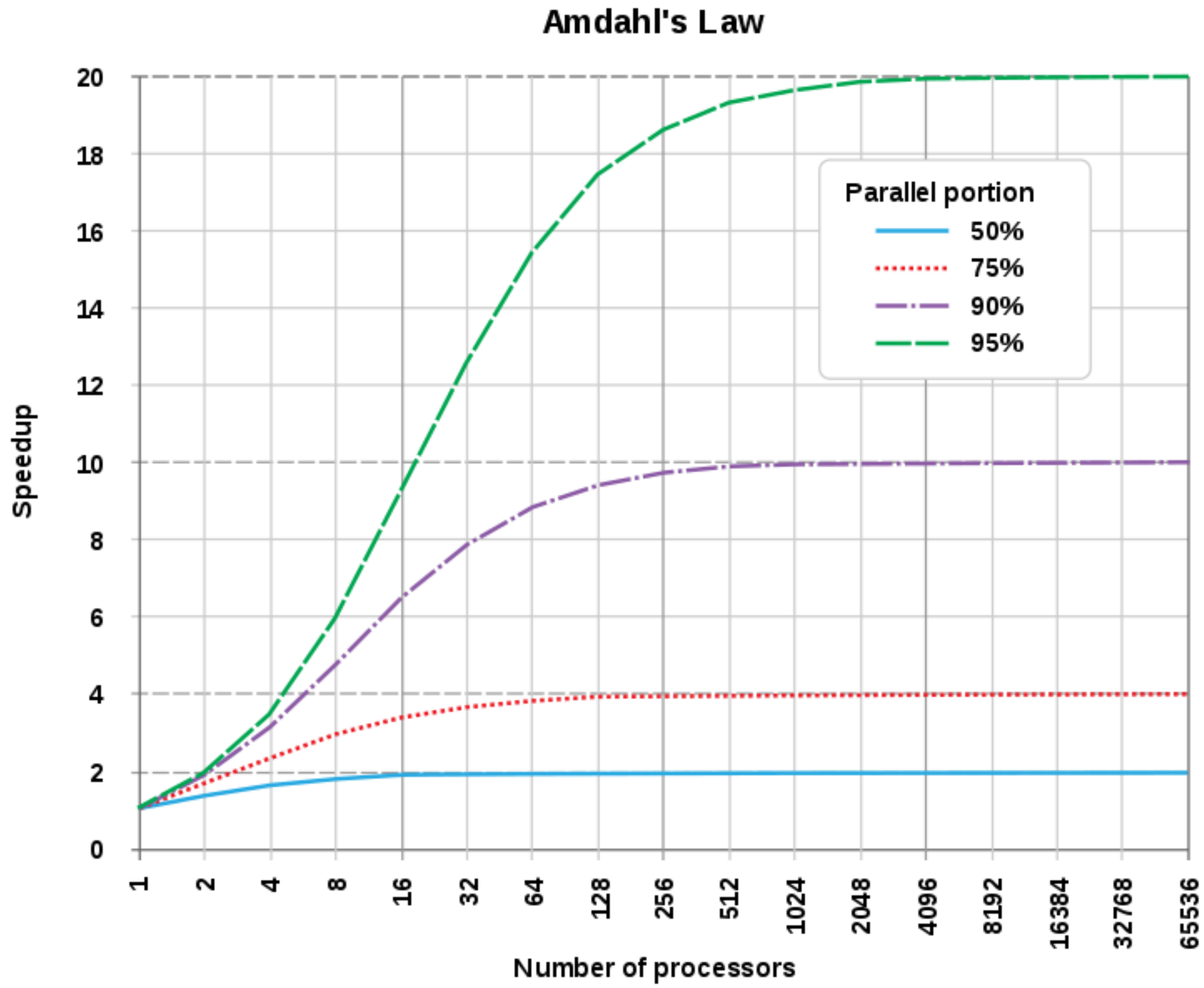
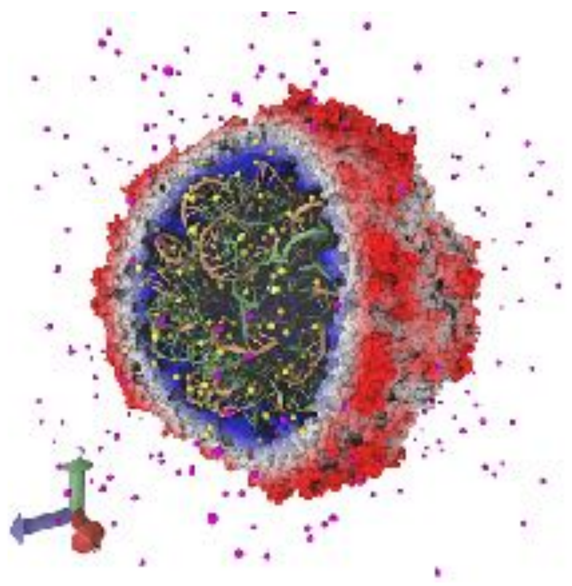
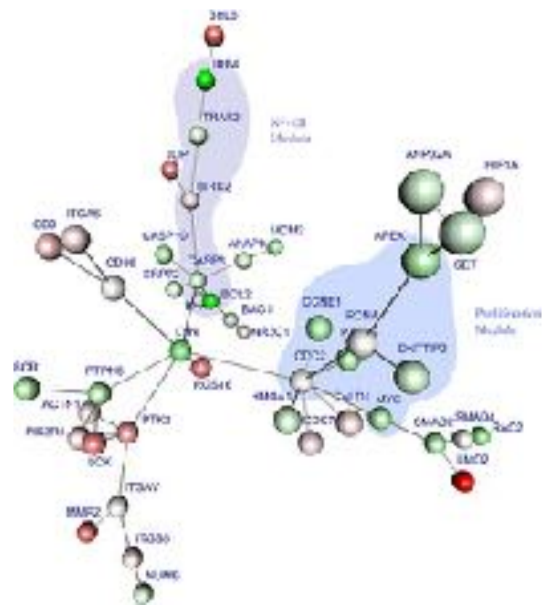
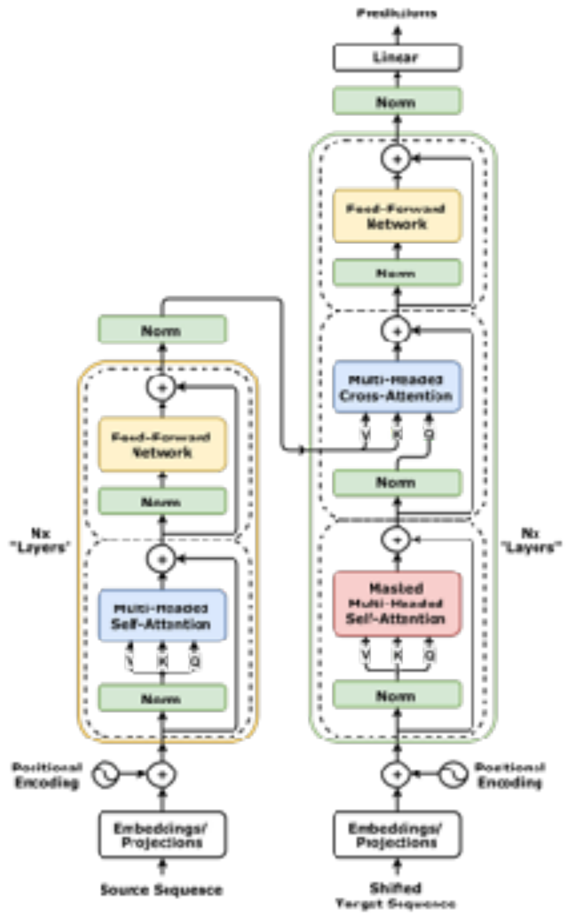
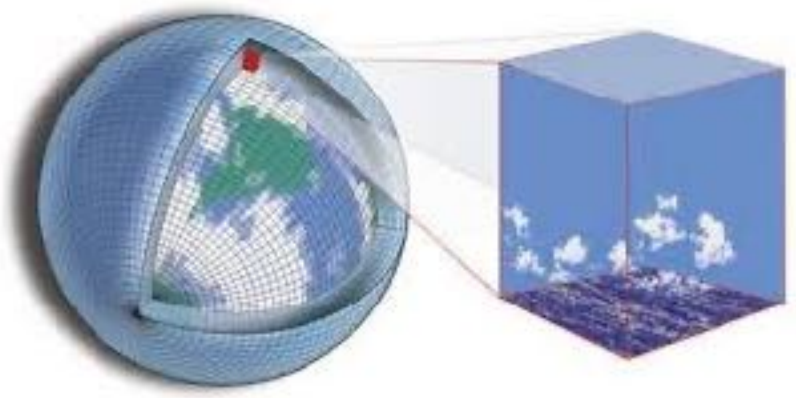


But many applications are data-rich









Plot from Wikipedia

Avoiding the von Neumann model of languages

Imperative Form

```
c := 0
for i := 1 step 1 until n do
  c := c + a[i] x b[i]
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rich world of expressions

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Functional Form

```
c = sum(a[0:n] * b[0:n])
```

produces a vector

Collection-oriented operations let us operate on collections as a whole

- A record-at-a-time user interface forces the programmer to do manual query optimization, and this is often hard.
- Set-a-time languages are good, regardless of the data model, since they offer much improved physical data independence.
- The programming language and database communities have long recognized that aggregate data structures and general operations on them give great flexibility to programmers and language implementors.

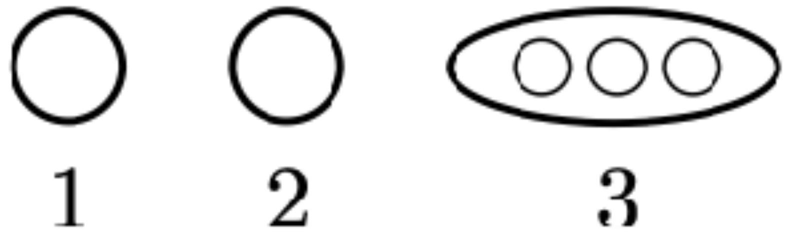
"What Goes Around Comes Around" Stonebraker and Hellerstein
"Collection-Oriented Languages" Sipelstein and Blelloch

Collection-Oriented Languages

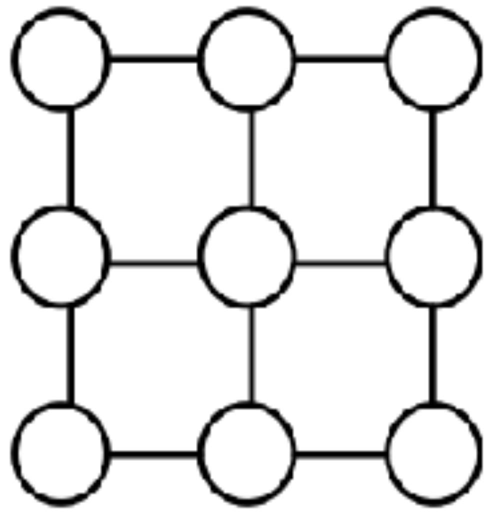
Lists
Lisp [1958]

Sets
SETL
[1970]

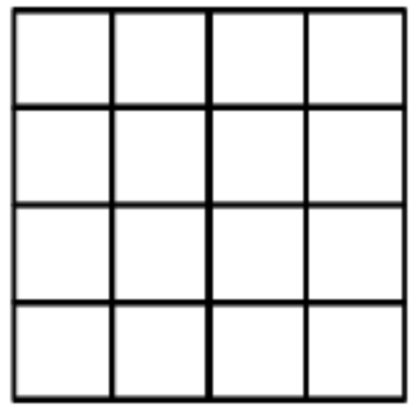
Nested Sequences
NESL [1994]



Grids
Halide [2012]



Arrays
APL [1962]
NumPy [2020]



Relations
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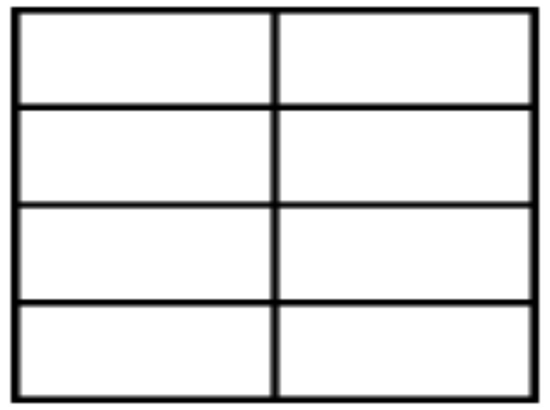
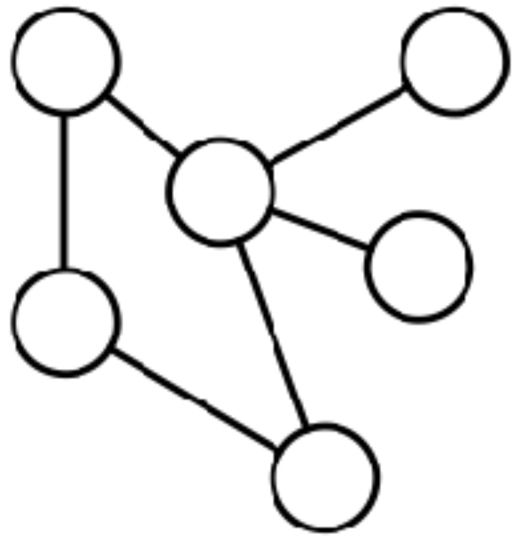
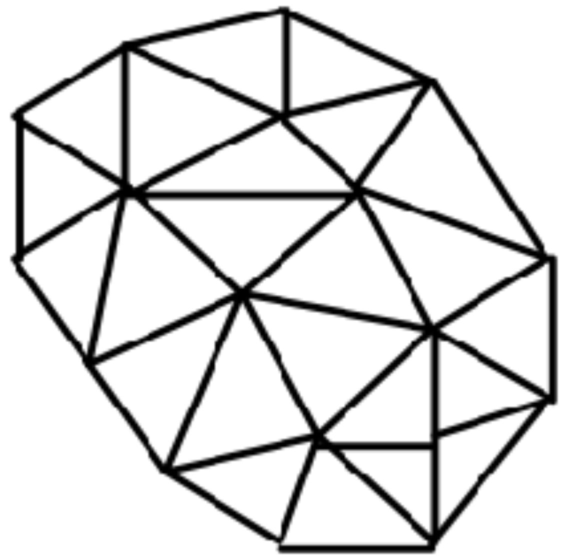


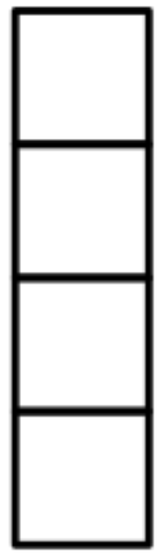
Graphs
GraphLab [2010]



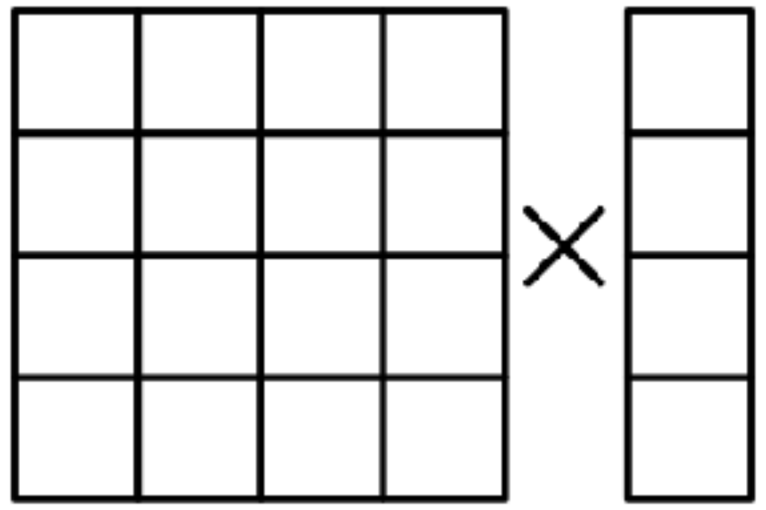
Meshes
2011



Vectors
Vector Model 1990



Matrices and Tensors
Matlab 1979], Taco 2017



A collection-oriented programming model provides collective operations
on some collection/abstract data structure

Collection-Oriented Languages

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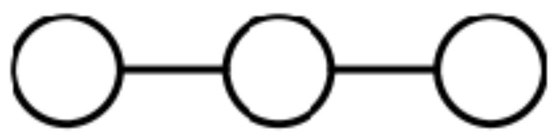

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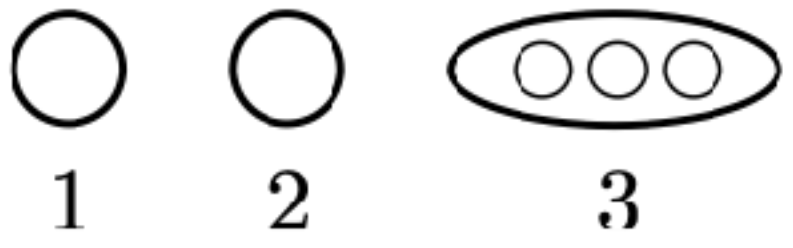

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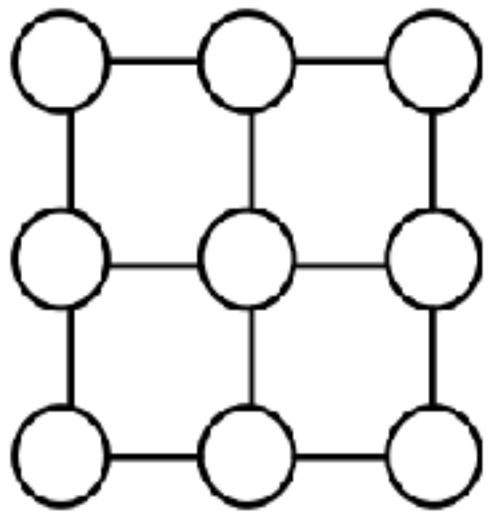

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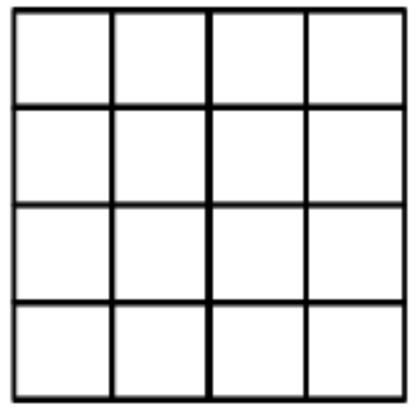

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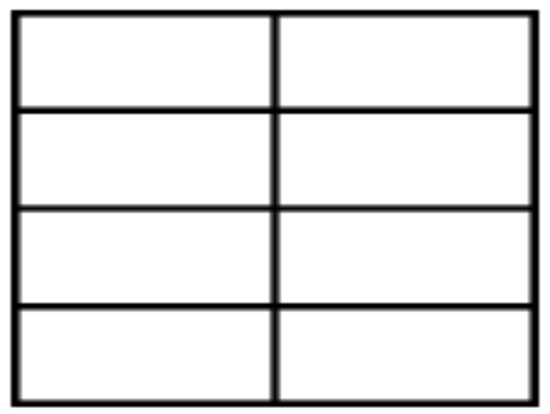

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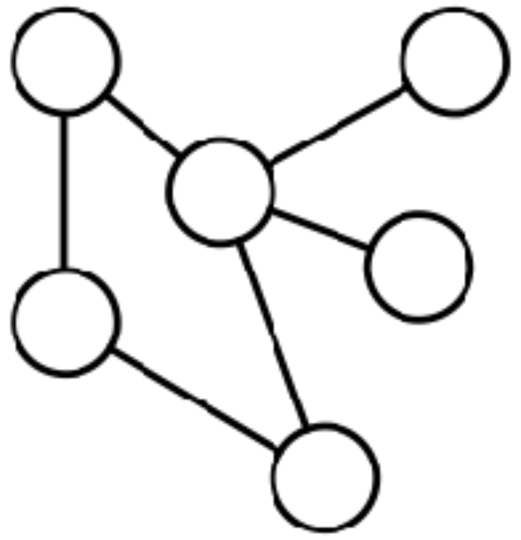

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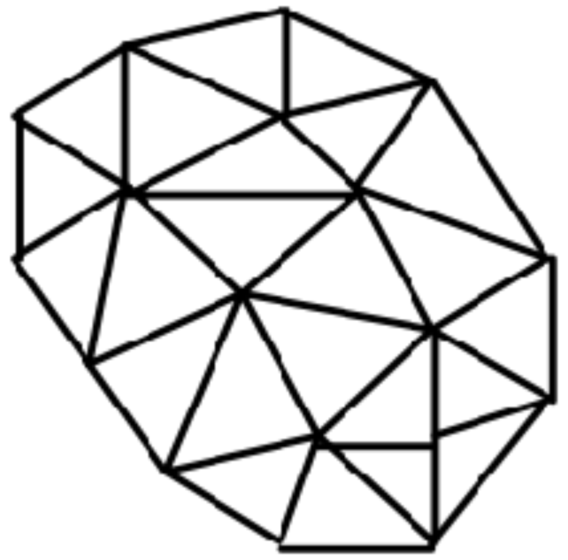

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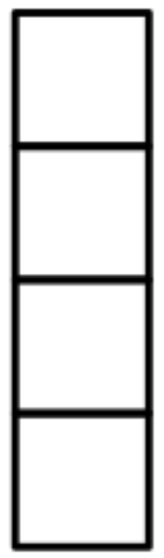

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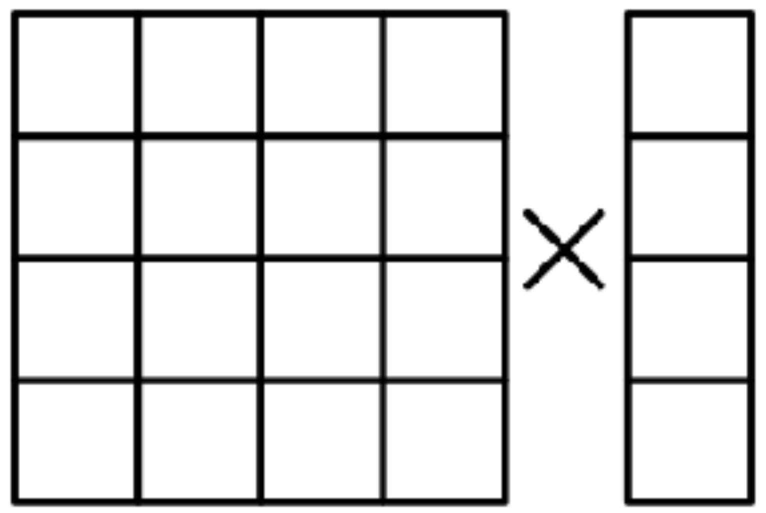

A collection-oriented programming model provides collective operations on some collection/abstract data structure

Object Orientation vs.
Collection Orientation

Features of collections

- Ordering: unordered, sequence, or grid-ordered?
- Regularity: Can the collection represent irregularity/sparsity?
- Nesting: nested or flat collections?
- Random-access: can individual elements be accessed?

The APL Programming Language

```
n ← 4 5 6 7
```

i.e., mkArray

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n+4
8 9 10 11
```

4 is broadcast across each n

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n+⍳4
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element-wise addition
(⍳4 makes the array [1,2,3,4])

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i.e., mkArray

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n+4
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```

4 is broadcast across each n

```
n+ι4
5 7 9 11
```

element-wise addition
(ι4 makes the array [1,2,3,4])

```
+/n
22
```

$$\sum_{i=0}^{n} n_i$$

The APL Programming Language

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```
+/n
22
```

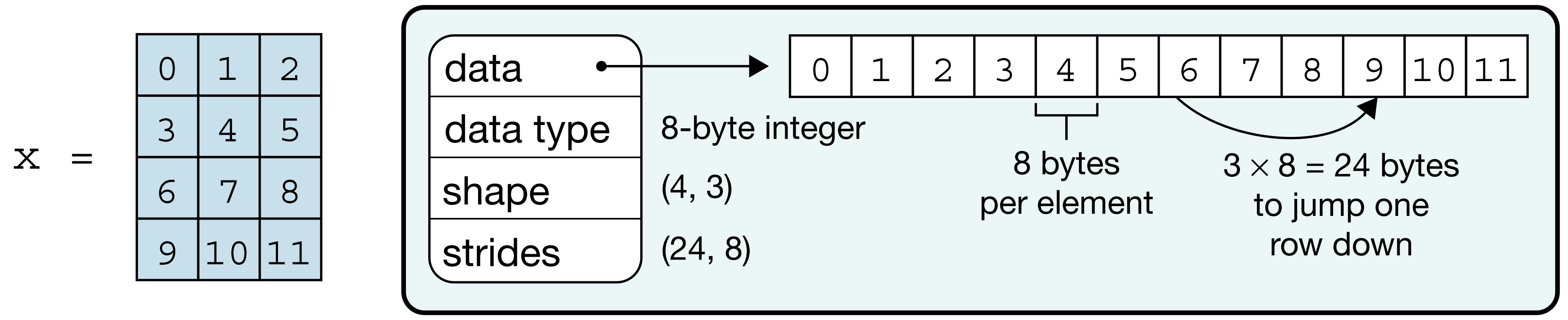
$$\sum_{i=0}^{n} n_i$$

```
+/(3+⍳4)
22
```

$$\sum_{i=1}^{4} (i+3)$$

Array Programming with NumPy

a Data structure



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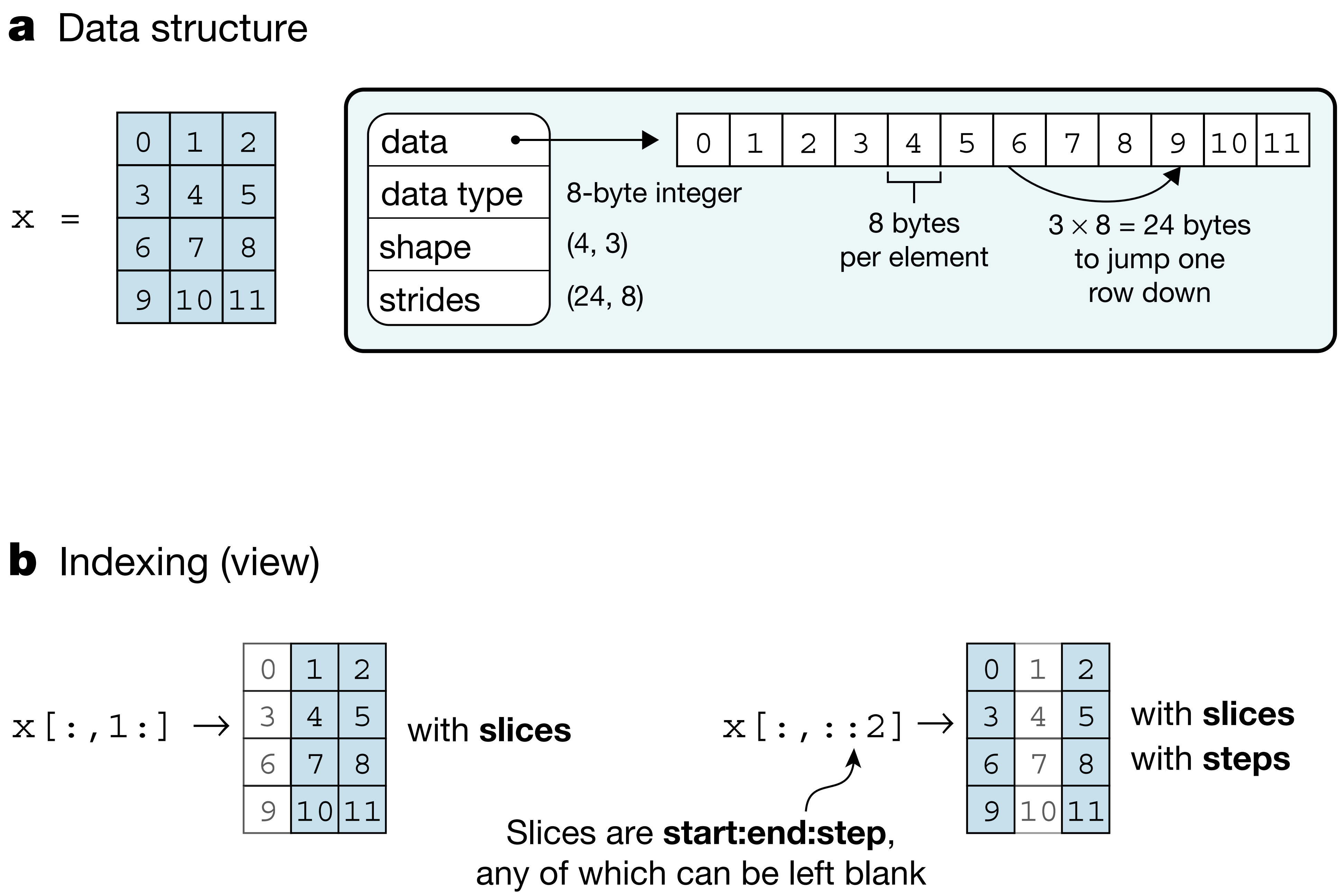

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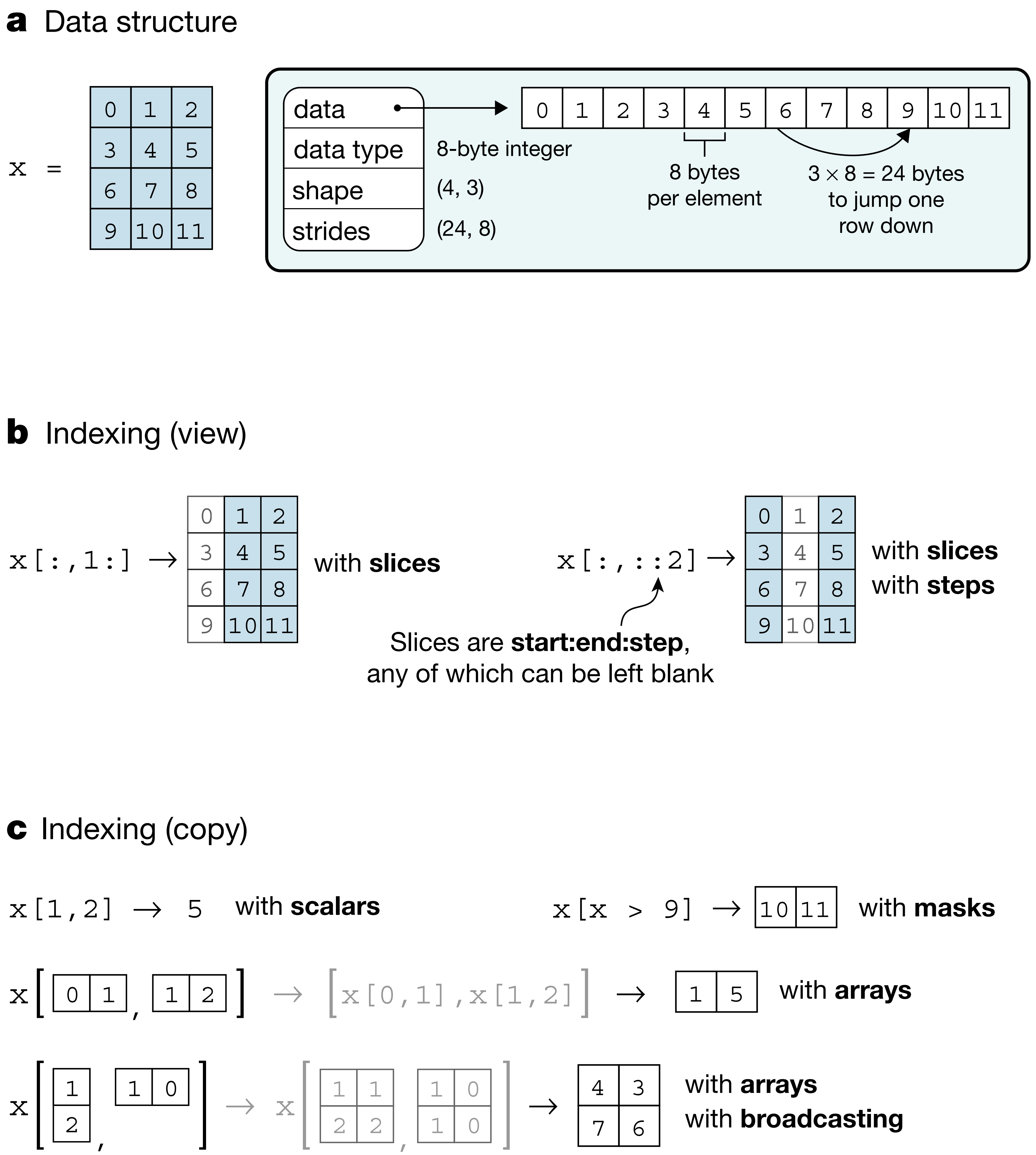

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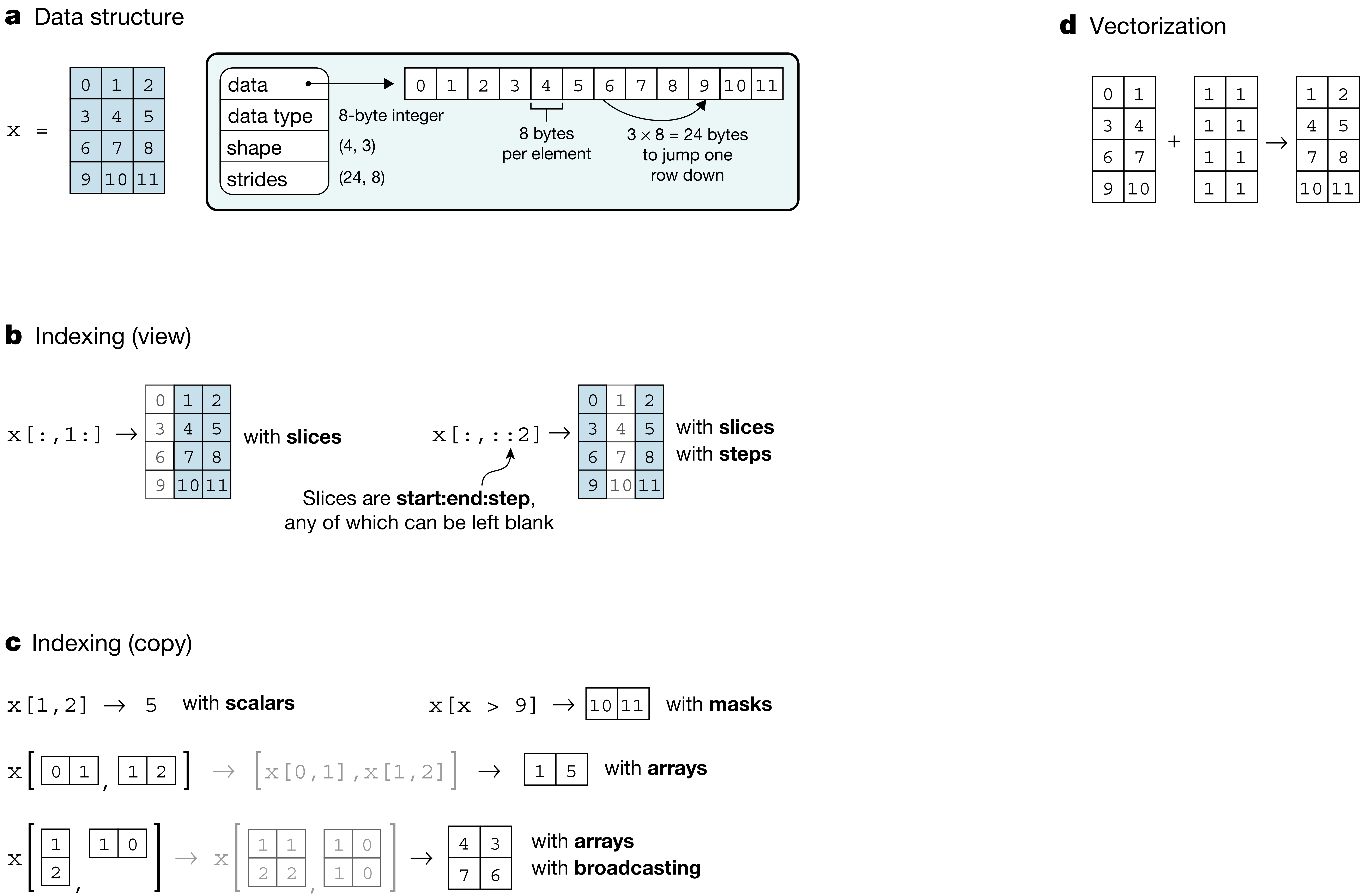

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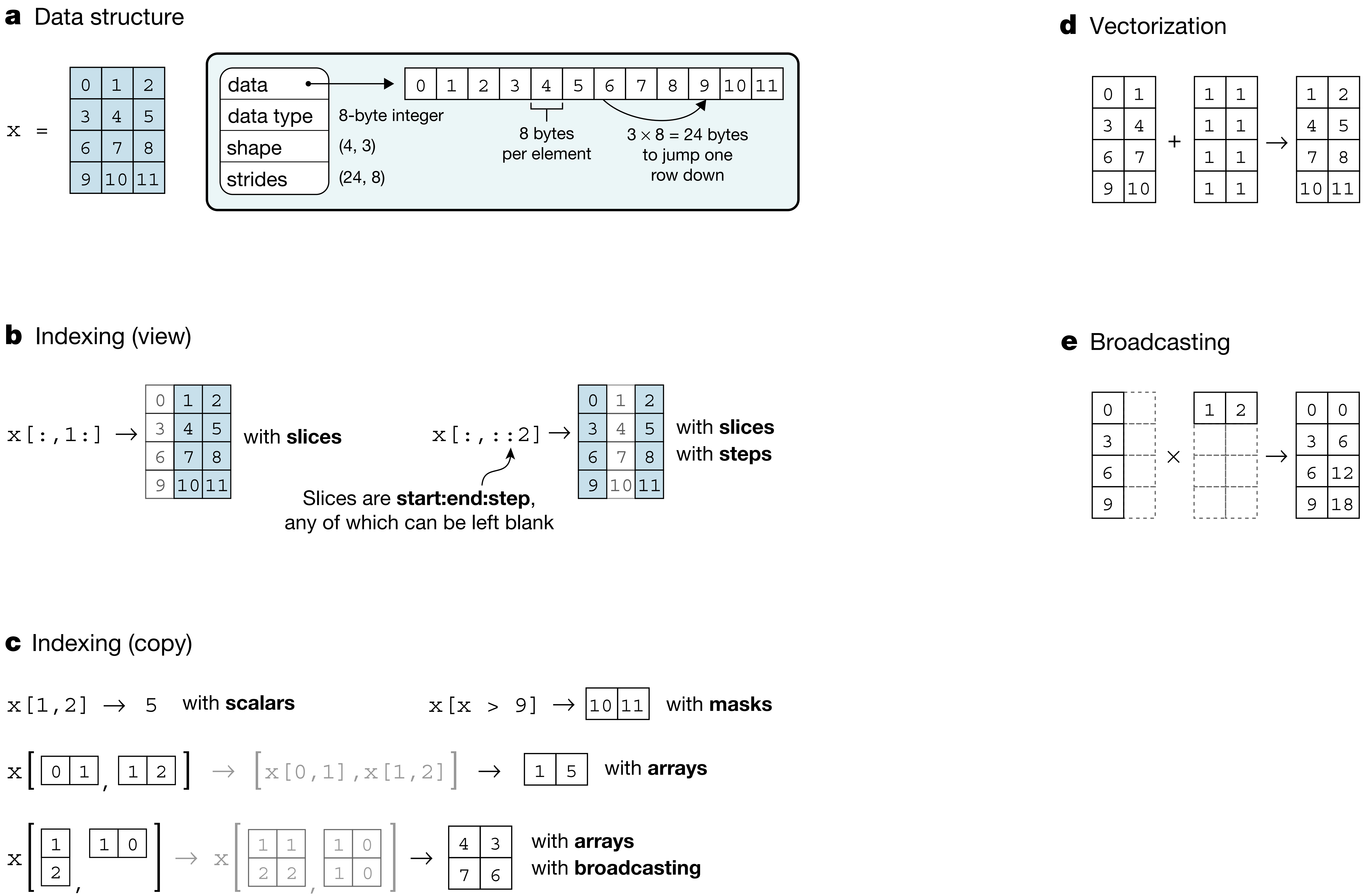

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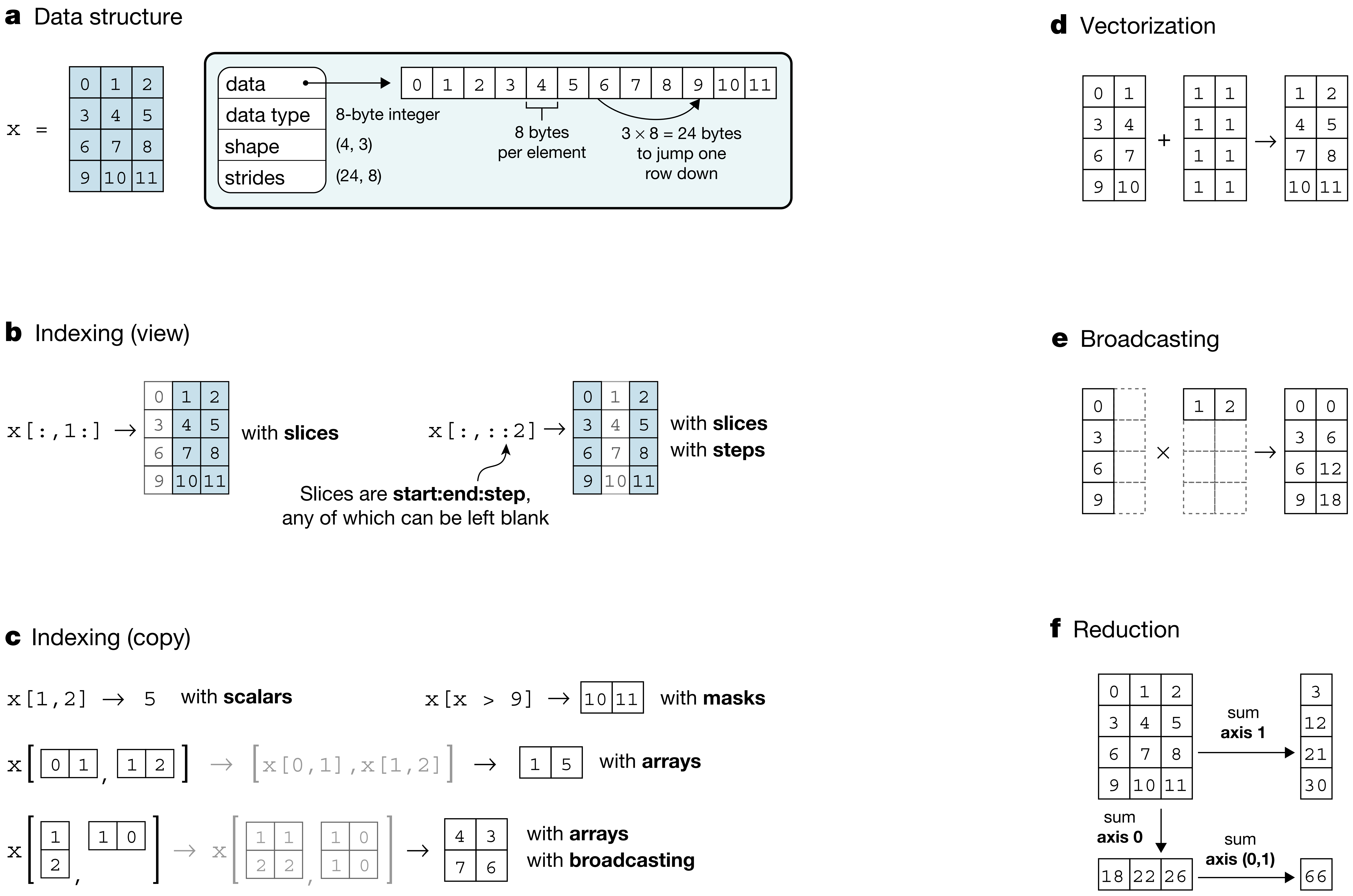

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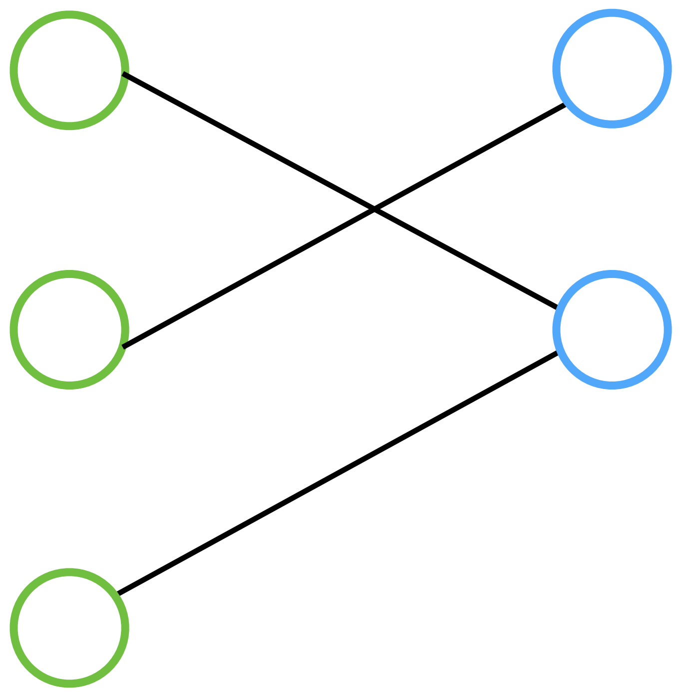
The SETL Language

Sets

Tuples

Functions



SETL Set Former Notation

Notation	Example
$\{x \in s \mid C(x)\}$	

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$\{x \in s \mid C(x)\}$	$\{x \in \{1,5,10,32\} \mid x \textbf{ lt } 10\} \rightarrow \{1,5\}$

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$[+ : x \in s_1, y \in s_2]\{ < x, y > \}$	$[+ : x \in \{1,2\}, y \in \{a, b\}]\{ < x, y > \} \rightarrow$ $\{ < 1,a > , < 1,b > , < 2,a > , < 2,b > \}$

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$\{e(x), x \in s \mid C(x)\}$	$\{i * i, i \in \{1{,}3{,}5\}\} \rightarrow \{1{,}9{,}25\}$
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$[\text{op} : x \in s \mid C(x)]e(x)$	$[+ : x \in \{1{,}2{,}3\}](x * x) \rightarrow 14$
$\forall x \in s \mid C(x)$	$\forall x \in 1{,}2{,}4 \mid (x//2) \textbf{ eq } 1 \rightarrow \textbf{f}$
$[+ : x \in s_1, y \in s_2]\{ < x, y > \}$	$[+ : x \in \{1{,}2\}, y \in \{a, b\}]\{ < x, y > \} \rightarrow$ $\{ < 1{,}a > , < 1{,}b > , < 2{,}a > , < 2{,}b > \}$

Standard set operations like union, intersection, and set difference are also supported

SETL Table Functions

$$f = \{ < 1,1 > , < 2,4 > , < 3,9 > \}$$

SETL Table Functions

$$f = \{ < 1,1 > , < 2,4 > , < 3,9 > \}$$

$$f(2) \rightarrow 4$$

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$$f \ + \{ < 2,5 > \} \rightarrow \{ < 1,1 > , < 2,5 > , < 3,9 > \}$$

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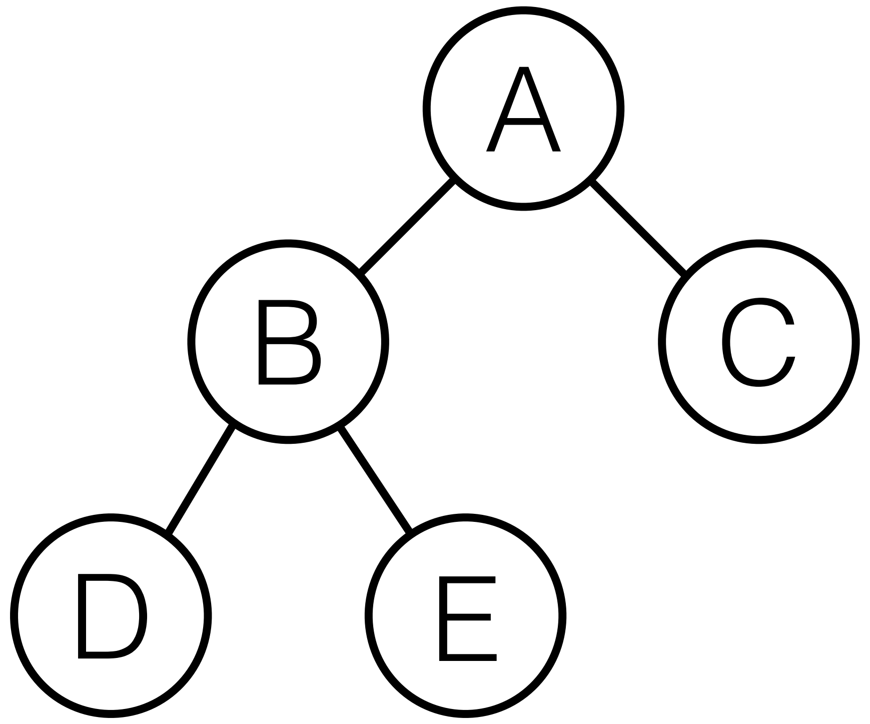

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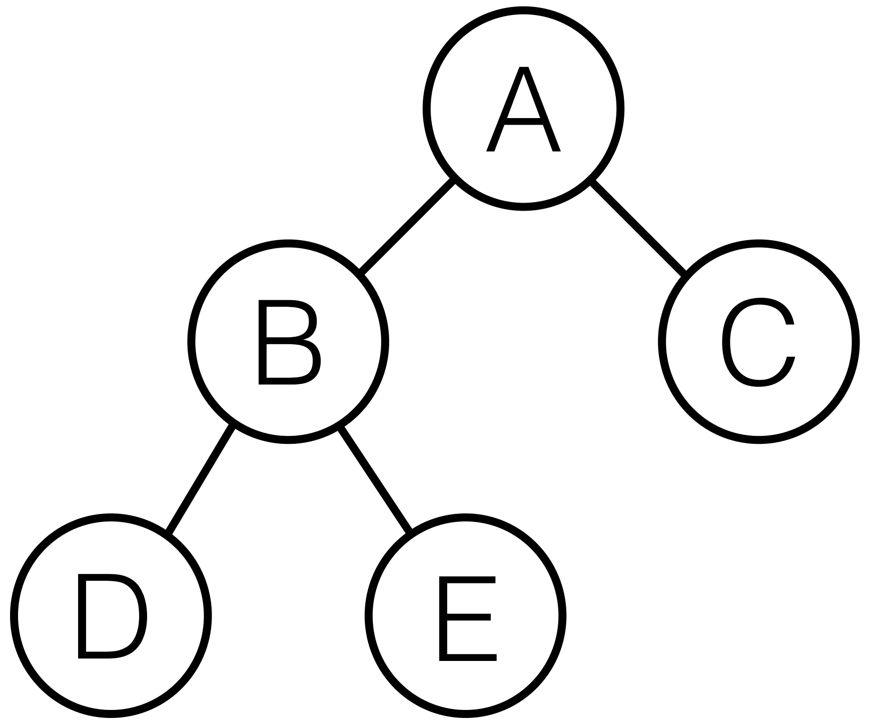

left $= \{ < A, B > , < B, D > \}$

right $= \{ < A, C > , < B, E > \}$

Relational Algebra

employees

name	id	department
Harry	3245	CS
Sally	7264	EE
George	1379	CS
Mary	1733	ME
Rita	2357	CS

departments

department	manager
CS	George
EE	Mary

Relational Algebra

employees

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department	manager
CS	George
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Projection (Π)

$\Pi_{name,department}$ employees

name	department
Harry	CS
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Relational Algebra

employees

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$$\Pi_{name,department}\ \text{employees}$$

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George	CS
Mary	ME
Rita	CS

Selection (σ)

$$\sigma_{department=CS}\ \text{employees}$$

Namename	id	department
Harry	3245	CS
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Rita	2357	CS

Relational Algebra

employees

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Selection (σ)

$\sigma_{department=CS}$ employees

Namename	id	department
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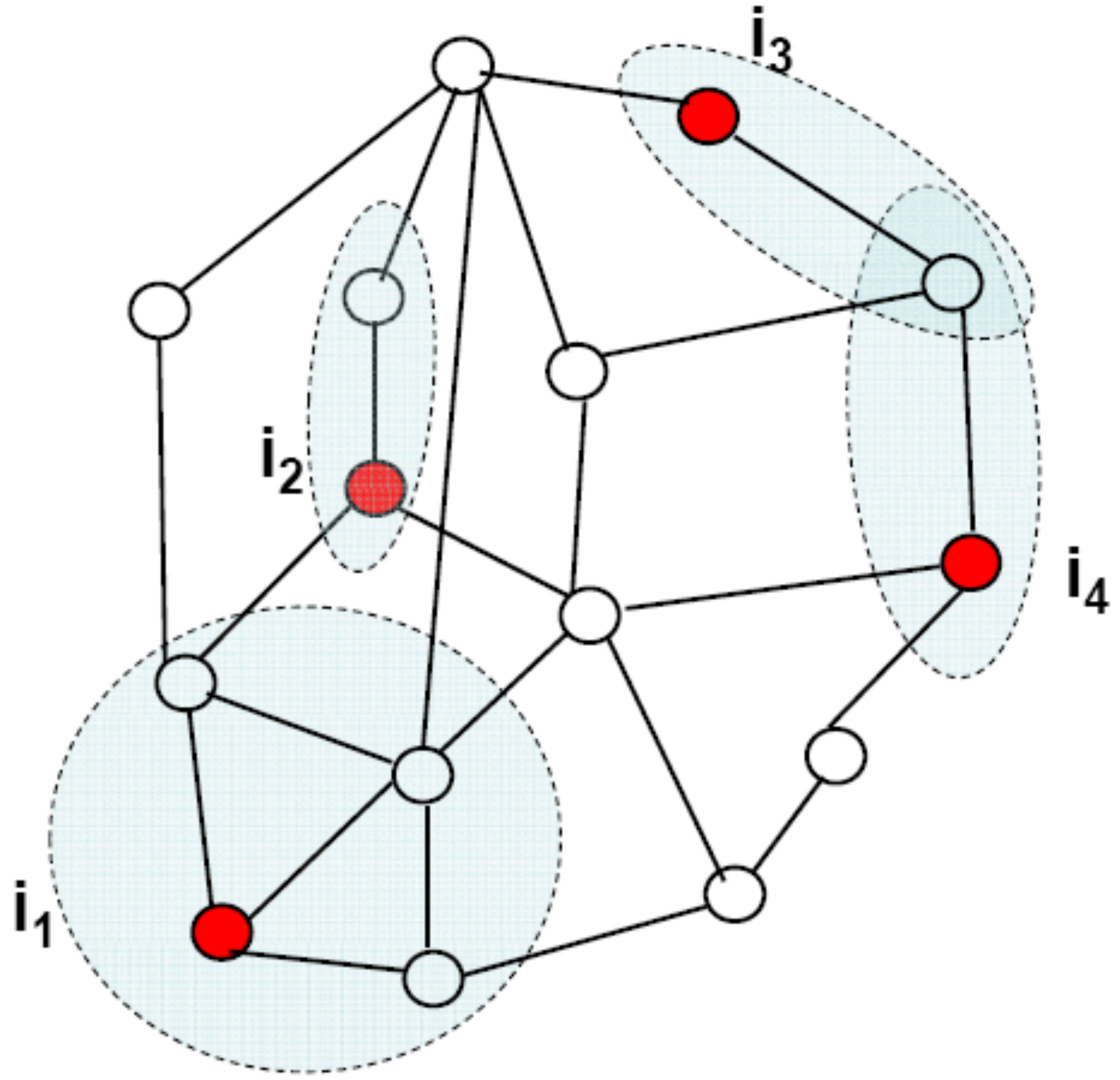
Natural join ($\bowtie$)

employees $\bowtie$ departments

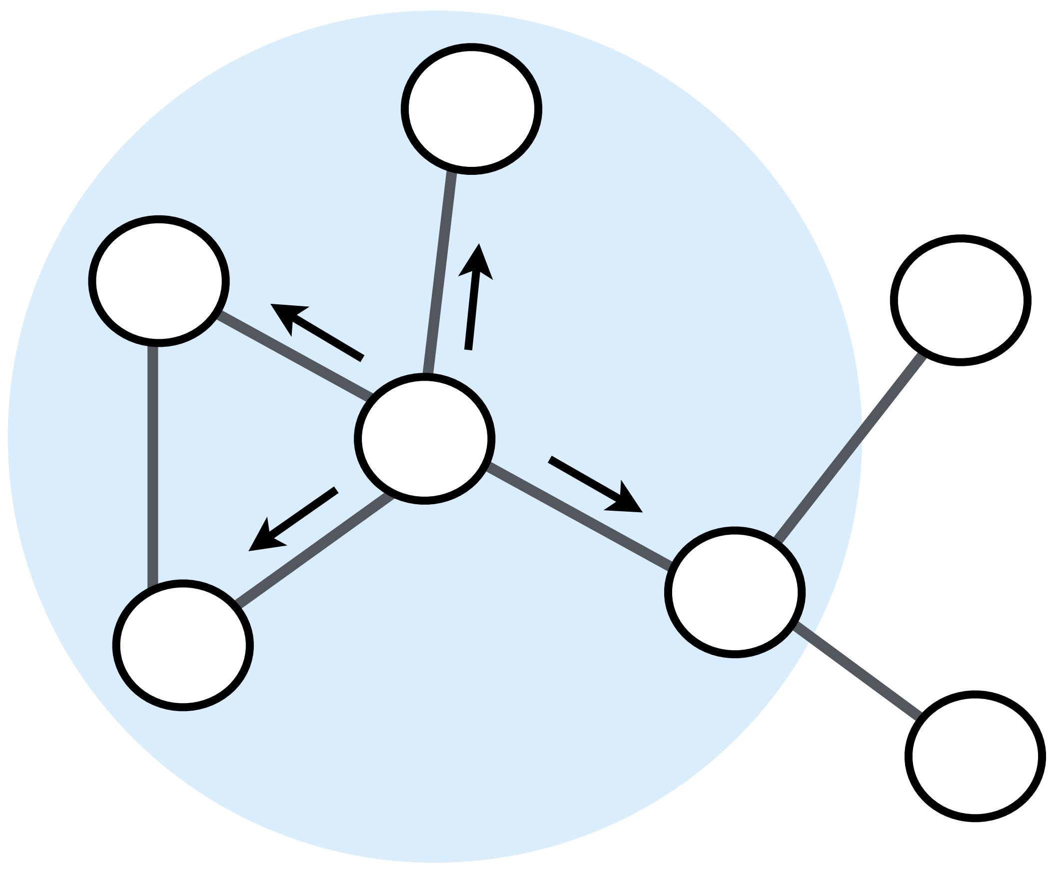
name	id	department	manager
Harry	3245	CS	George
Sally	7264	EE	Mary
George	1379	CS	George
Rita	2357	CS	George

Graph operations

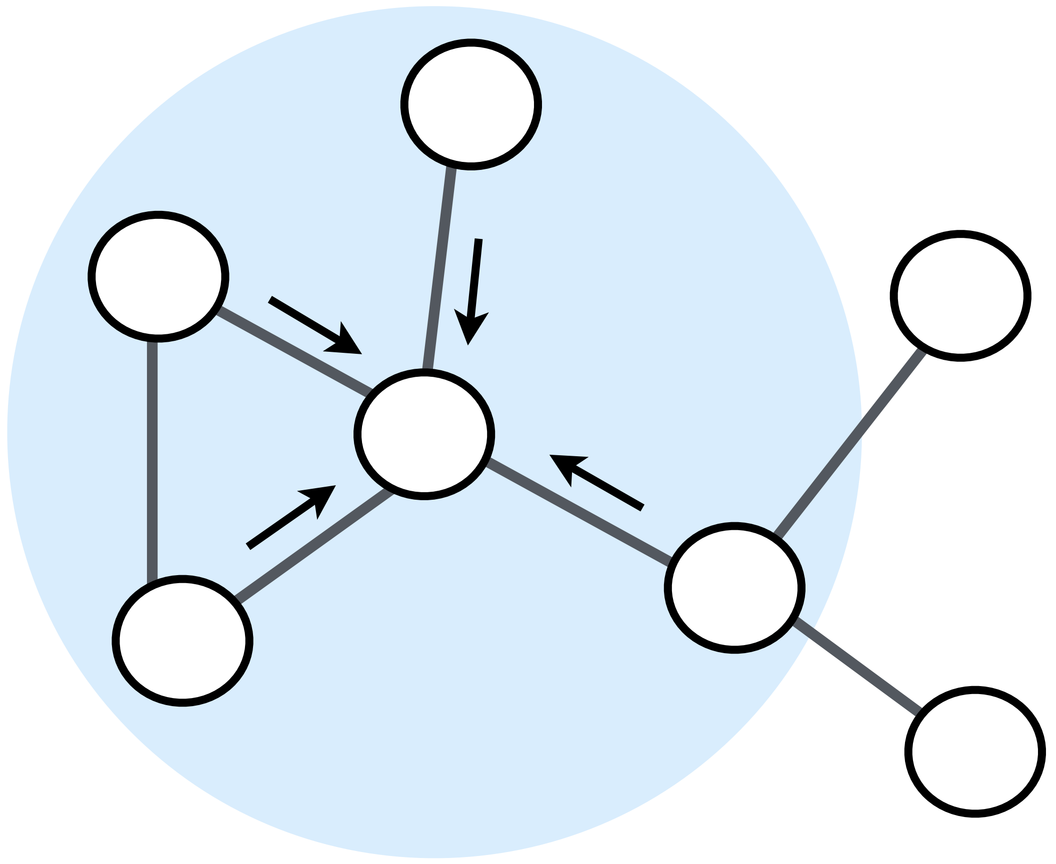
Simultaneous operations on different parts of the graph



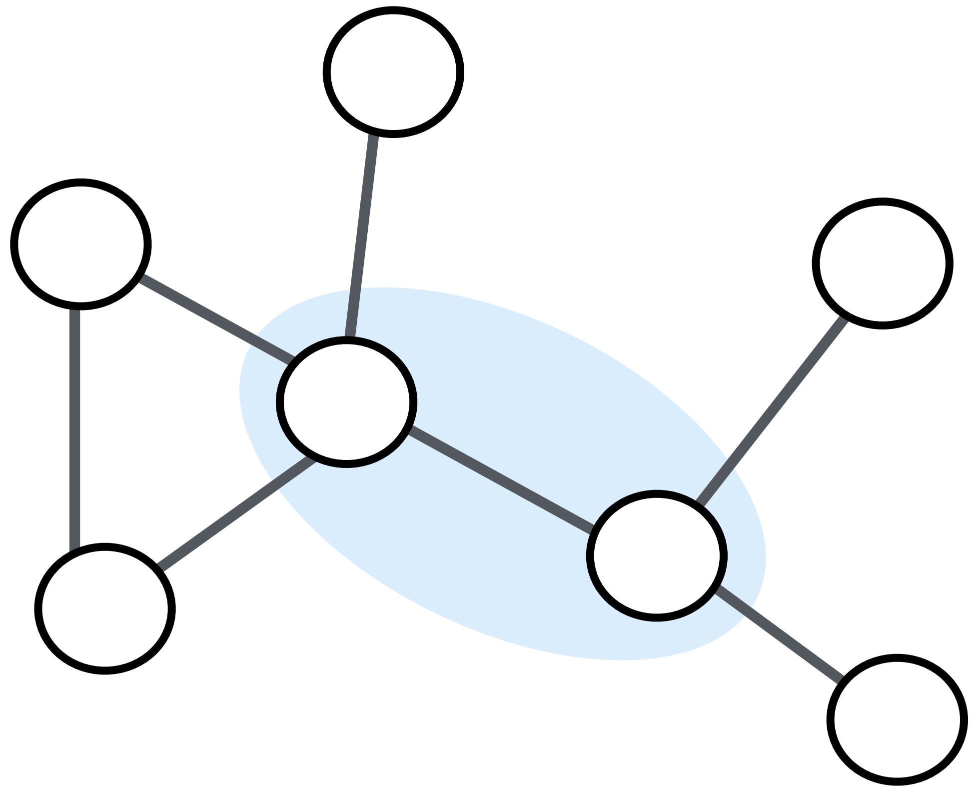
push



pull



edge functions



Left figure from Tao of Parallelism in Algorithms (Pingali et al.)

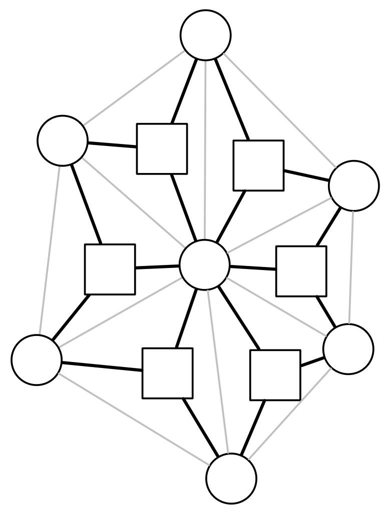
Relations, Graphs, and Algebra: No glove fits all

Relations

Names	City	Age
Peter	Boston	54
Mary	San Fransisco	35
Paul	New York	23
Adam	Seattle	84
Hilde	Boston	19
Bob	Chicago	76
Sam	Portland	32
Angela	Los Angeles	62

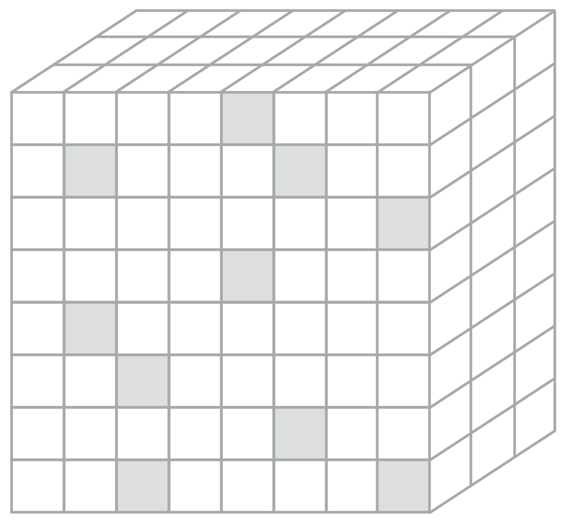
Ideal for combining data to form systems

Graphs



Ideal for local operations

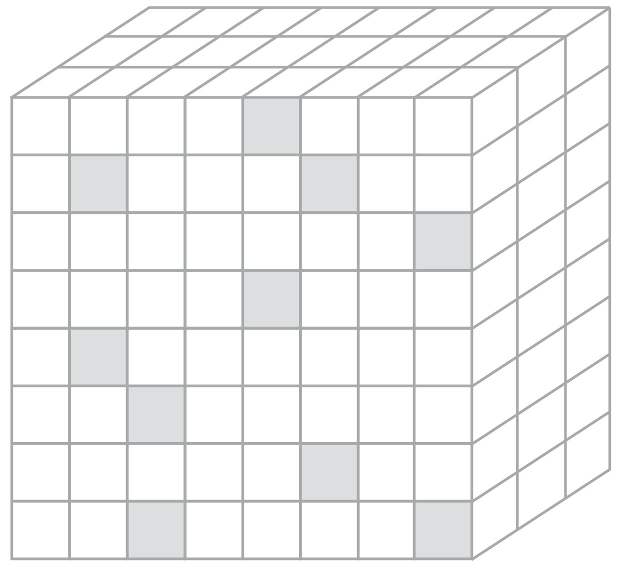
Tensors



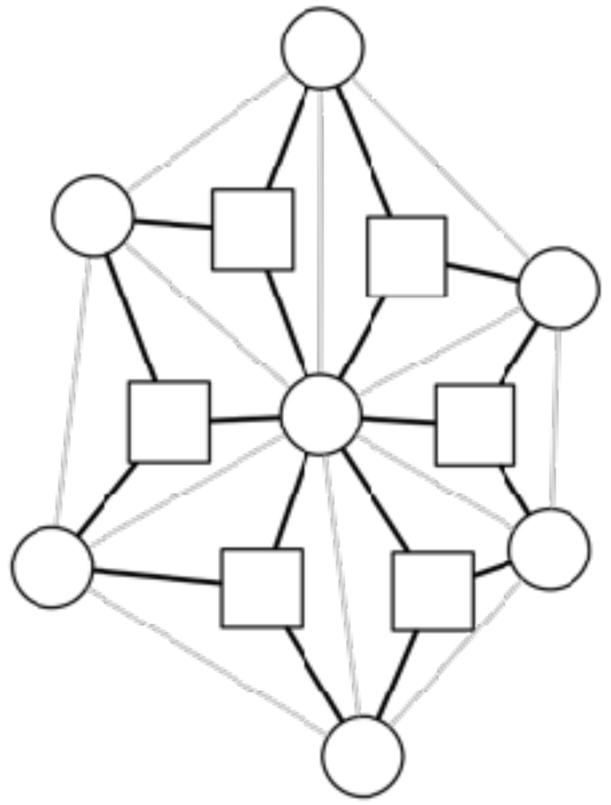
Ideal for global operations

It is critical to be able to compose languages and abstractions

Tensors



Graphs



Relations

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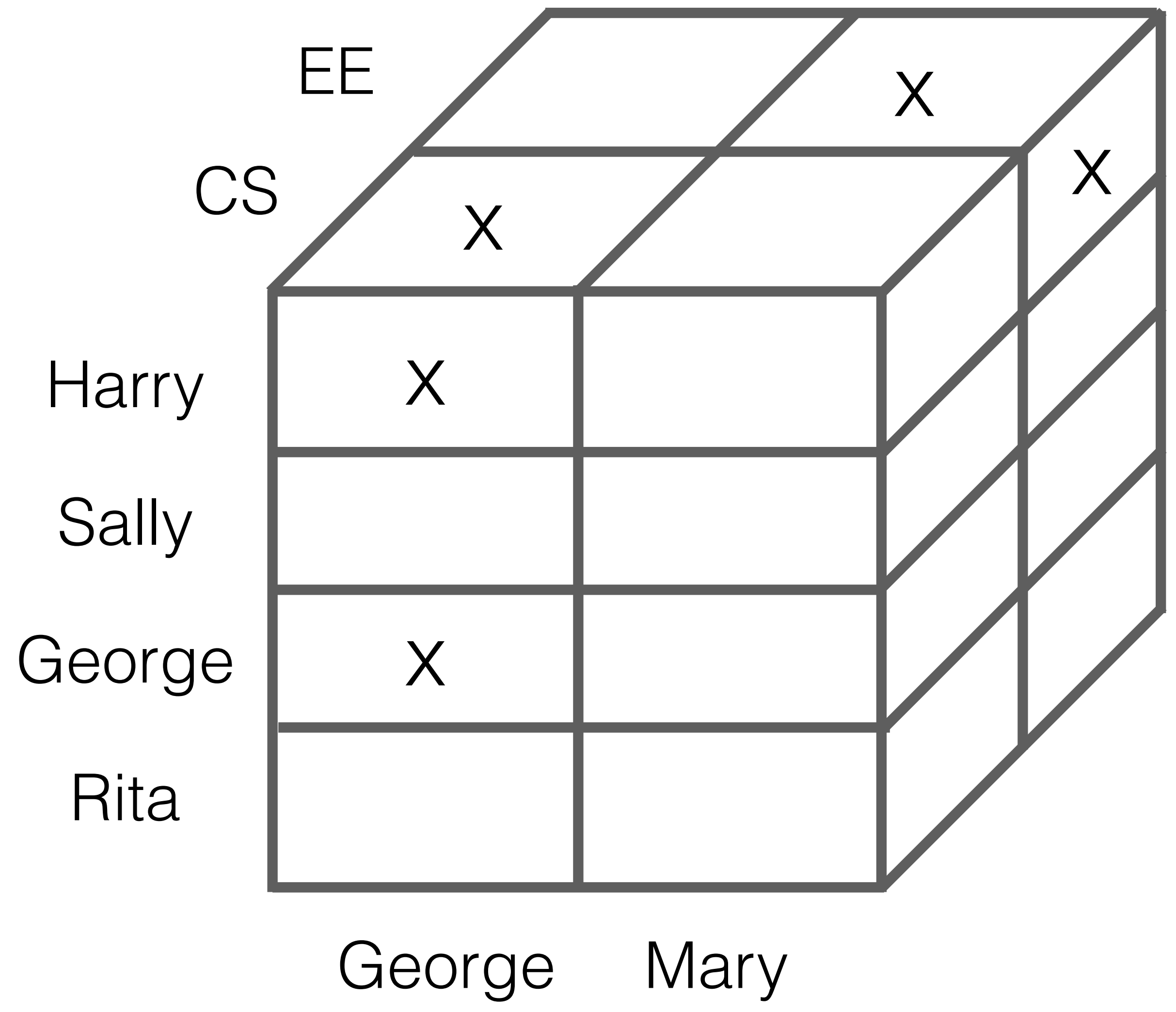
Example: Relations and Tensors

name	department	manager
Harry	CS	George
Sally	EE	Mary
George	CS	George
Rita	CS	George

Tensor Assembly

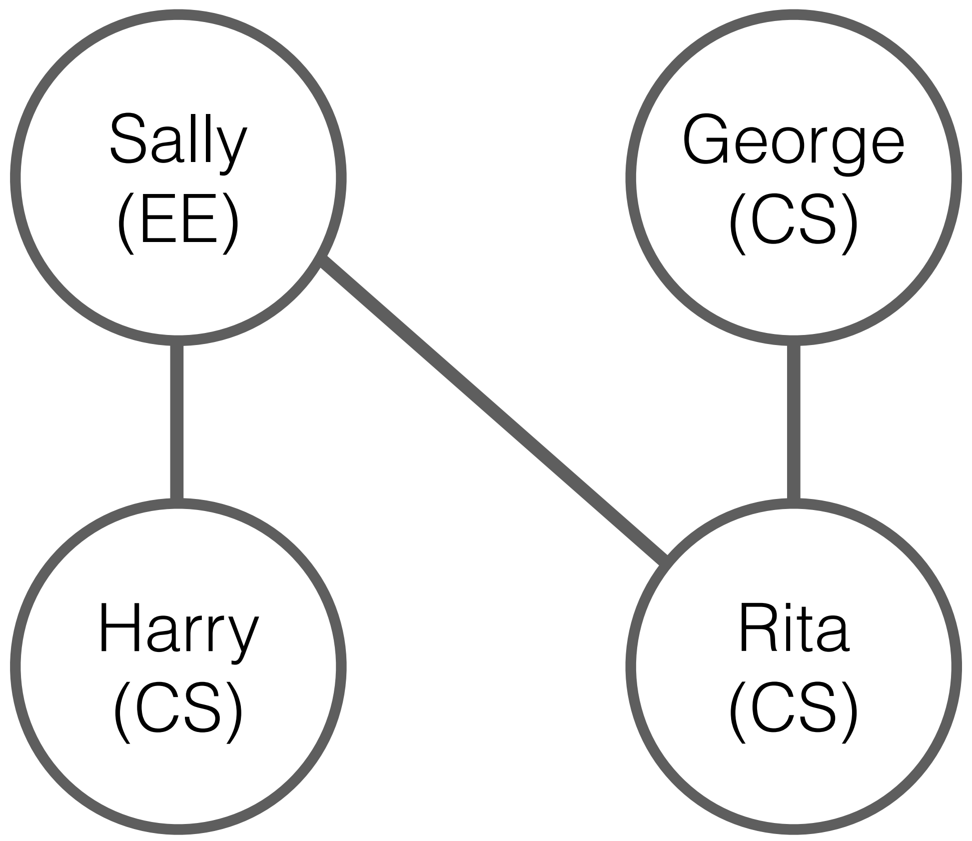


Example: Relations and Graphs

name	department
Harry	CS
Sally	EE
George	CS
Rita	CS

$\bowtie$

name1	name2
Harry	Sally
Sally	Harry
George	Rita
Rita	George
Sally	Rita
Rita	Sally



Example: Graphs and Tensors (Simit)

Dynamics Tetrahedral Neo-Hookean FEM Simulation

Example: Graphs and Tensors (Simit)

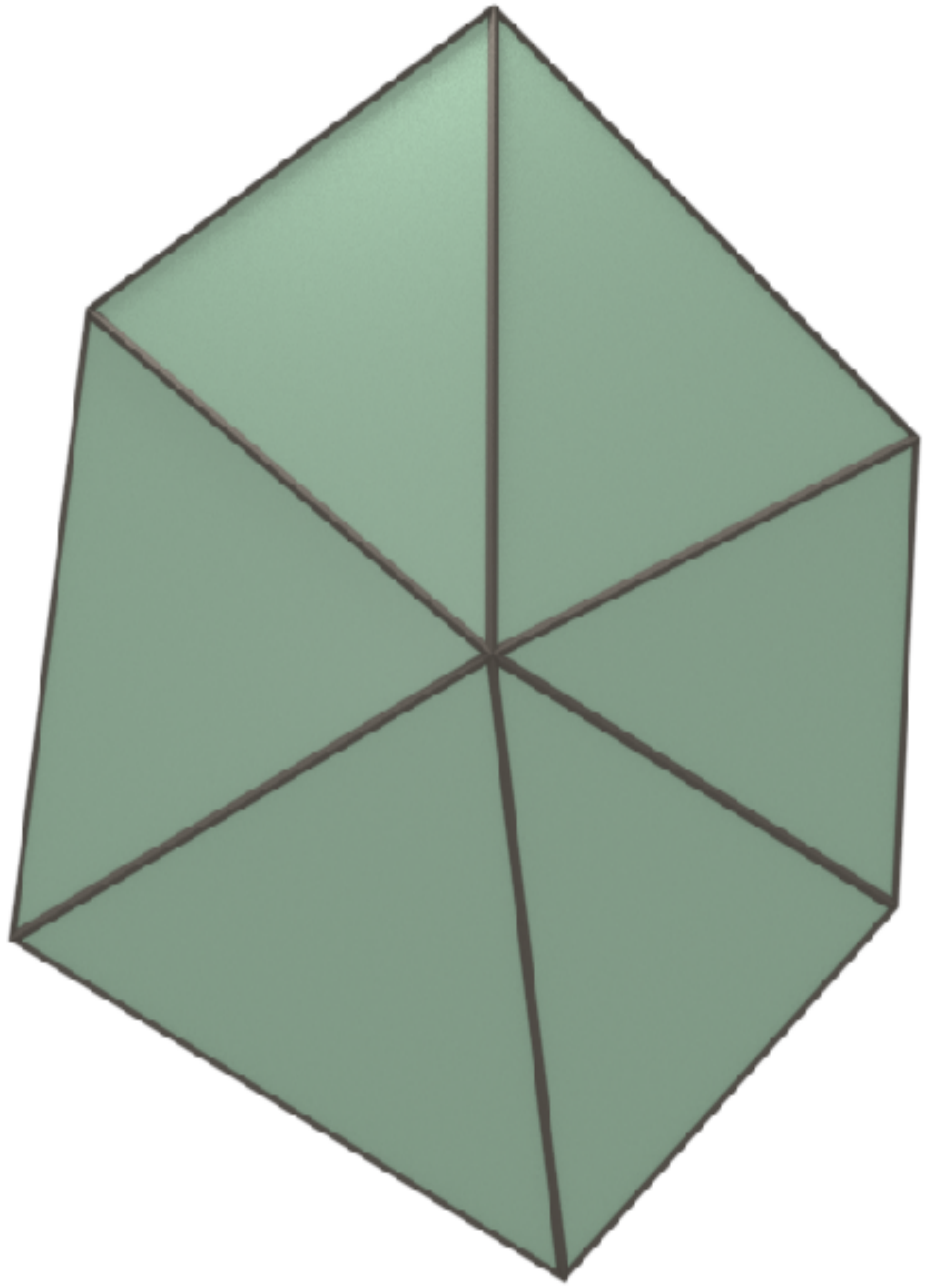
Dynamics Tetrahedral Neo-Hookean FEM Simulation

Example: Graphs and Tensors (Simit)

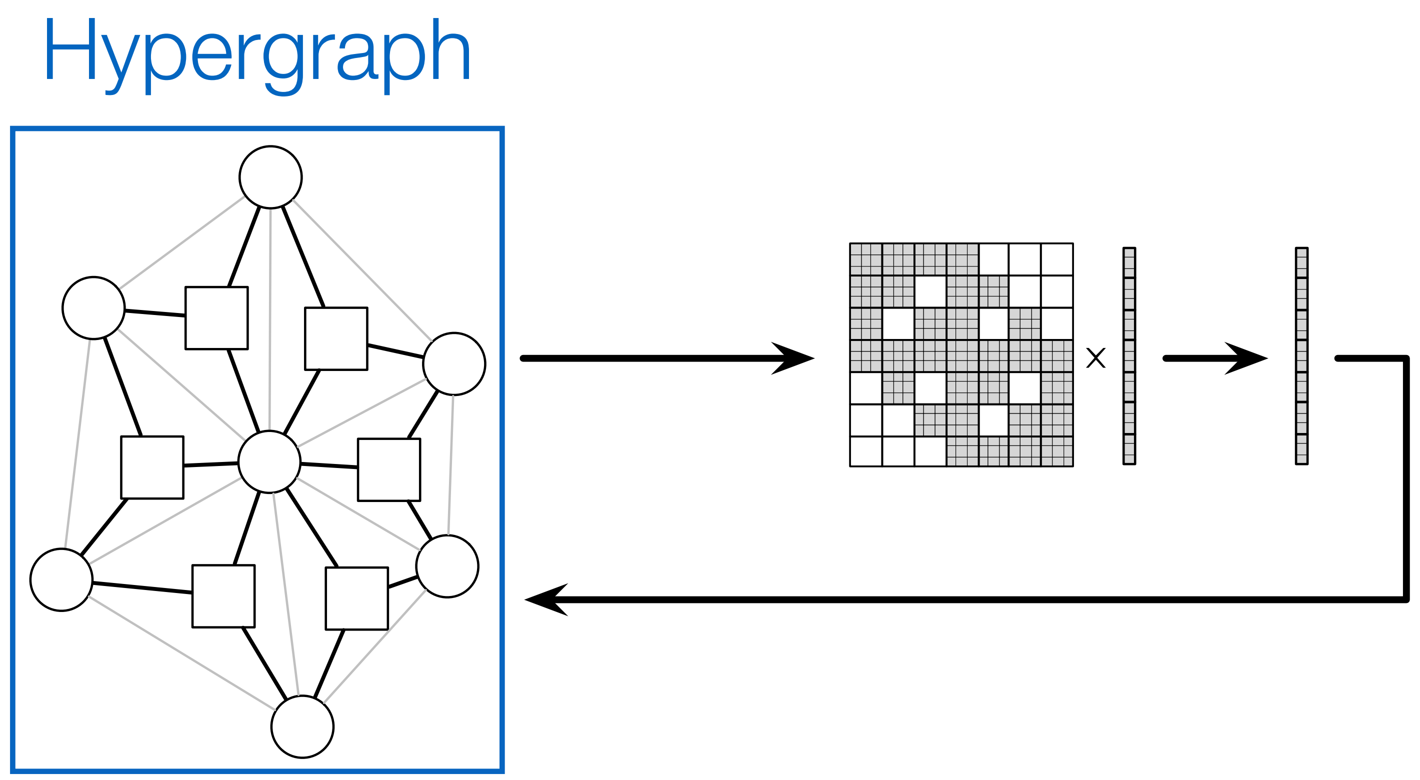
Statics Tetrahedral Neo-Hookean FEM Simulation

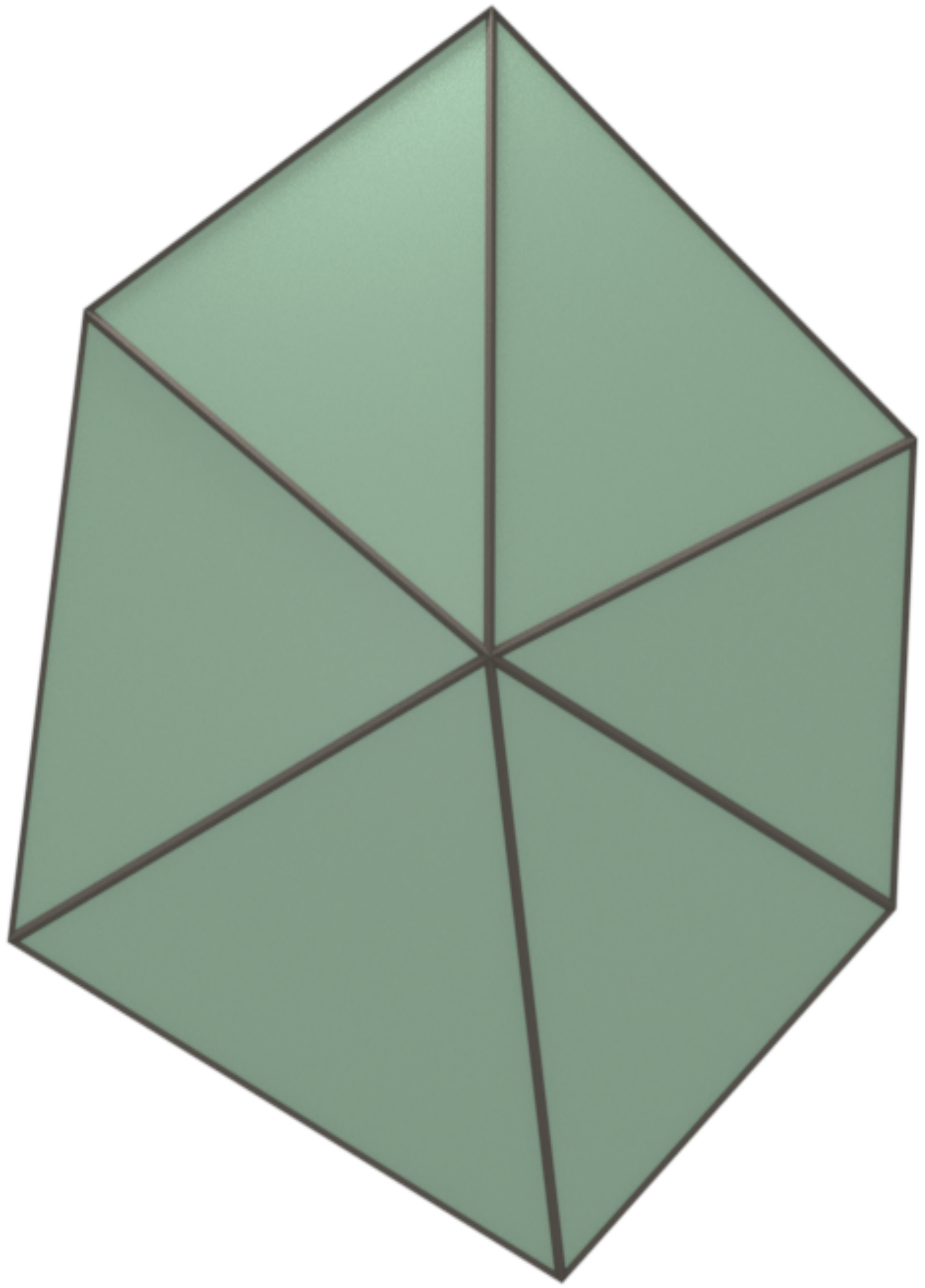
Example: Graphs and Tensors (Simit)

Statics Triangular Neo-Hookean FEM Simulation

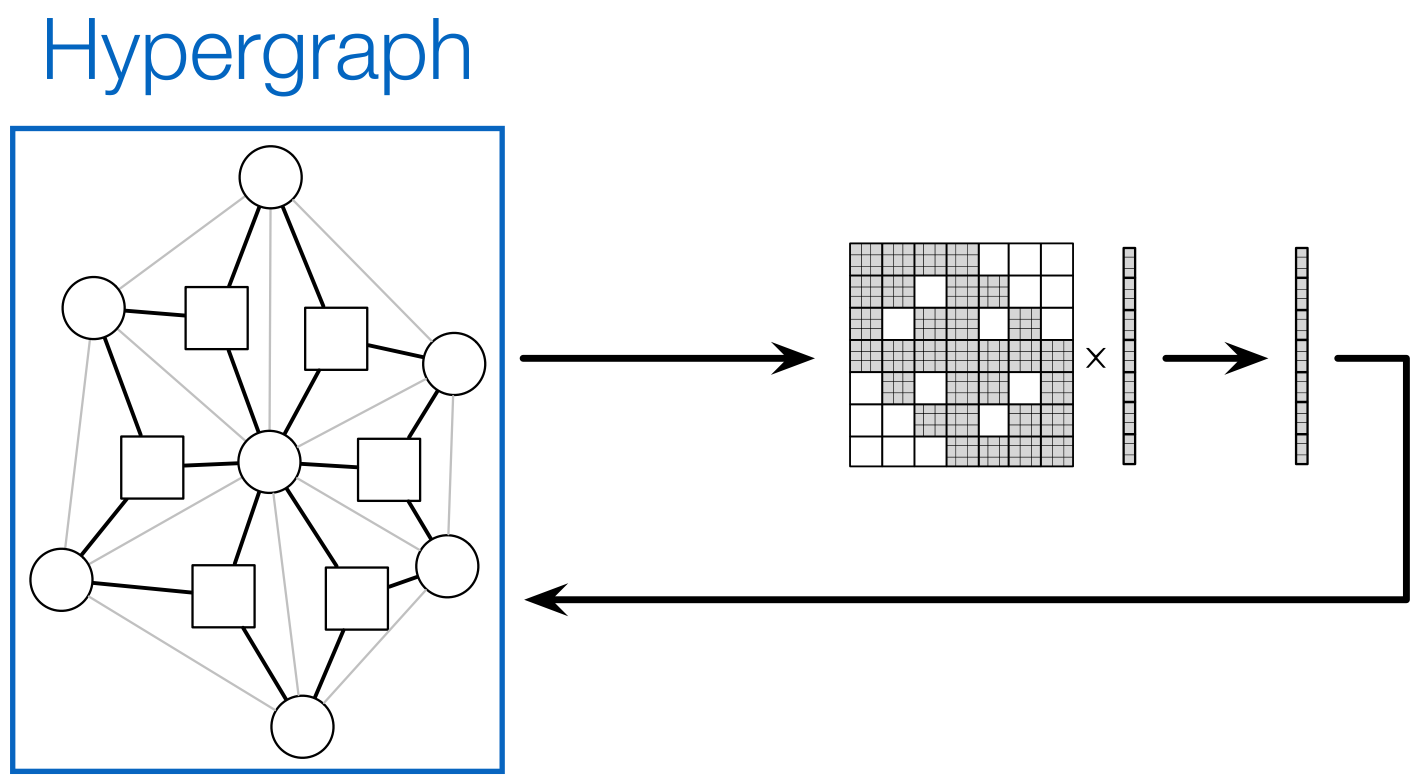


Statics Triangular Neo-Hookean FEM Simulation





Statics Triangular Neo-Hookean FEM Simulation




```
element Vertex
  x  : vector[3](float);   % position
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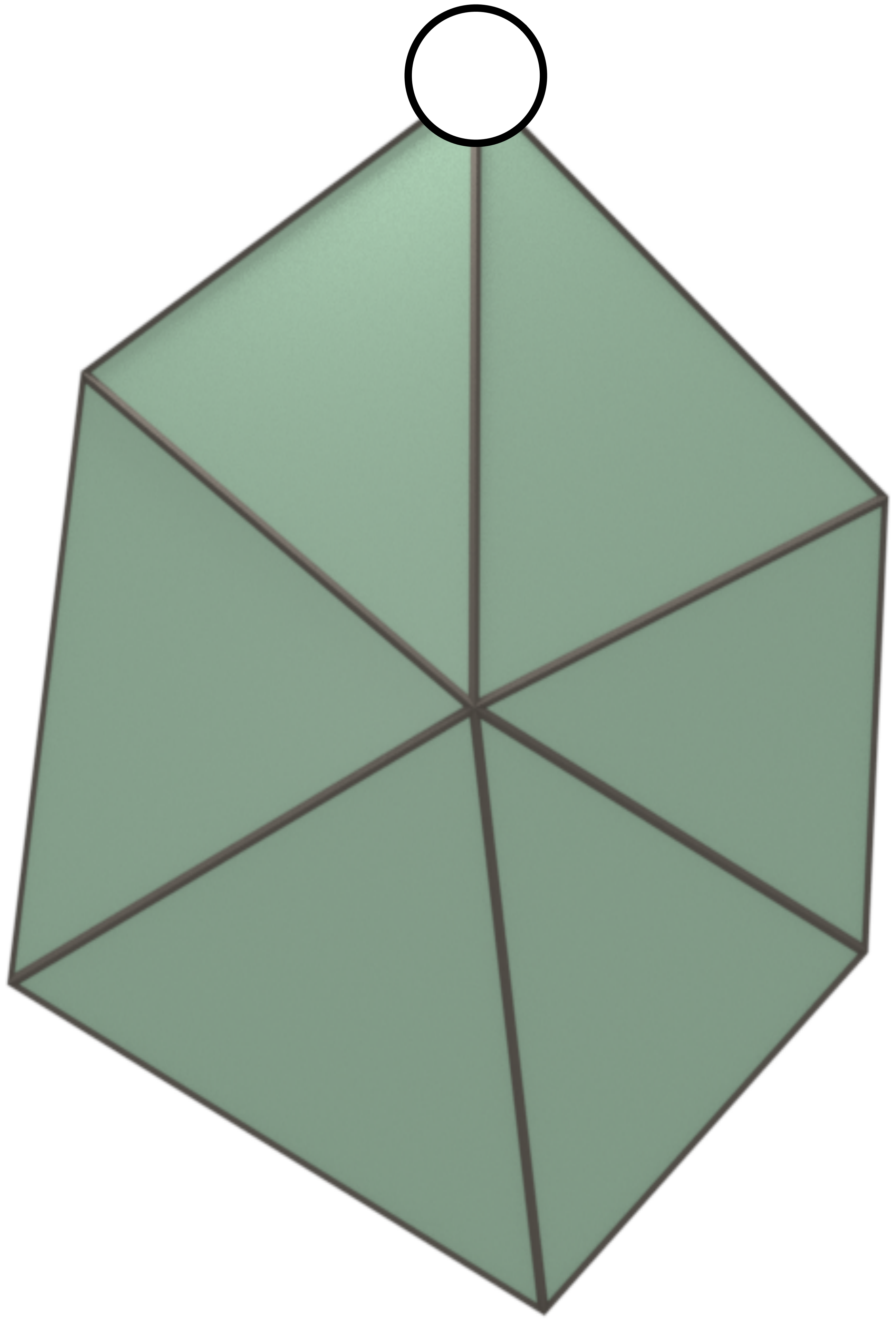
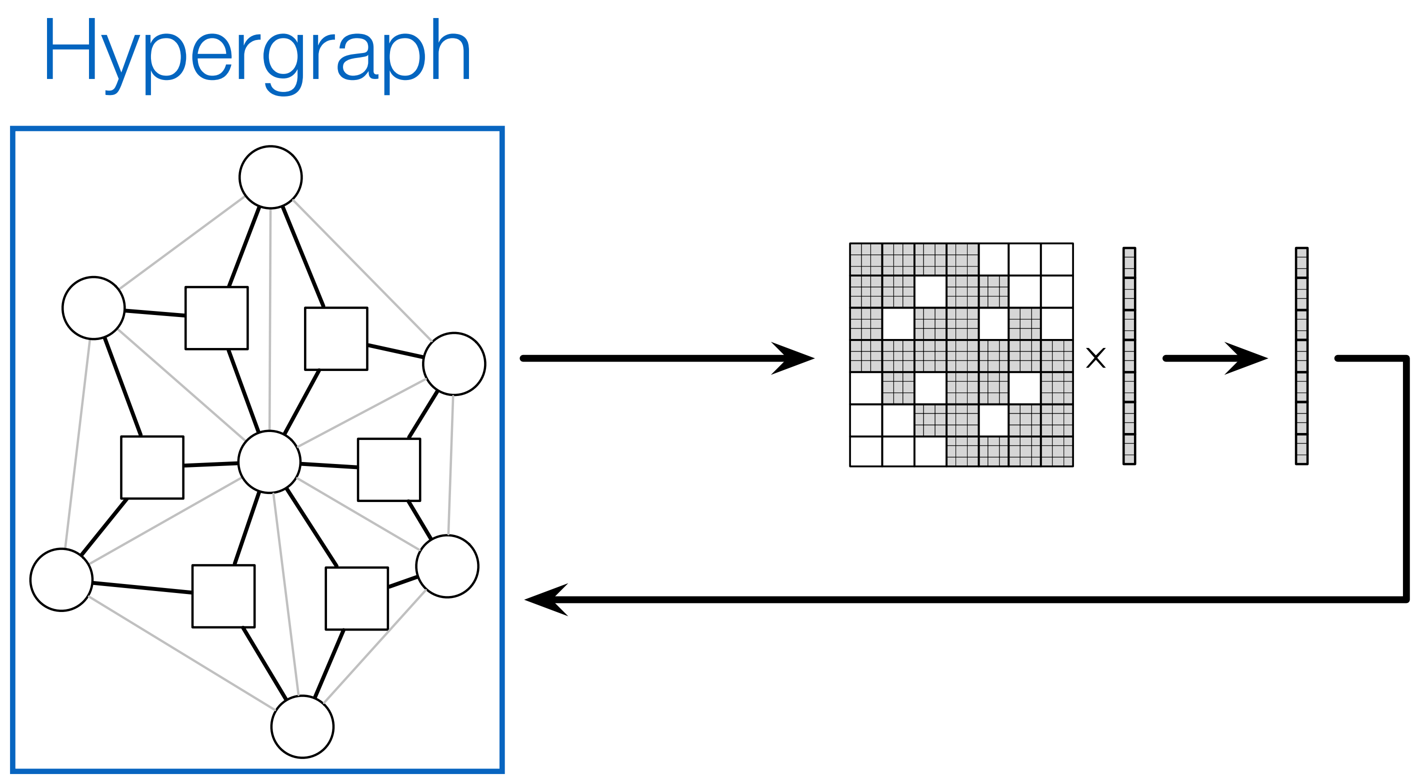
  fe : vector[3](float);   % external force
end
```



Statics Triangular Neo-Hookean FEM Simulation

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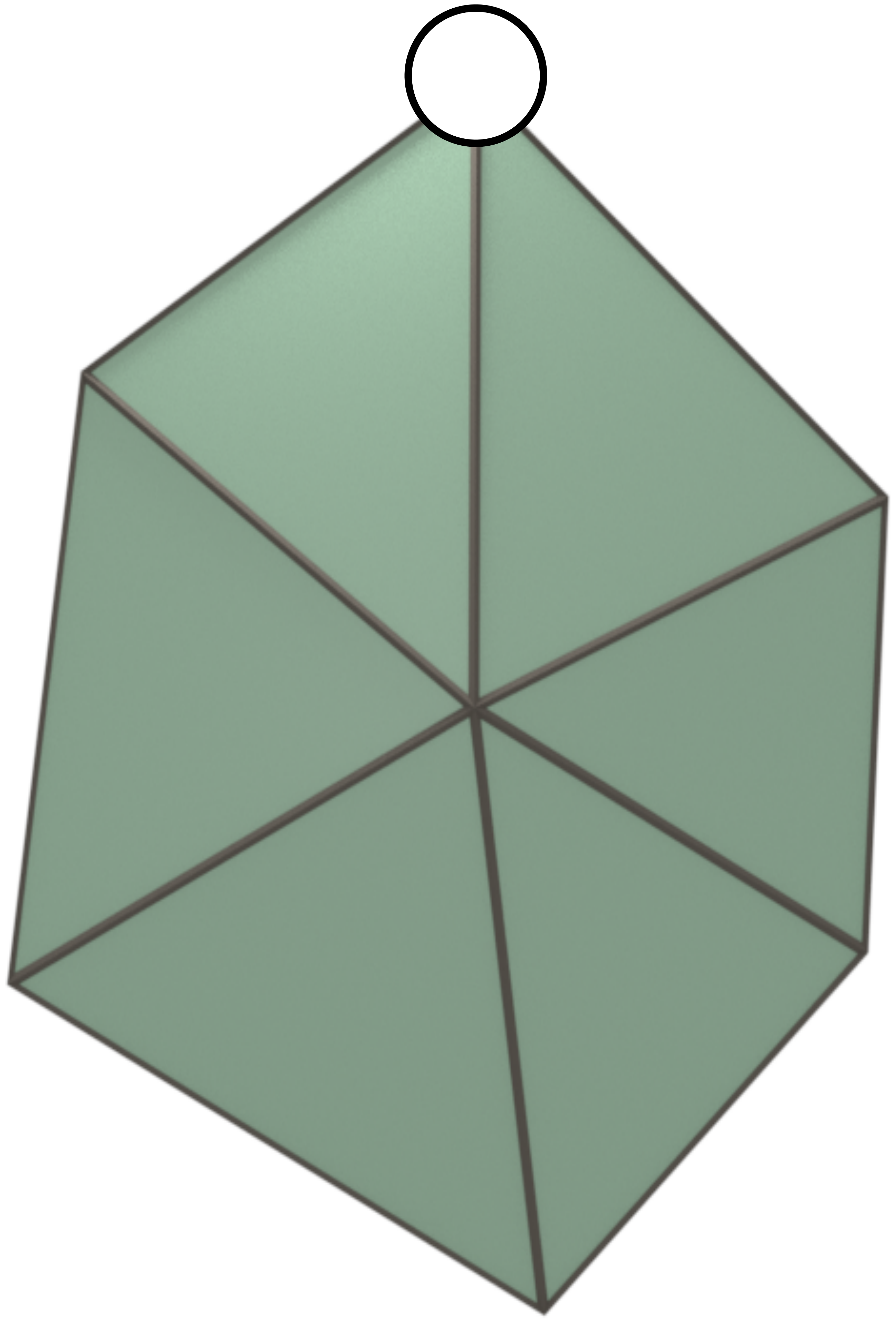

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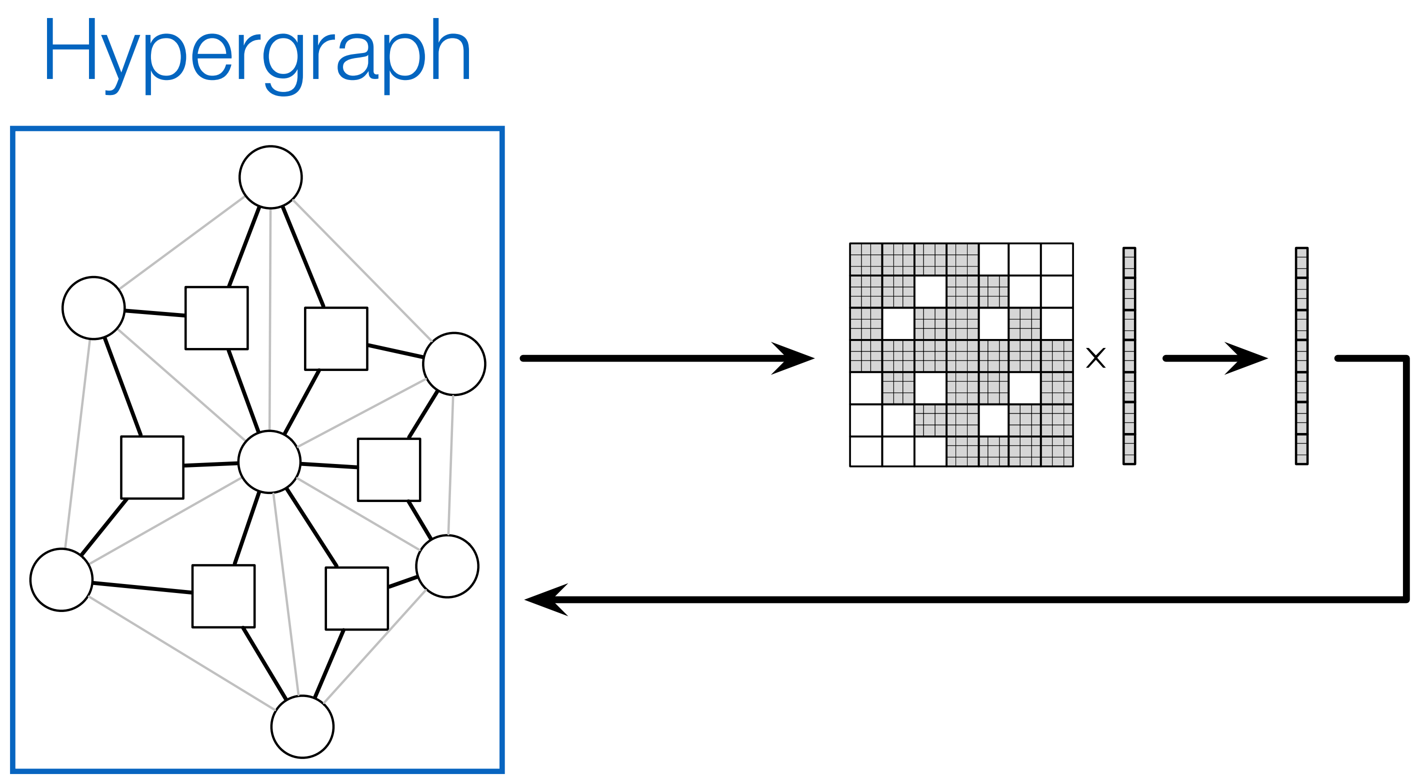


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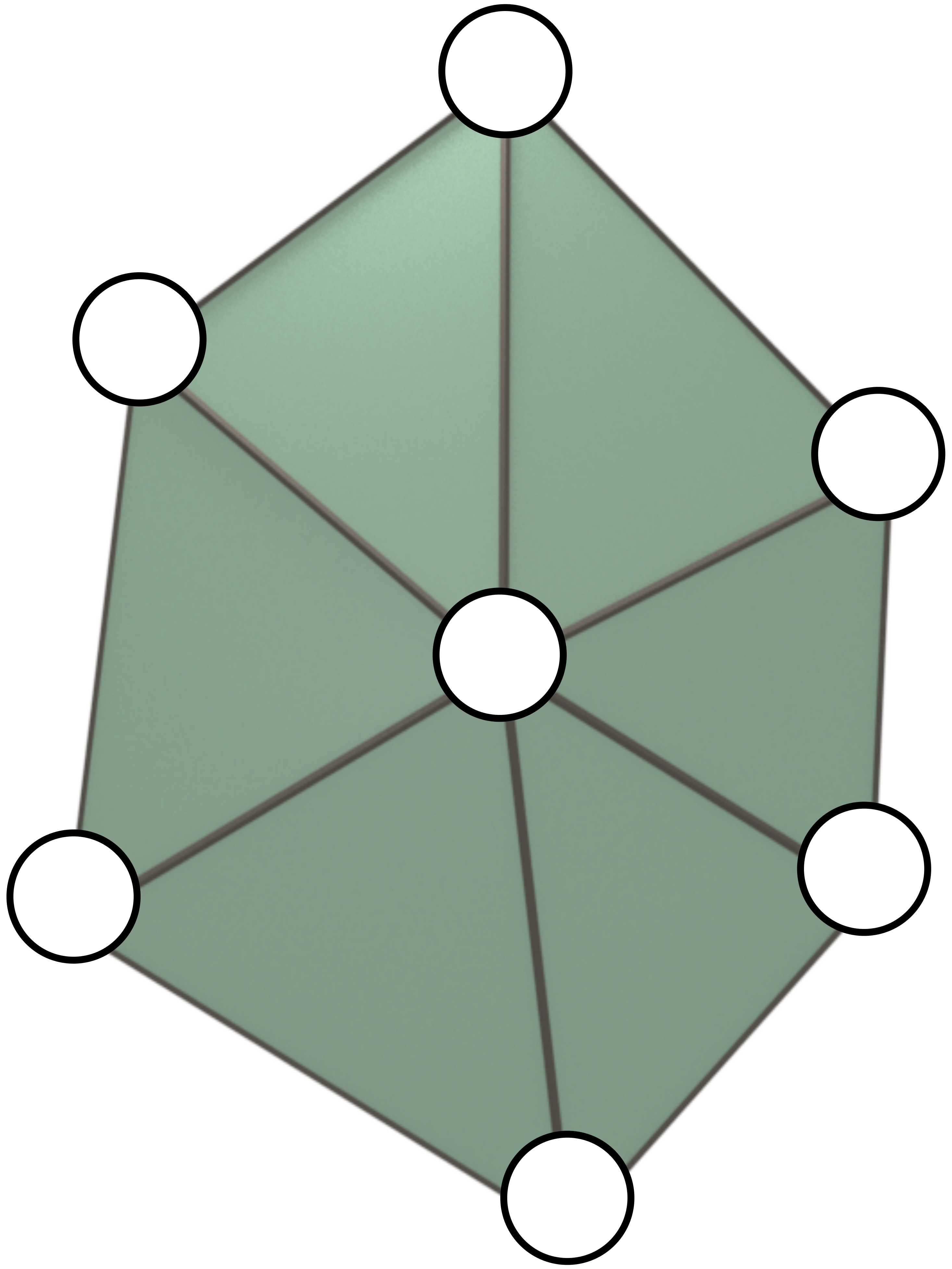
```



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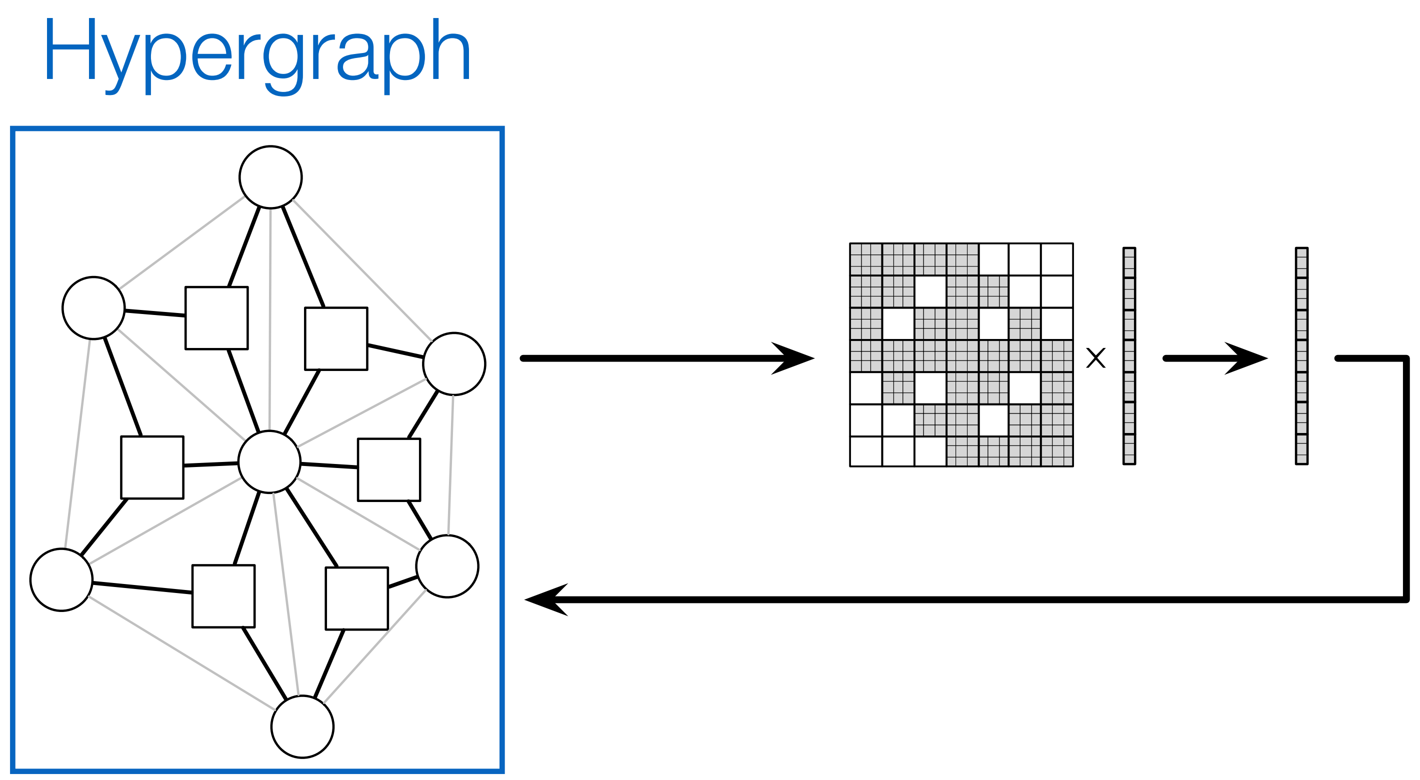


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```

```
% graph vertices and triangle hyperedges
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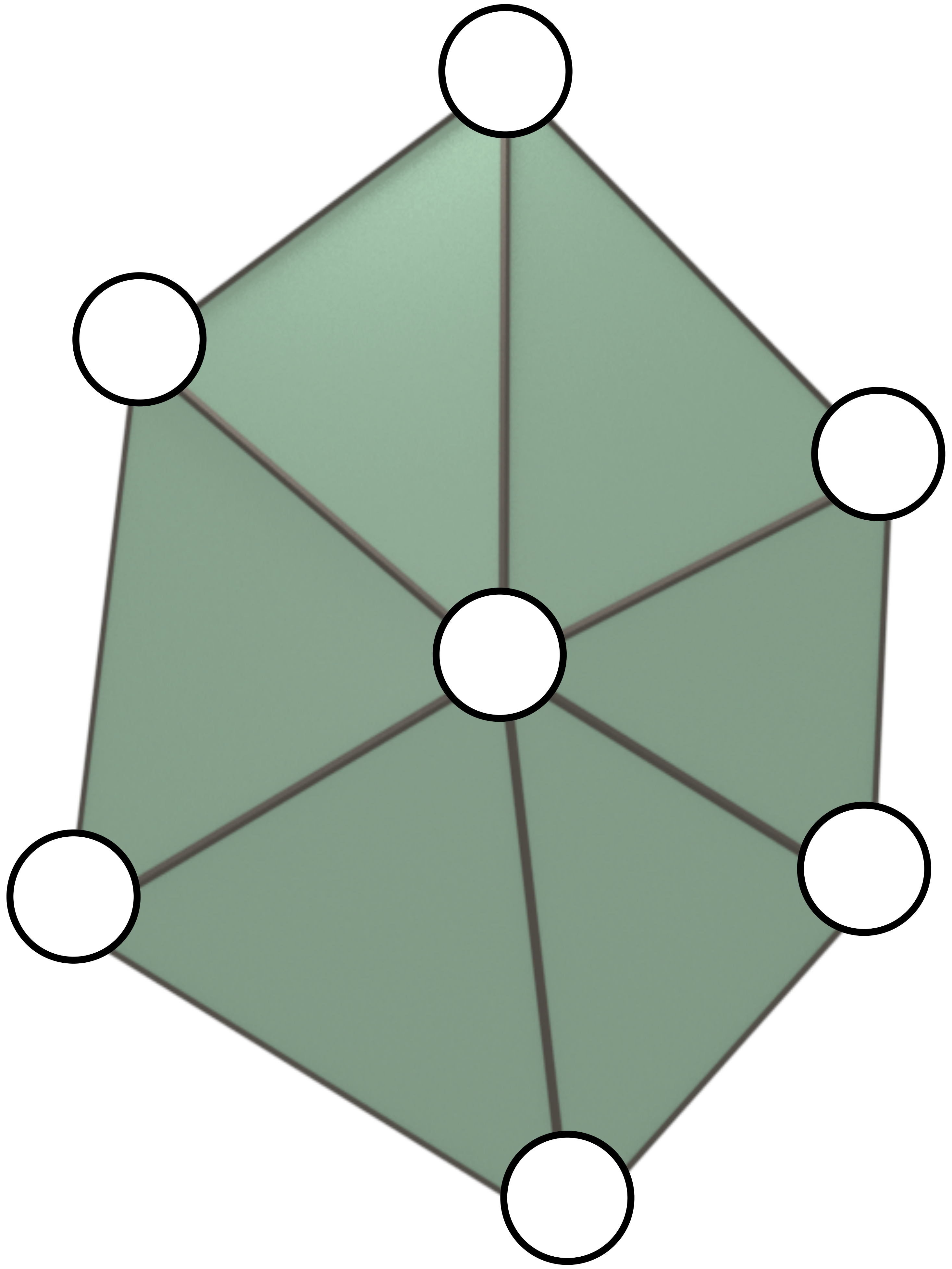
```



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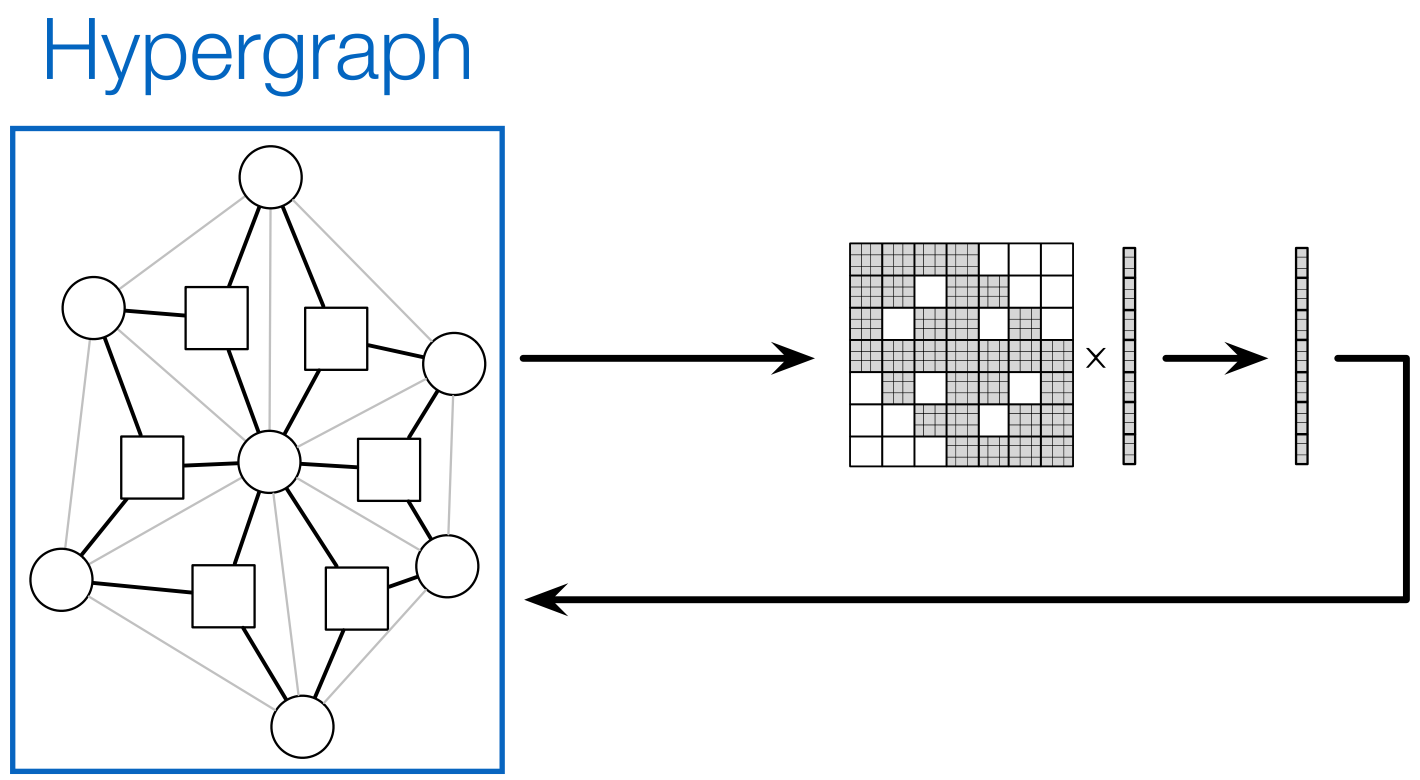
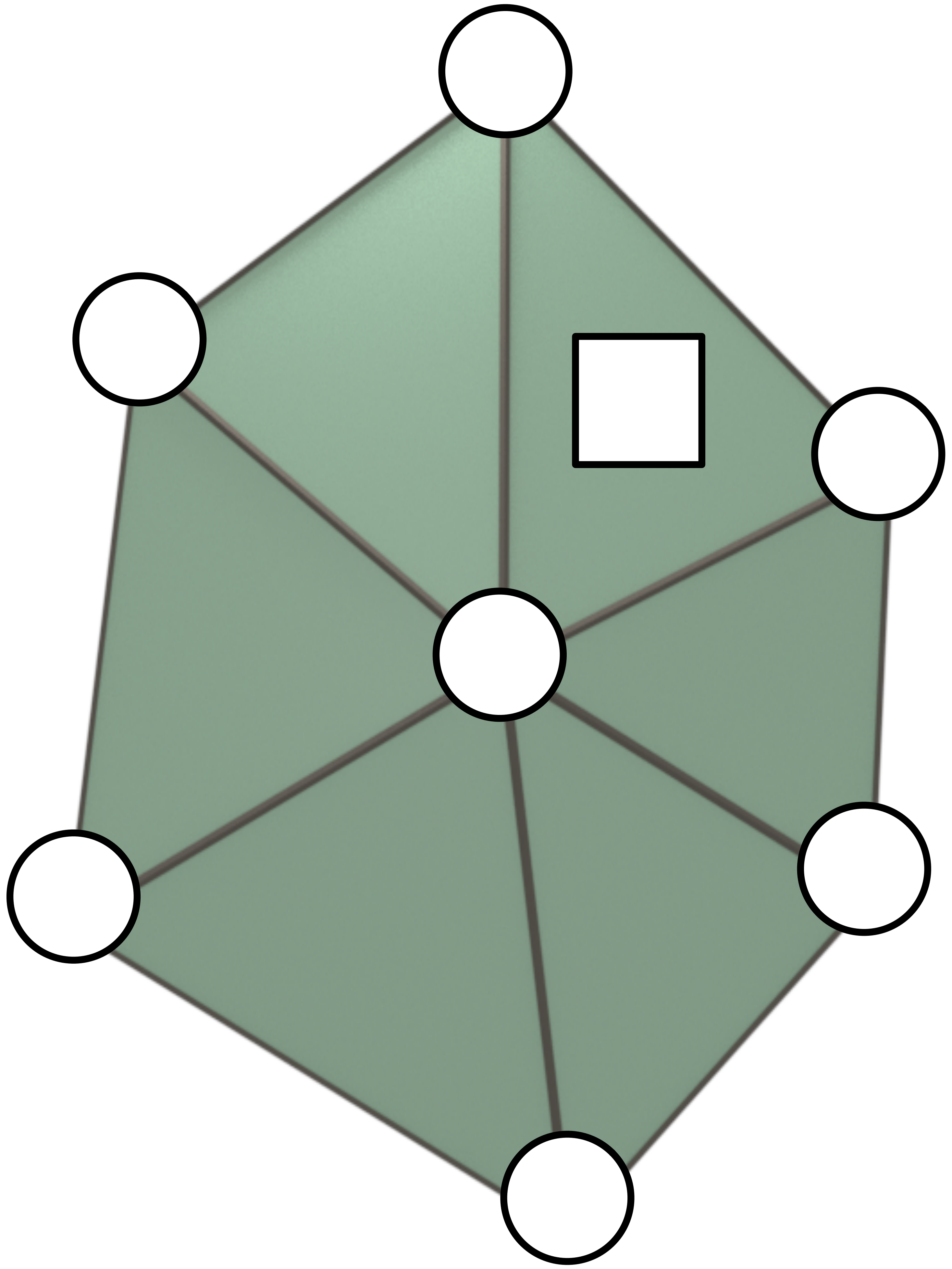
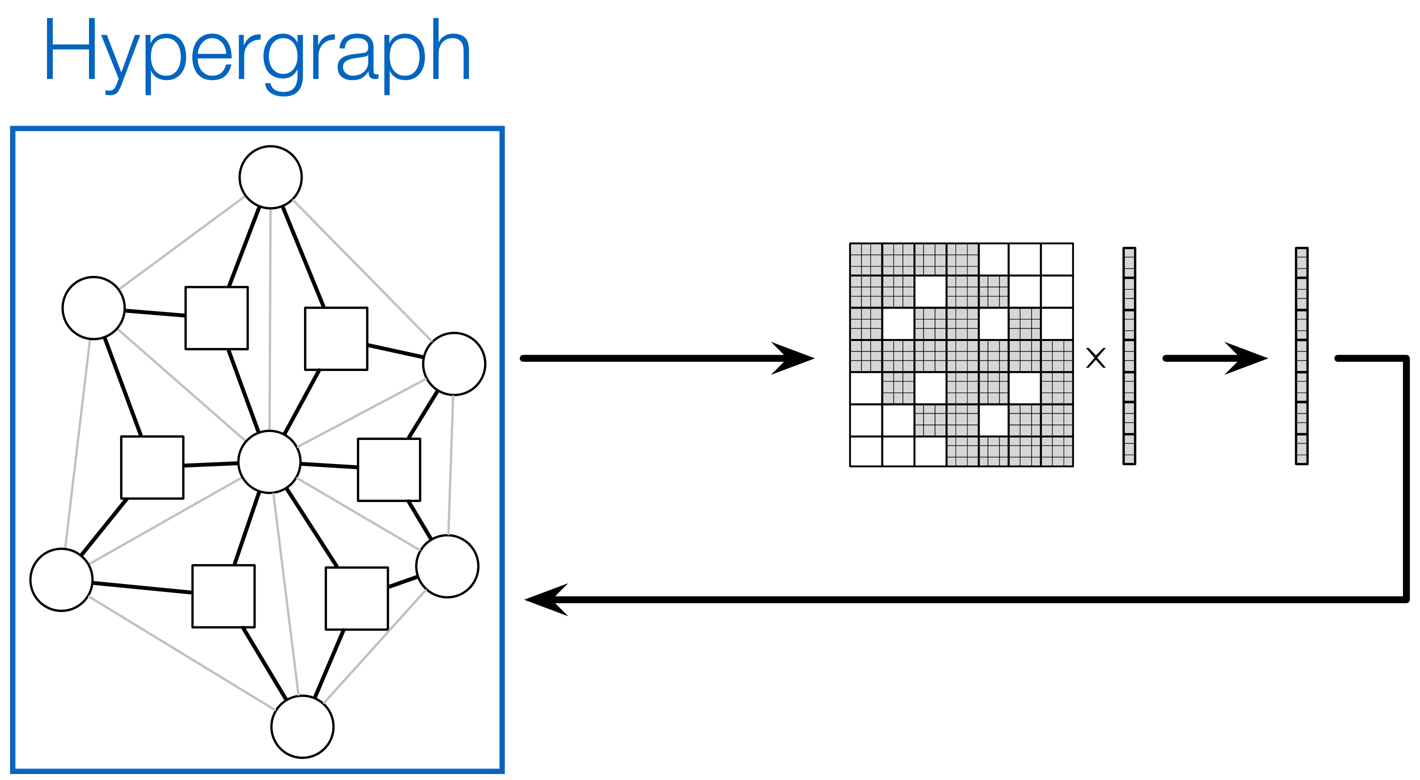



```
element Triangle
  u  : float;                  % shear modulus
  l  : float;                  % lame's first parameter
  W  : float;                  % volume
  B  : matrix[3,3](float);     % strain-displacement
end
```

```
% graph vertices and triangle hyperedges
extern verts     : set{Vertex};
```



Statics Triangular Neo-Hookean FEM Simulation

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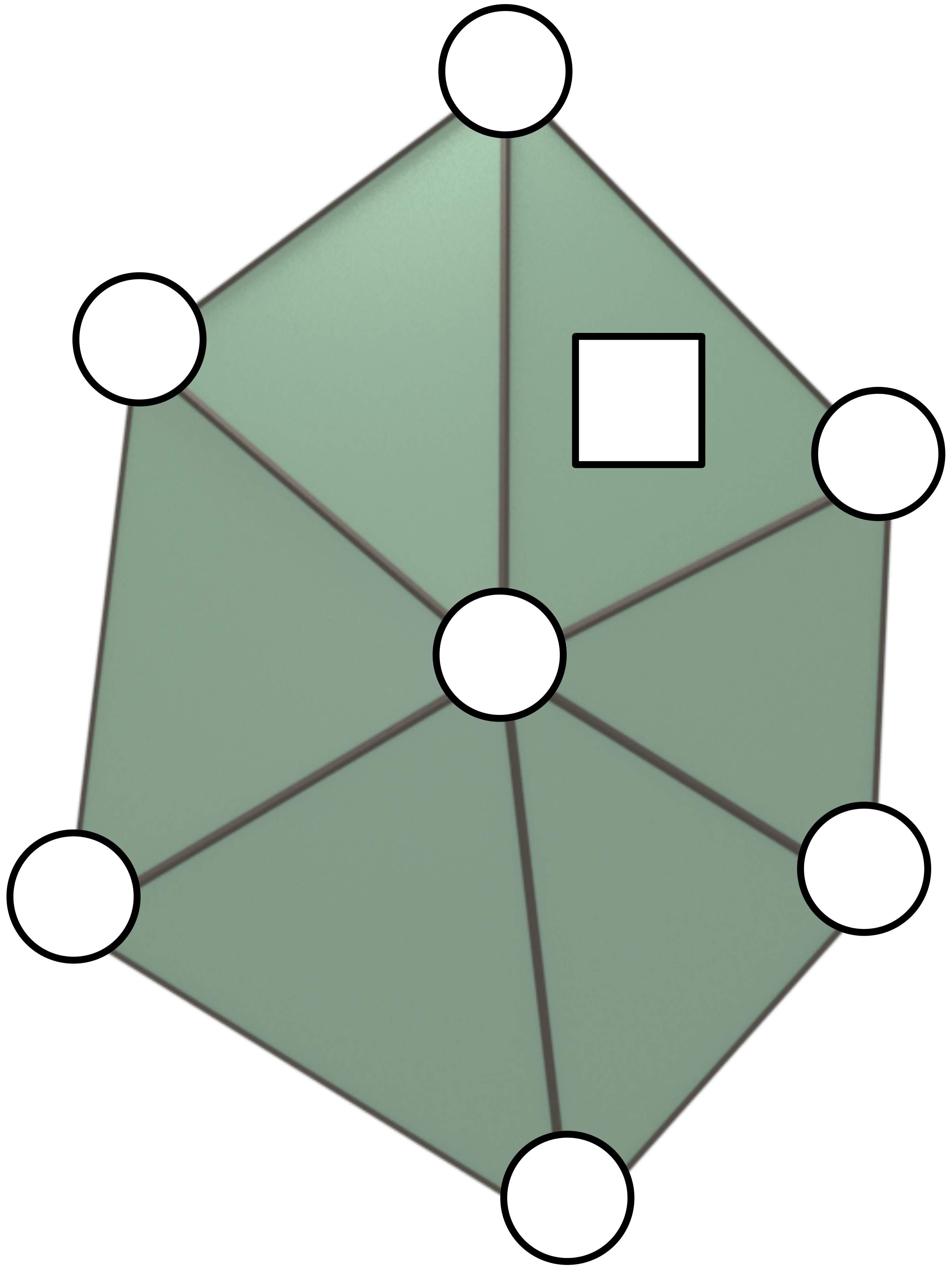
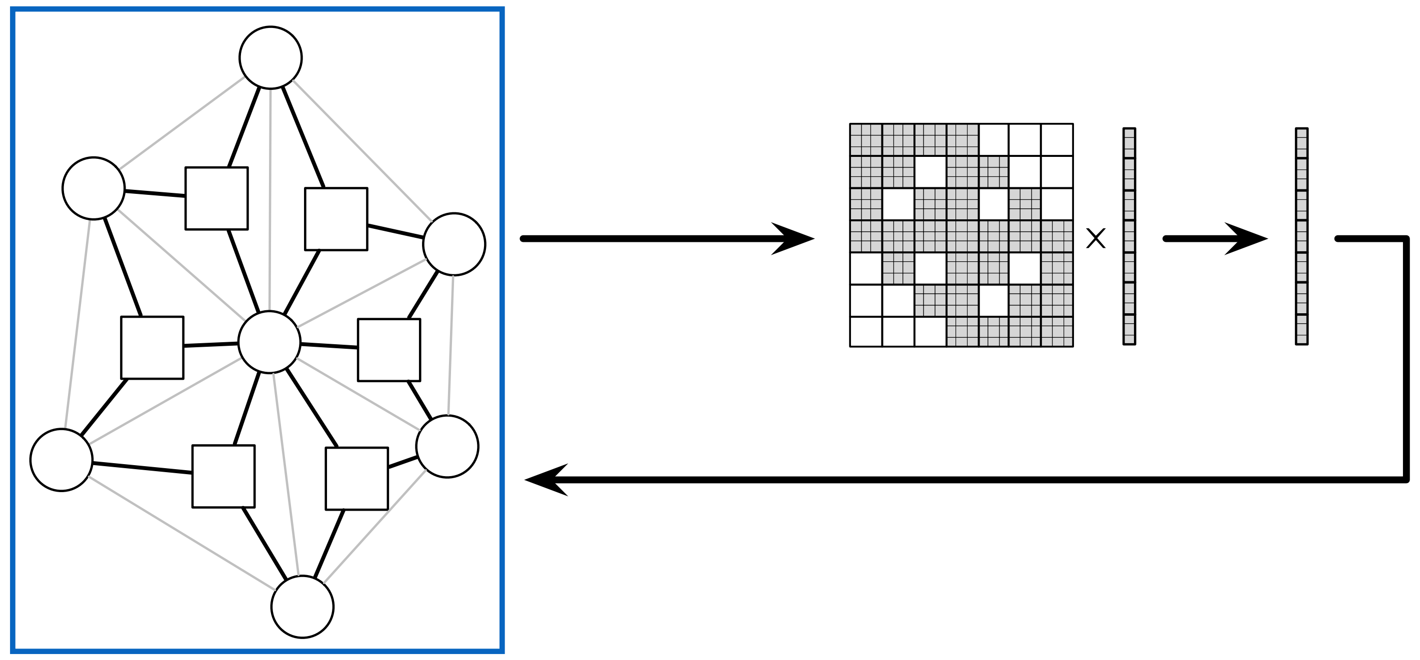


Statics Triangular Neo-Hookean FEM Simulation

Hypergraph



```
element Vertex
  x  : vector[3](float);   % position
  v  : vector[3](float);   % velocity
  fe : vector[3](float);   % external force
```

```
extern triangles : set{Triangle}(verts, verts, verts);
```

```
  u  : float;              % shear modulus
  l  : float;              % lame's first parameter
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  B  : matrix[3,3](float); % strain-displacement
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% graph vertices and triangle hyperedges
extern verts     : set{Vertex};
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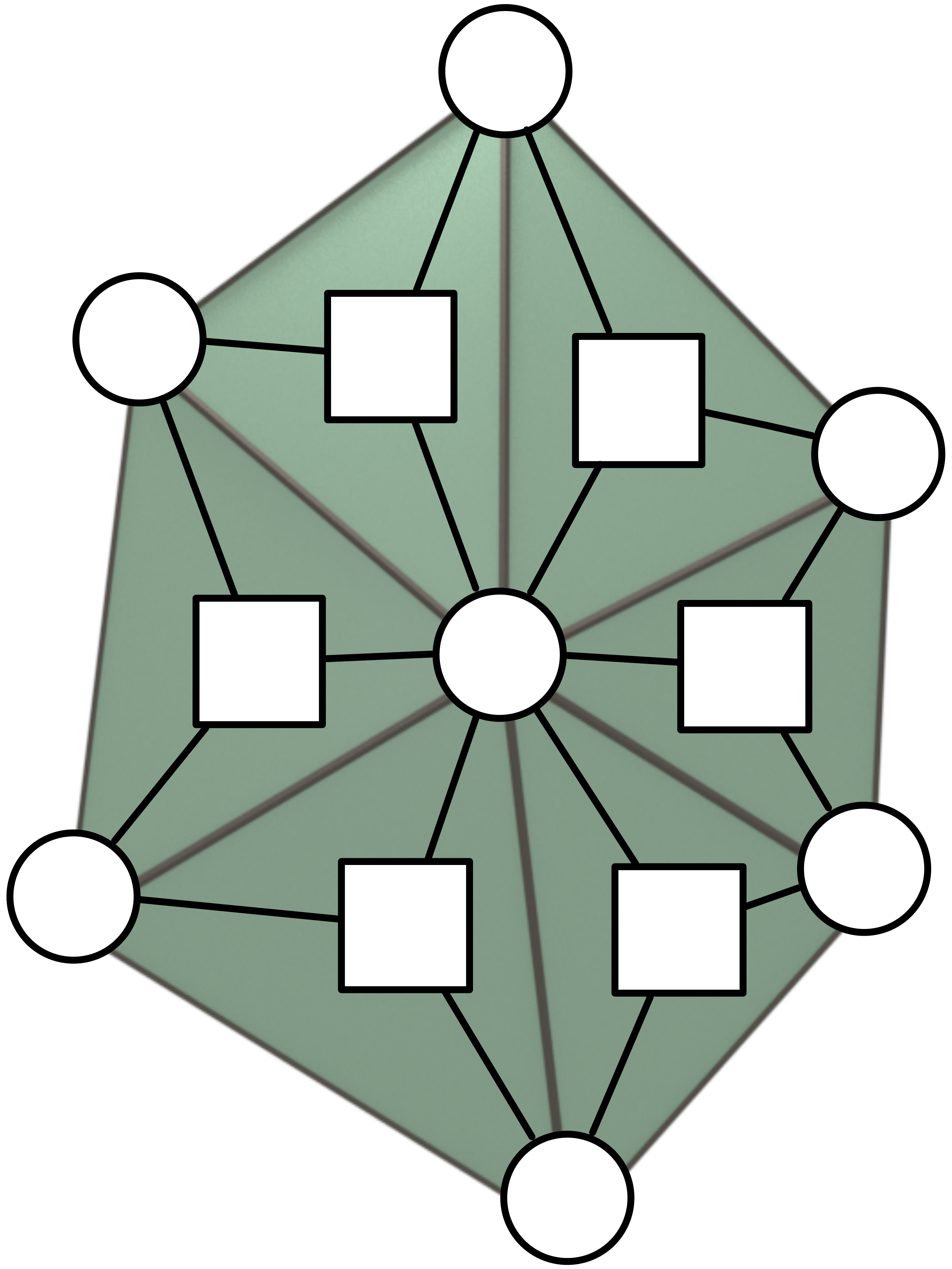

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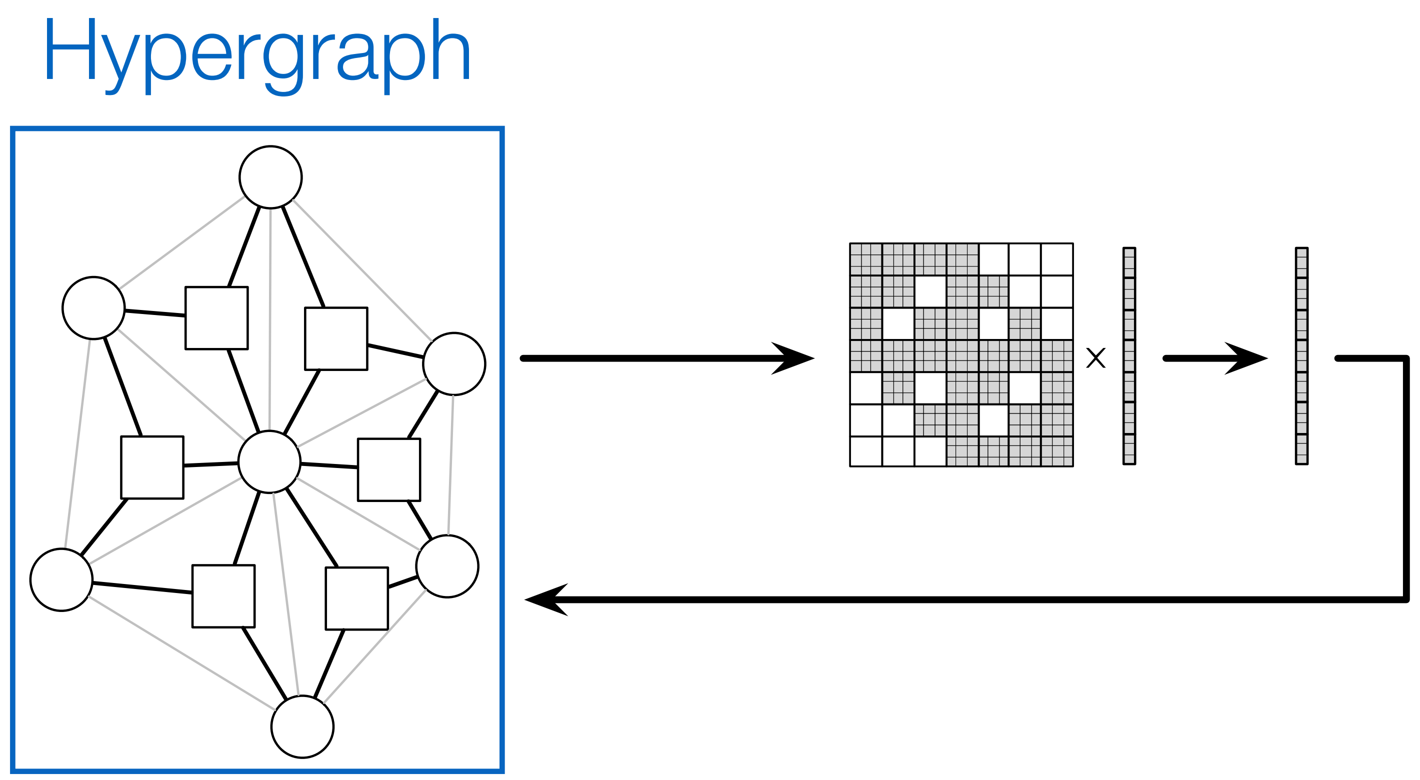


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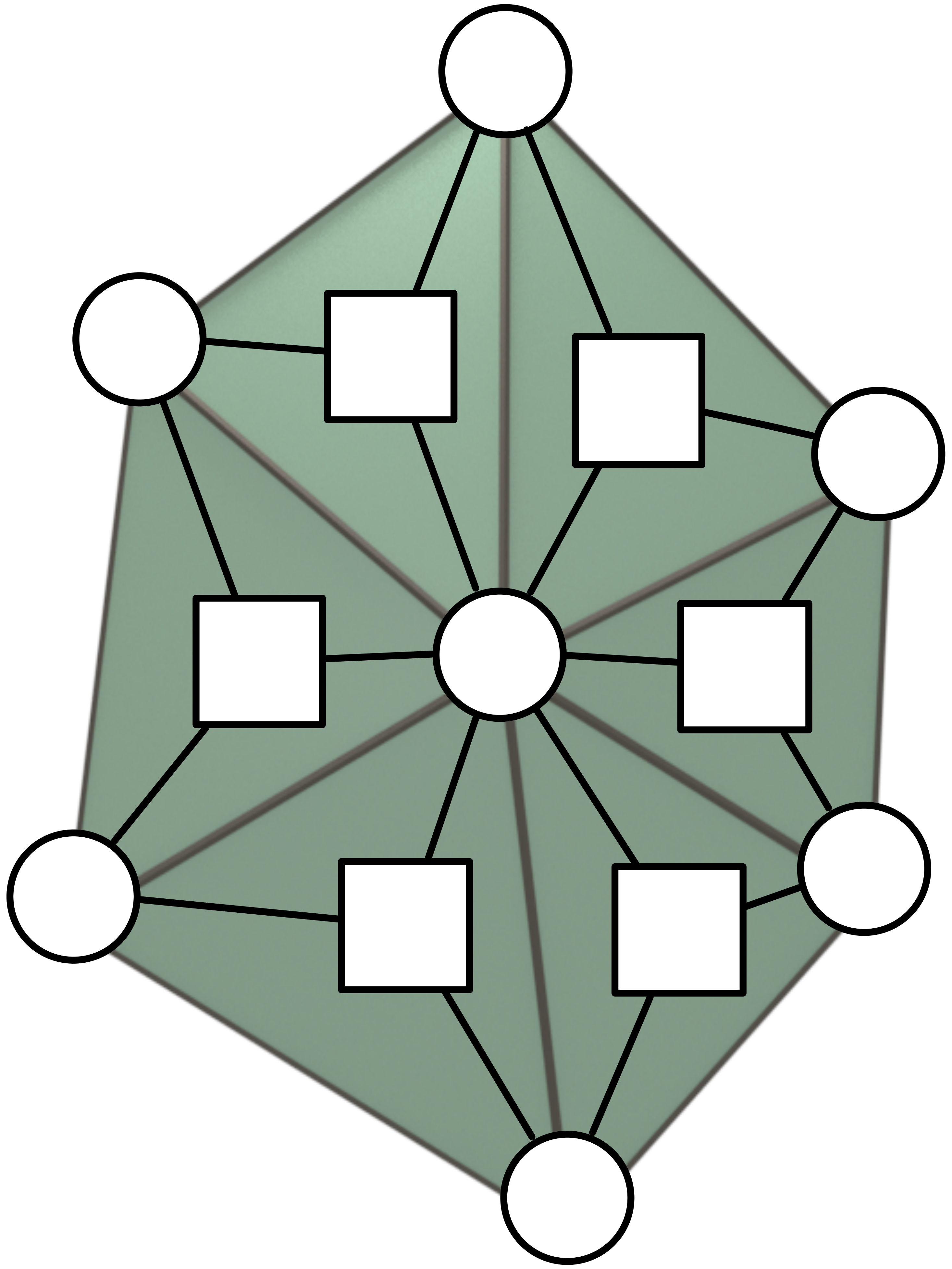
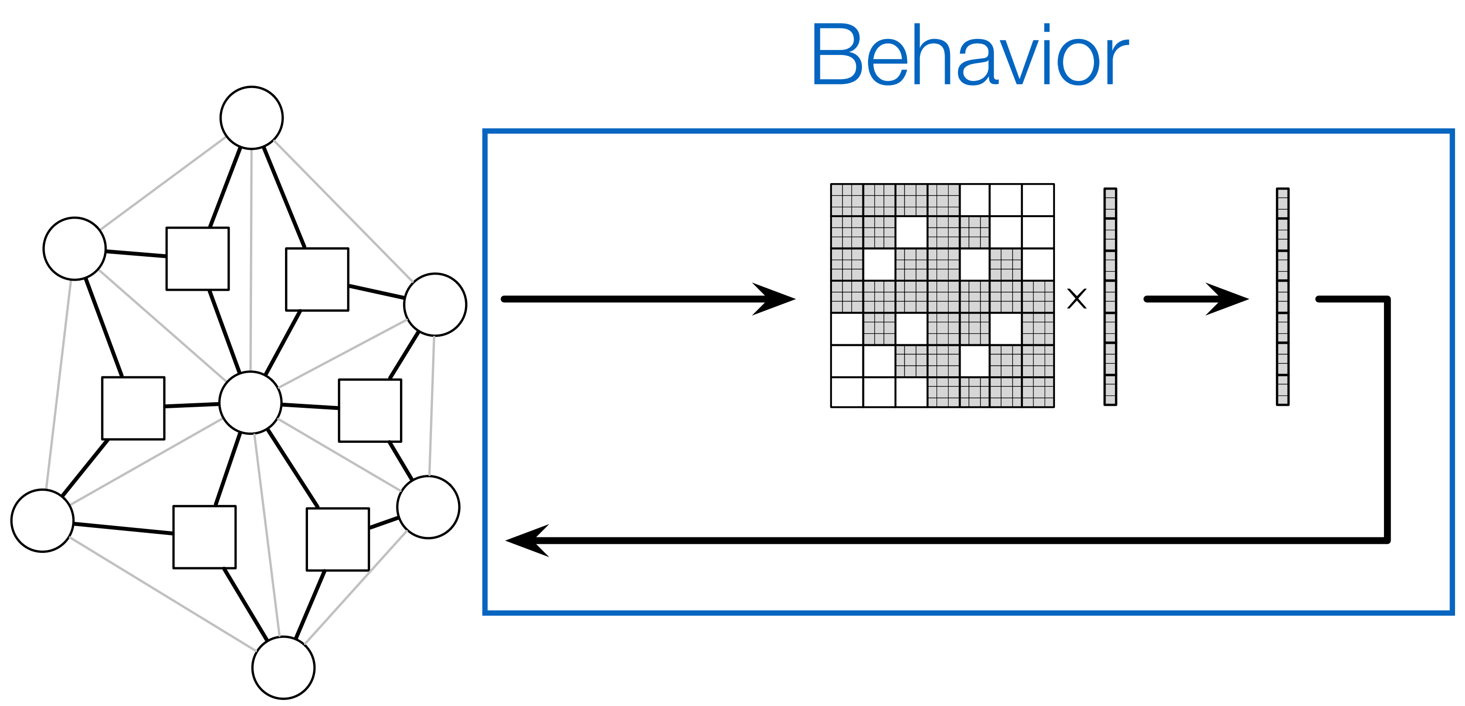


Statics Triangular Neo-Hookean FEM Simulation

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element Triangle
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  W  : float;
  B  : matrix[3,3](flo
end

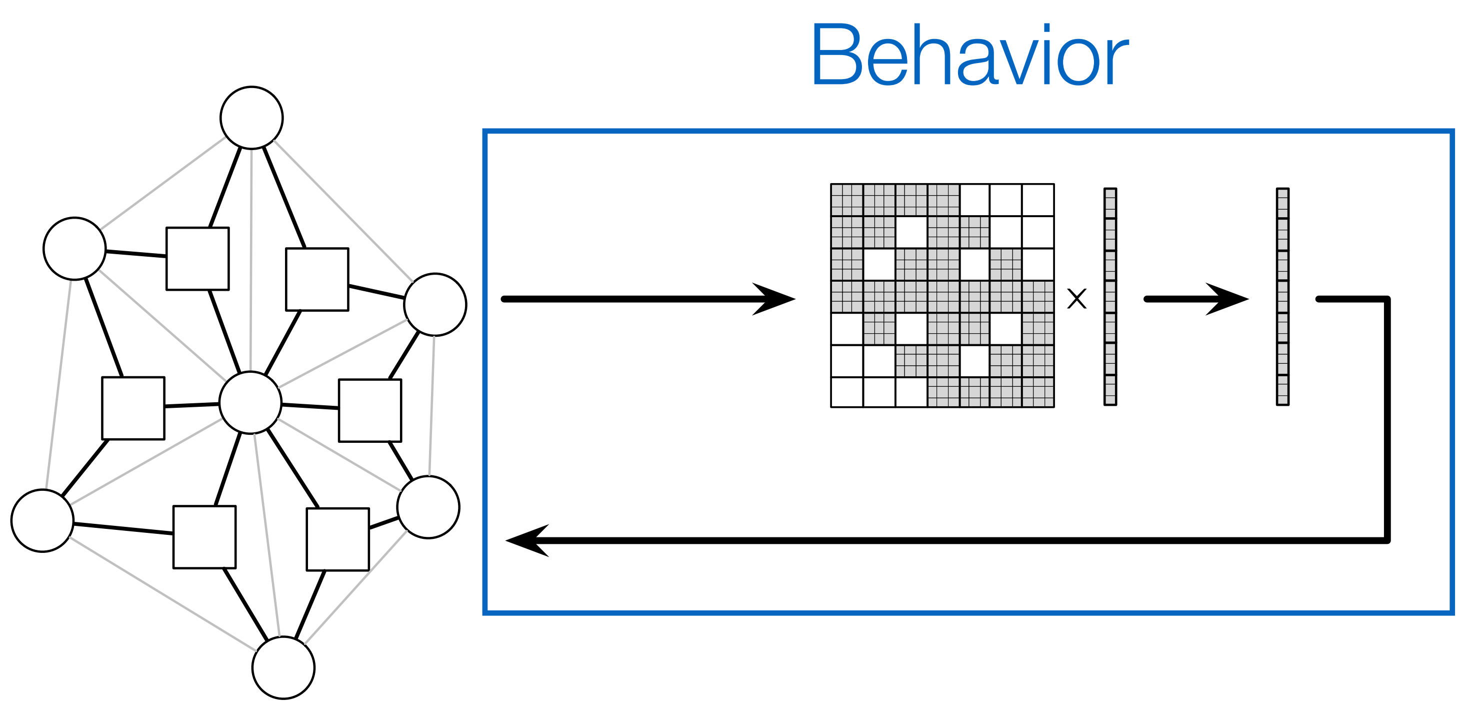
% graph vertices and t
extern verts     : set
extern triangles : set

% compute triangle area
func compute_area(inout t : Triangle, v : (Vertex*3))
  t.B = compute_B(v);
  t.W = det(B) / 2.0;
end

export func init()
  apply compute_area to triangles;
end
```

```
% newton's method
export func newton_method()
  while abs(f - verts.fe) > 1e-6
    // assemble stiffness matrix
    // assemble force vector
    // compute new position
  end
end
```

Statics Triangular Neo-Hookean FEM Simulation

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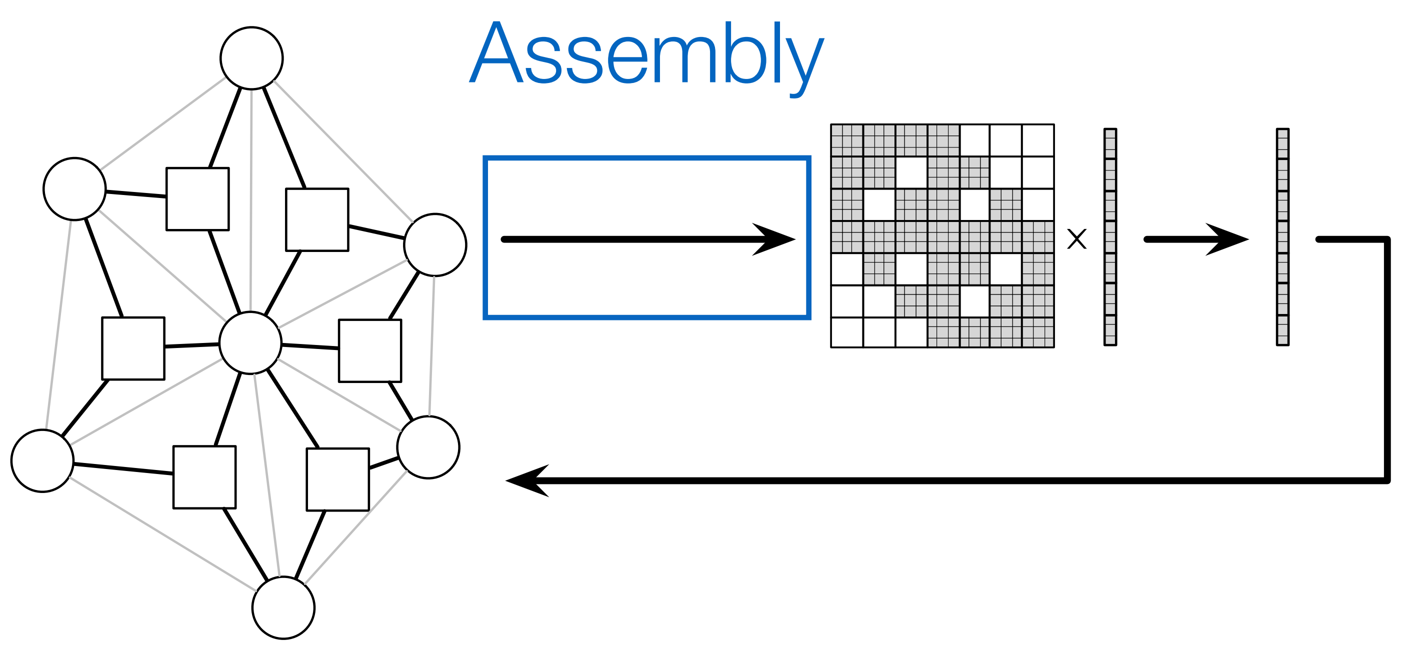


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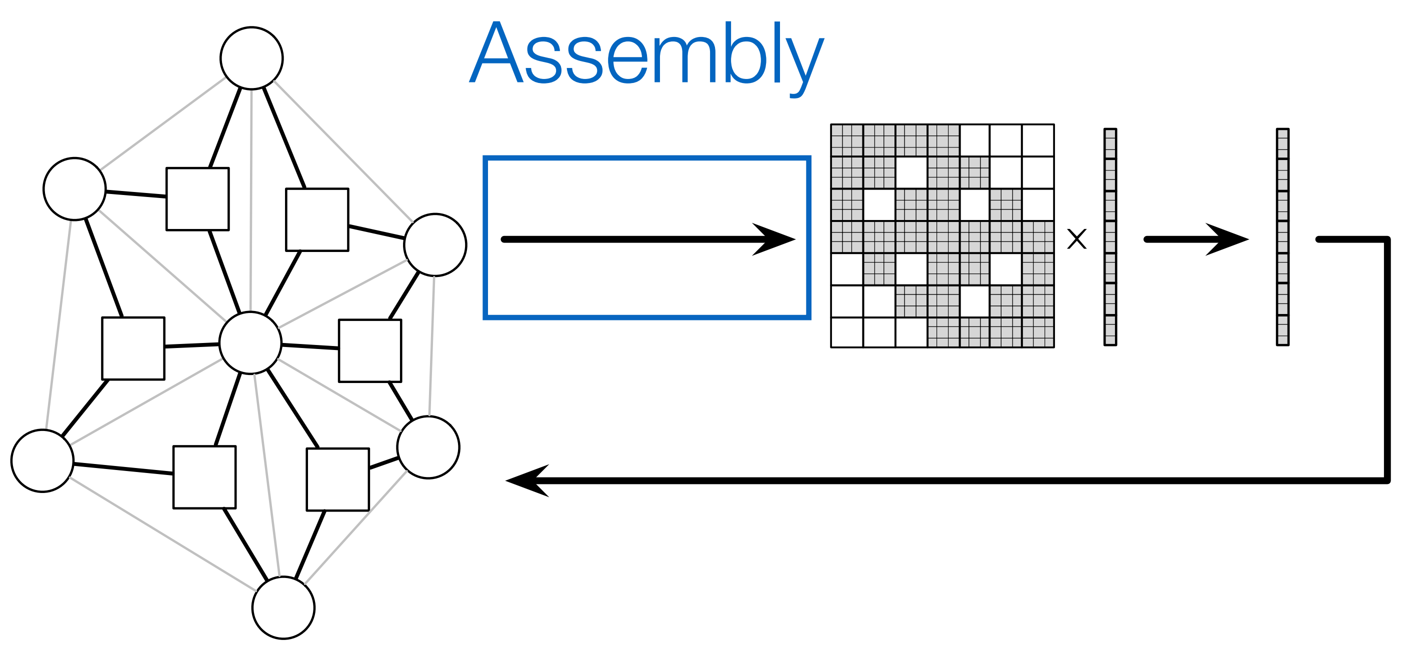

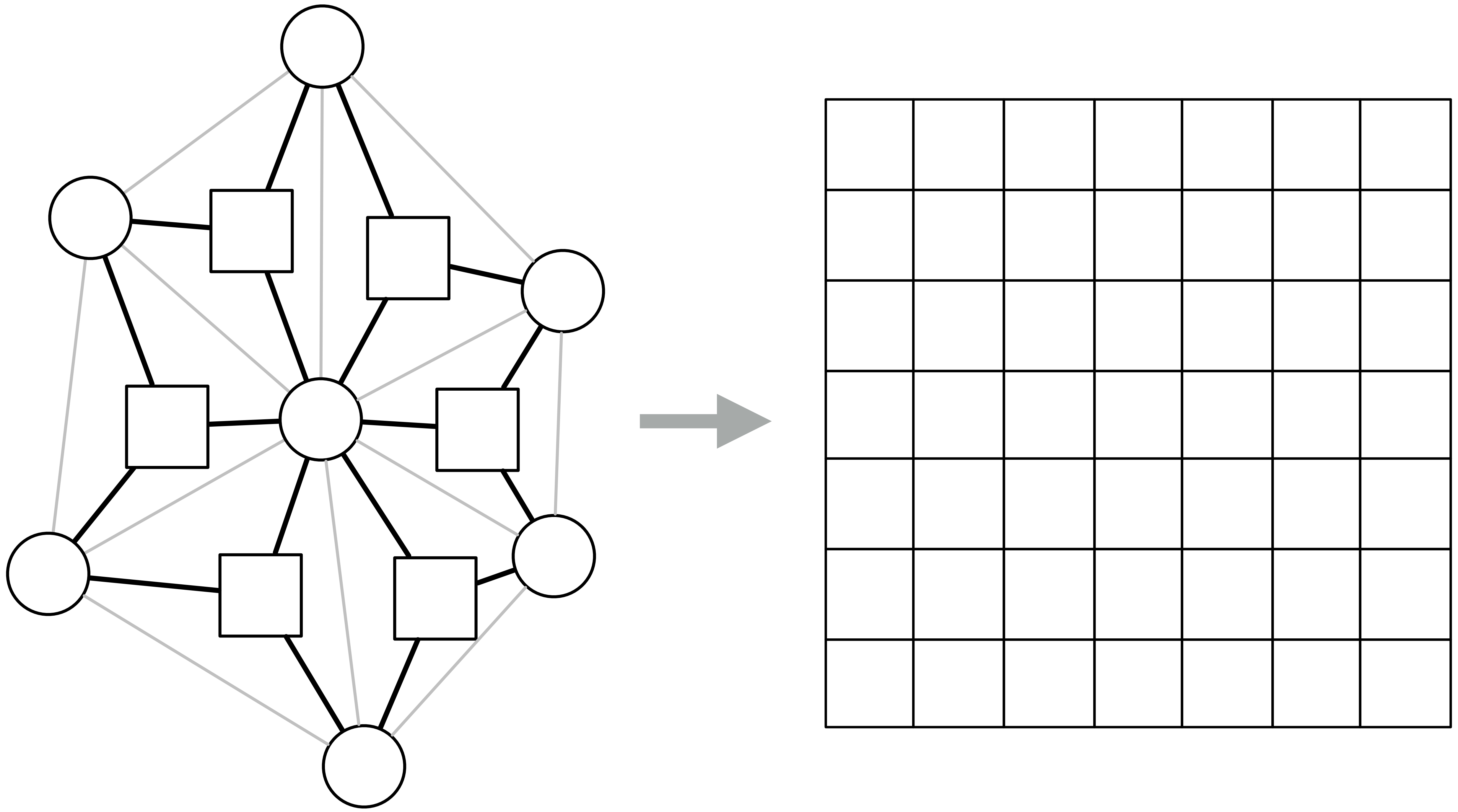

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end
```

```
K = map triangle_stiffness to triangles reduce +;
```

```
% newton's method
export func newton_method()
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    // assemble stiffness matrix
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    // compute new position
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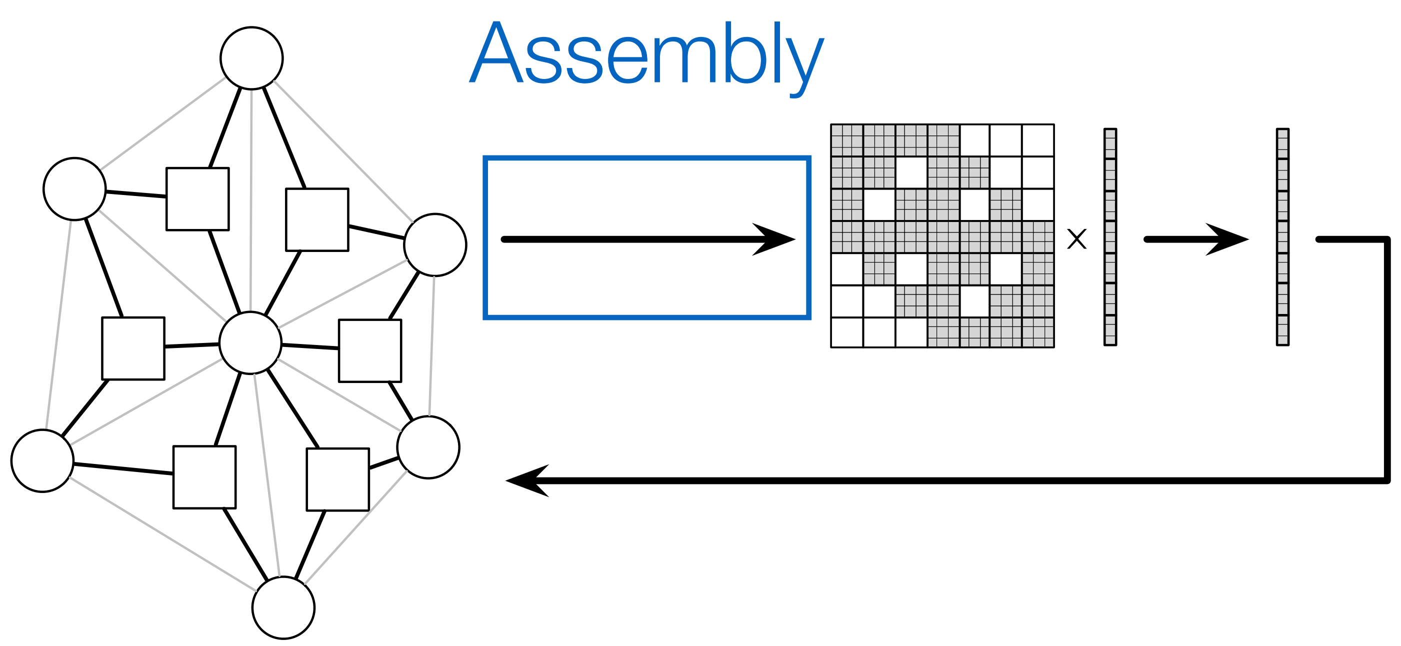
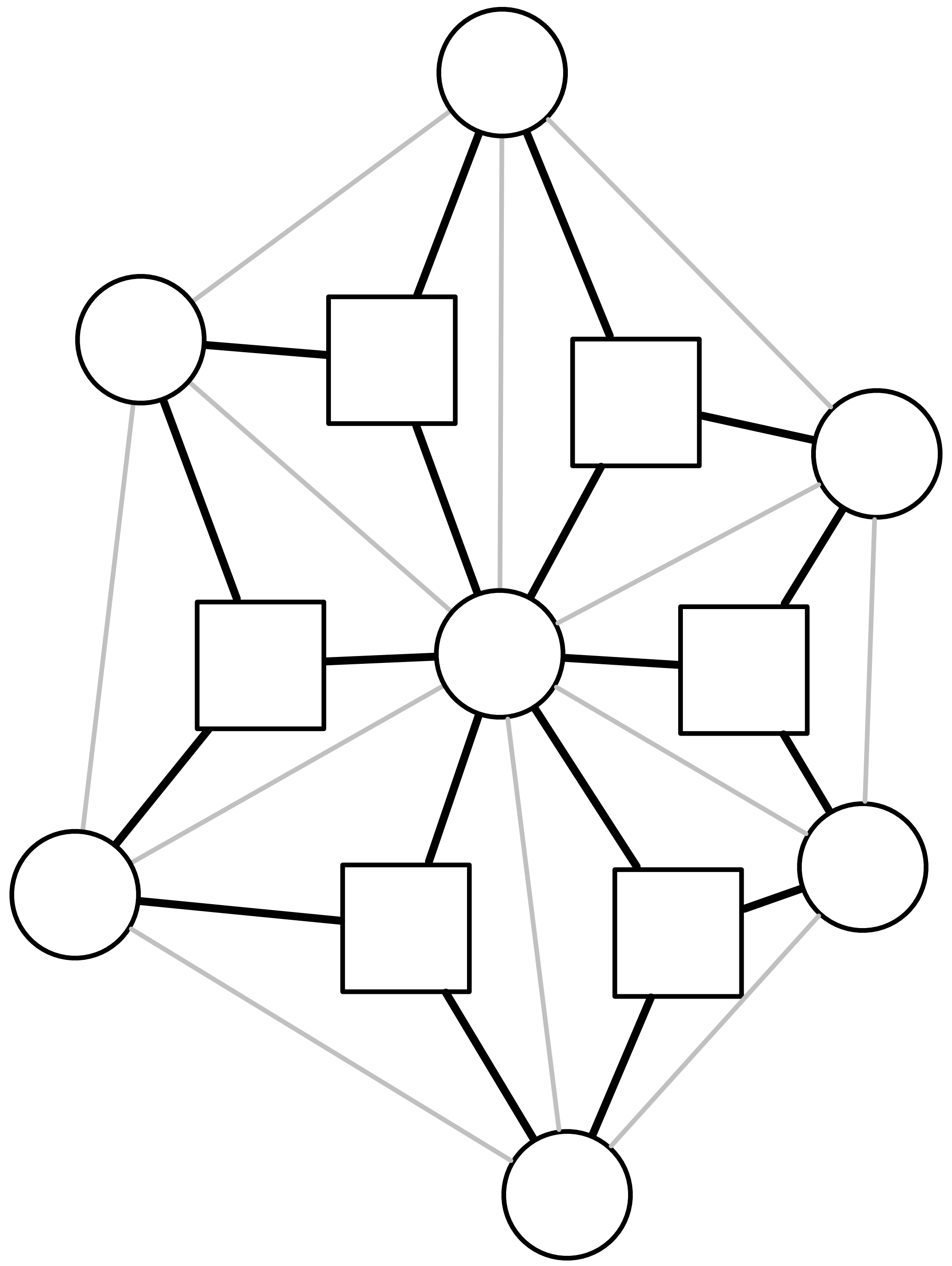
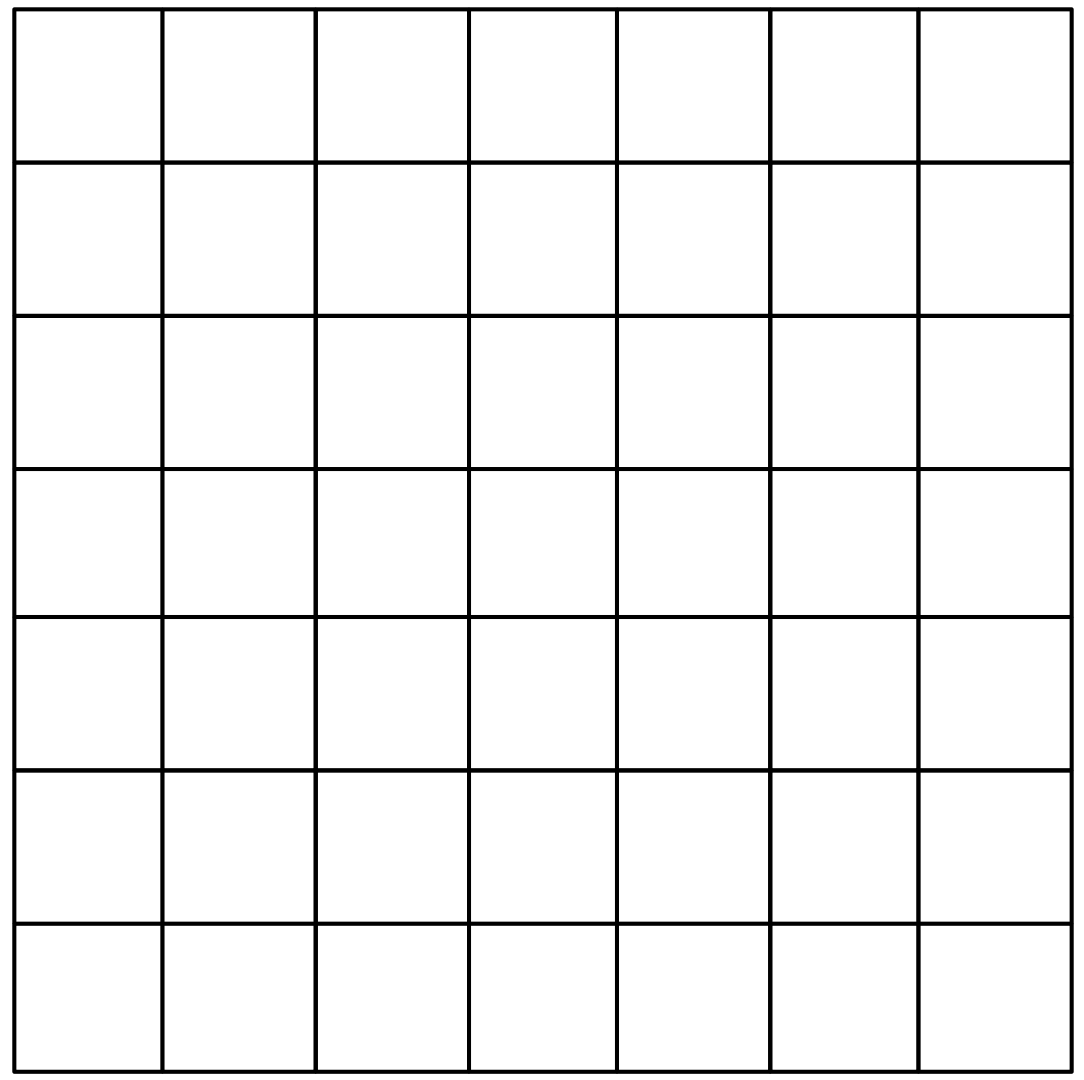
```

Statics Triangular Neo-Hookean FEM Simulation







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  W  : float;                % volume
  B  : matrix[3,3](float); % strain-displacement
end

% graph vertices and triangle hyperedges
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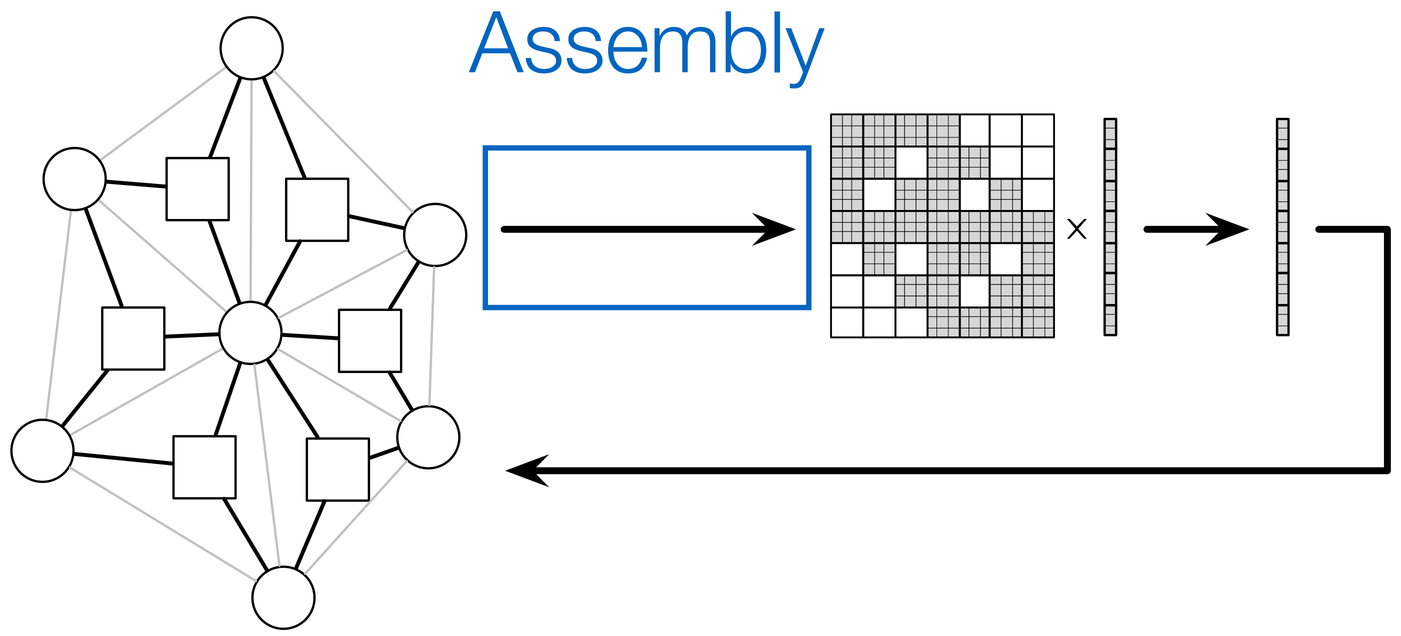
extern triangles : set{Triangle}(verts, verts, verts);

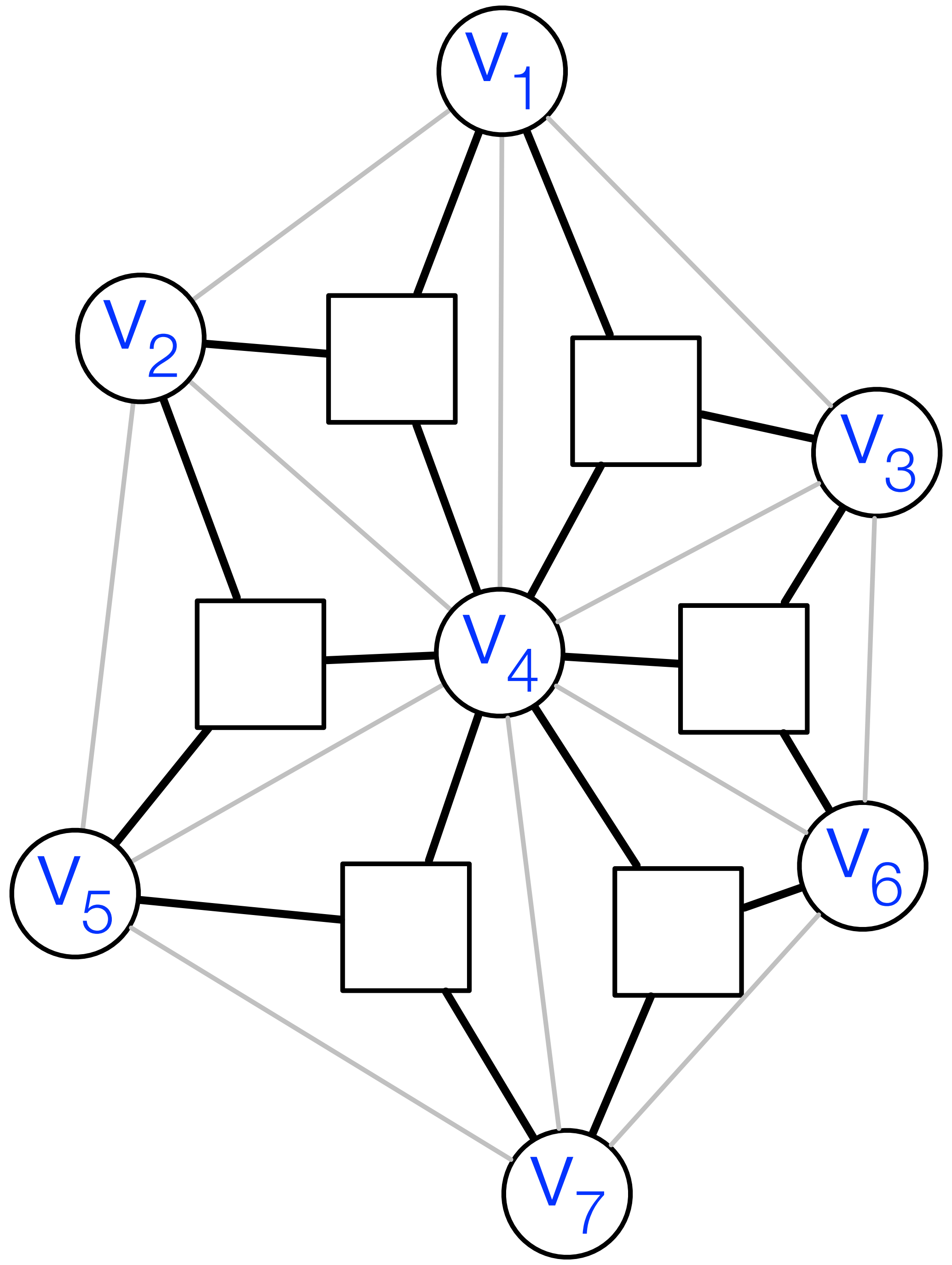
% compute triangle area
func compute_area(inout t : Triangle, v : (Vertex*3))
  t.B = compute_B(v);
  t.W = det(B) / 2.0;
end

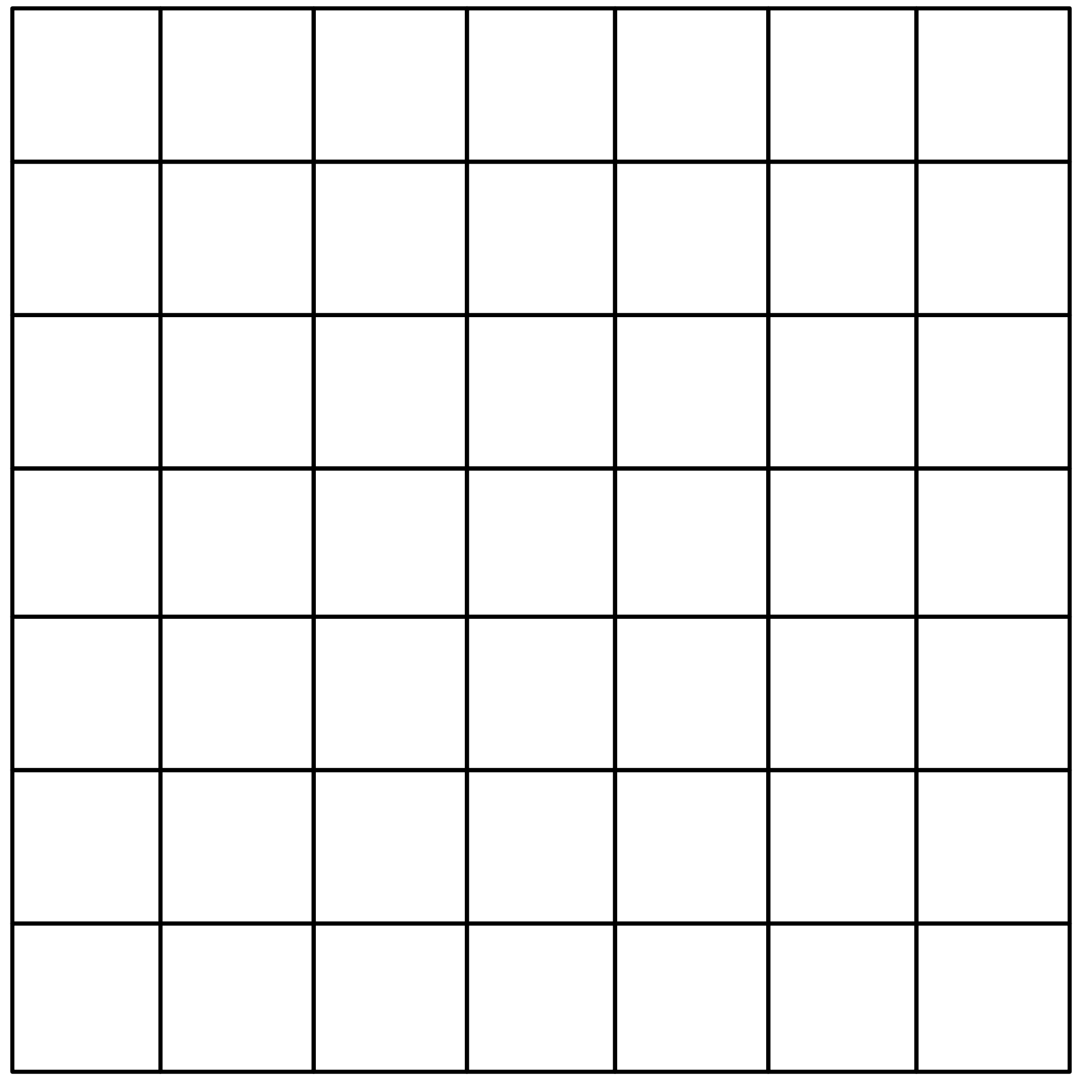
export func init()
  apply compute_area to triangles;
end
```

```
% newton's method
export func newton_method()
  while abs(f - verts.fe) > 1e-6
    K = map triangle_stiffness to triangles reduce +;
    // assemble force vector
    // compute new position
  end
end
```

Statics Triangular Neo-Hookean FEM Simulation







```
element Vertex
  x  : vector[3](float);    % position
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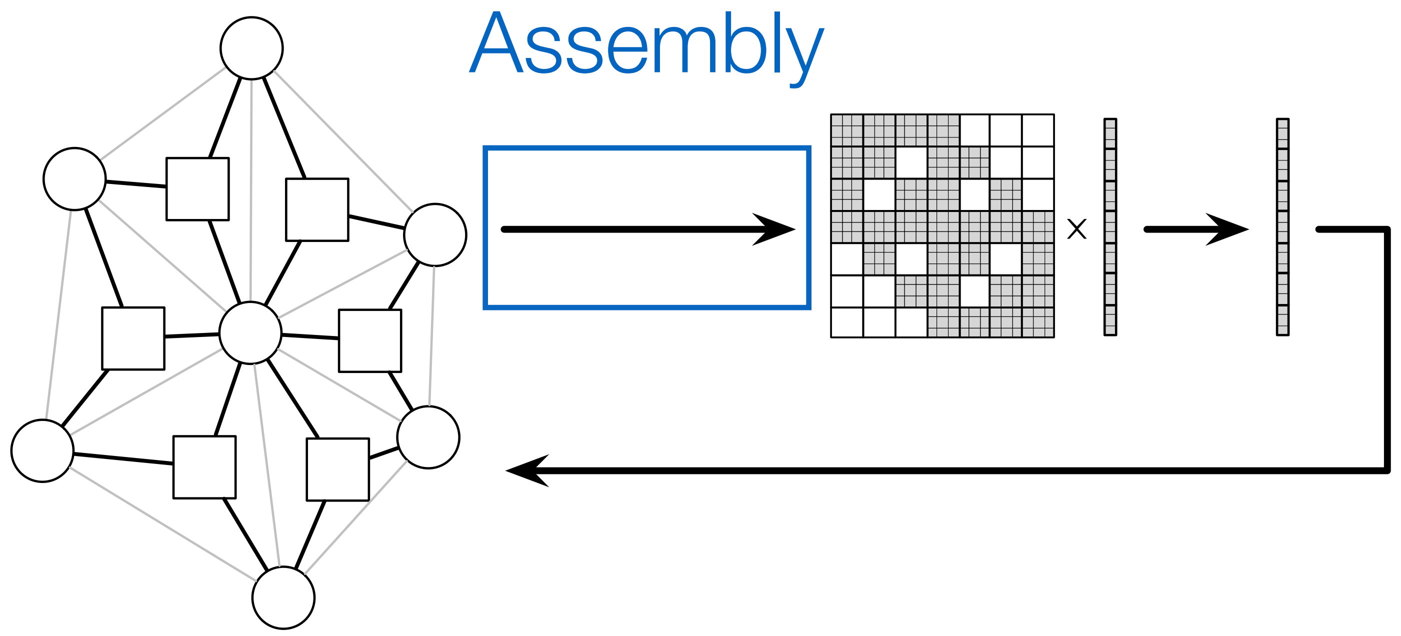
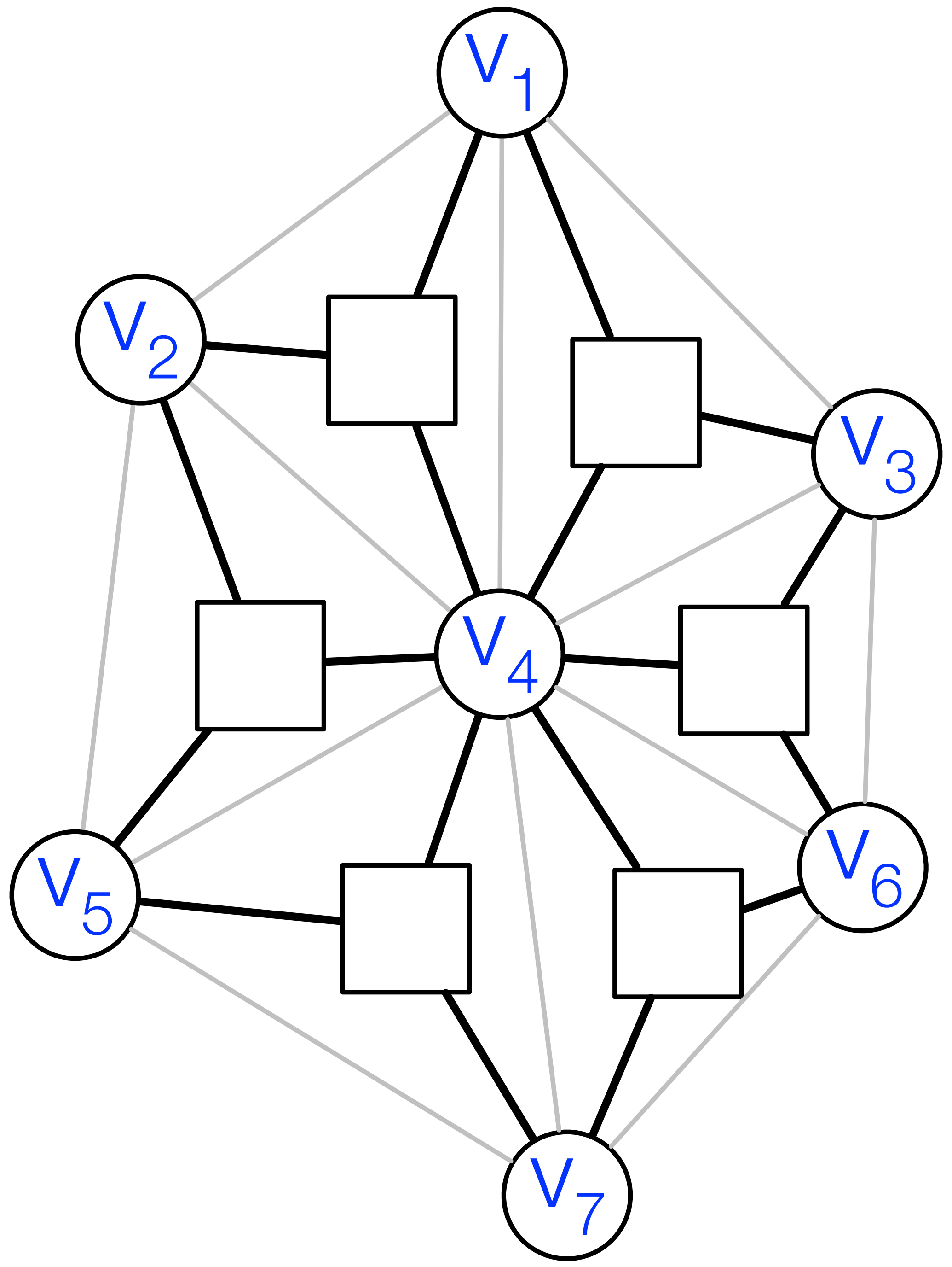
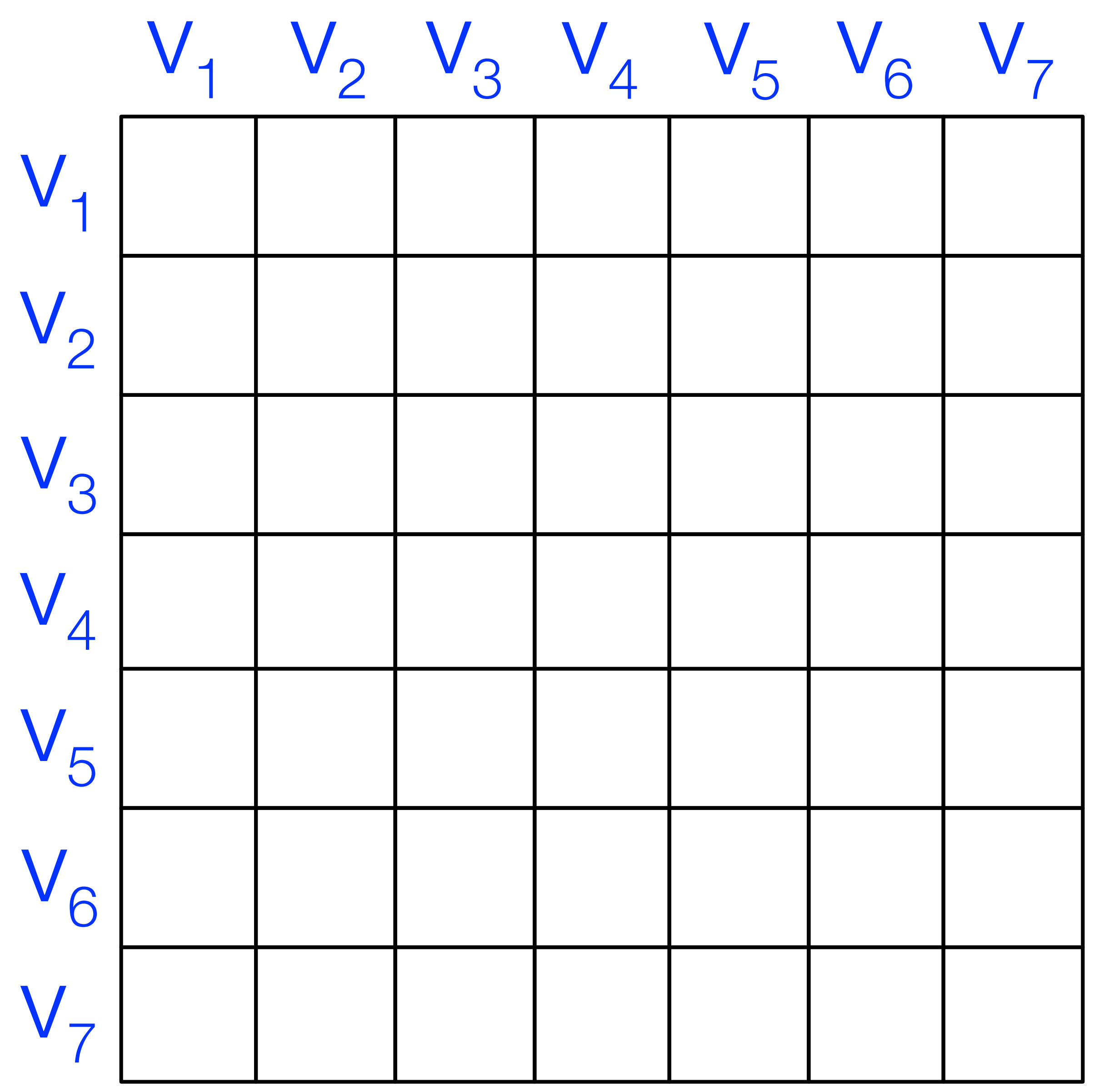
Statics Triangular Neo-Hookean FEM Simulation





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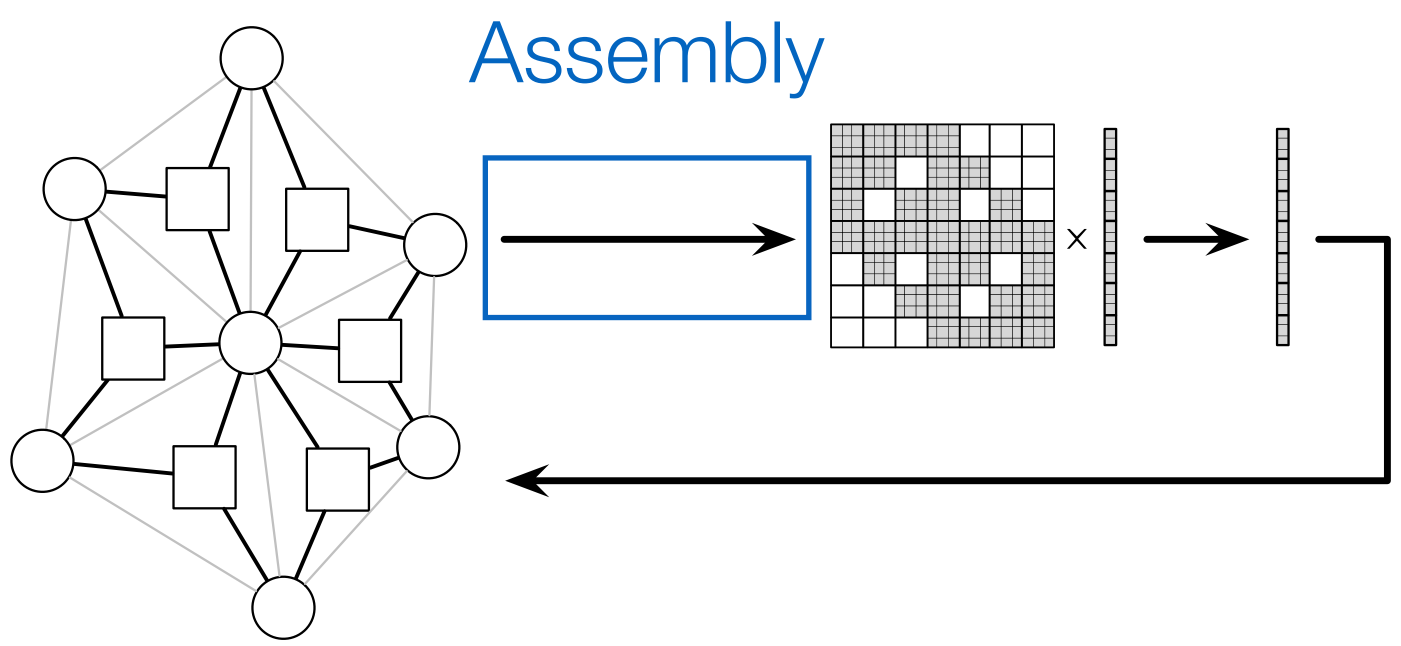
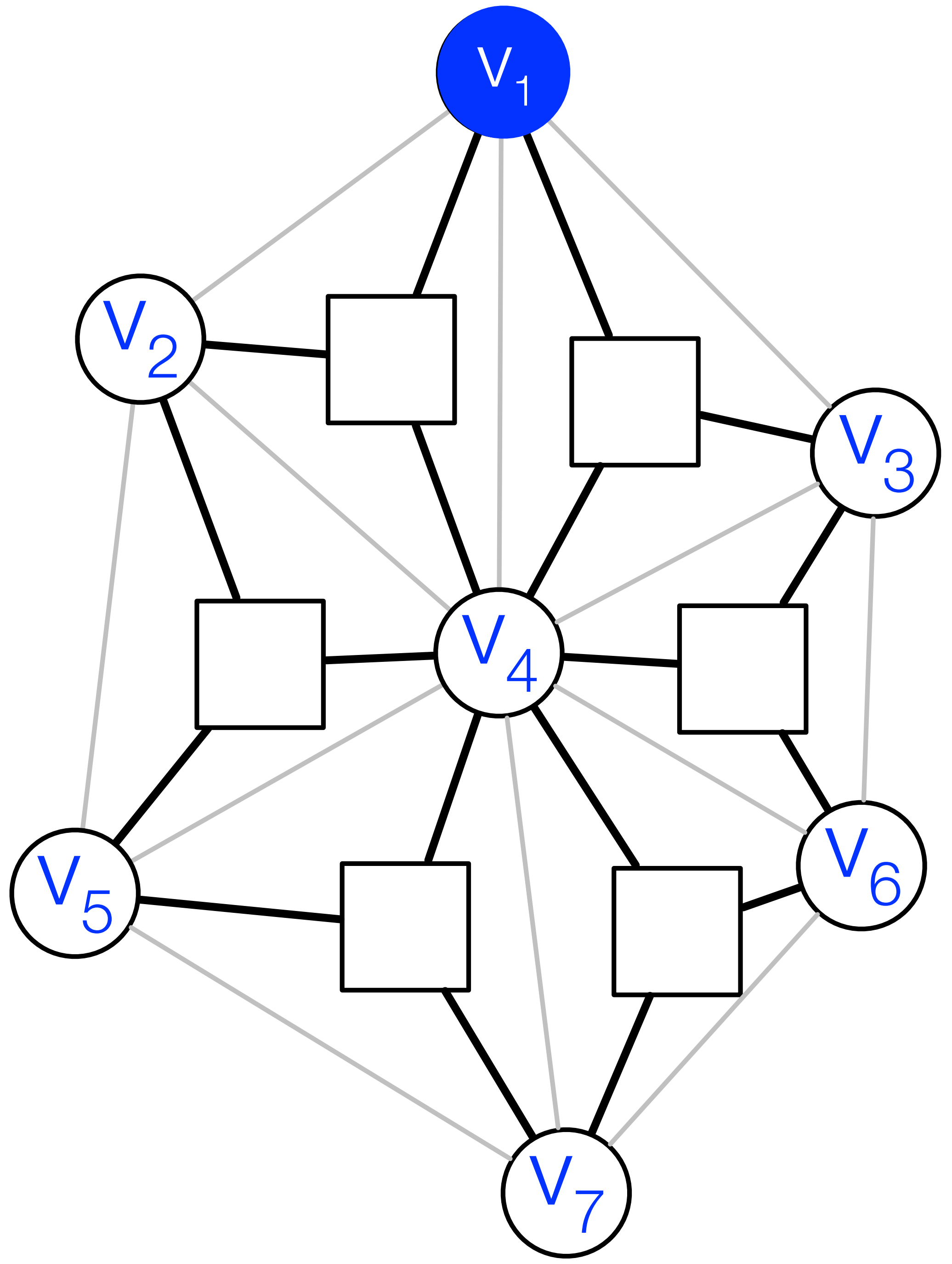
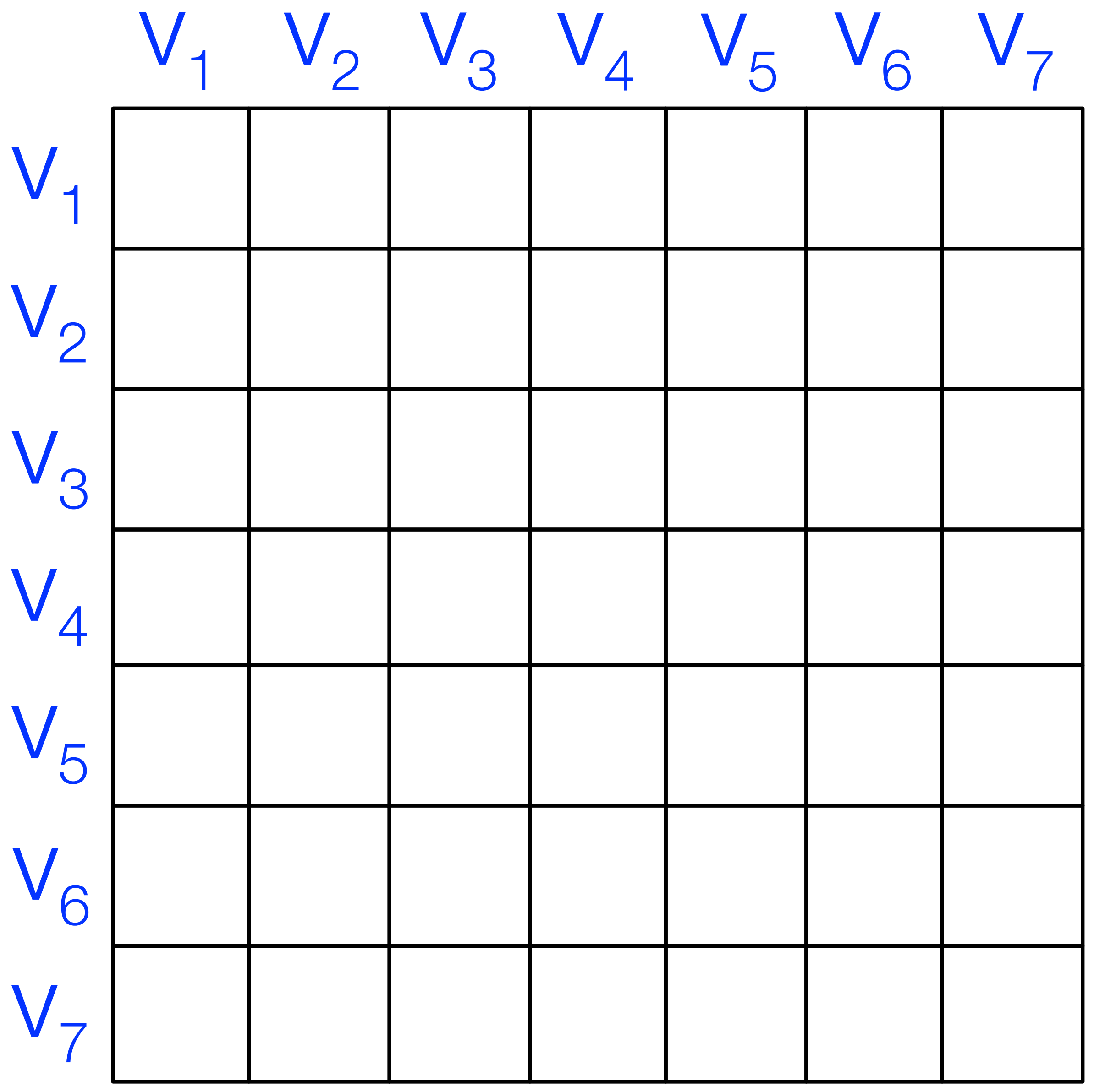
Statics Triangular Neo-Hookean FEM Simulation





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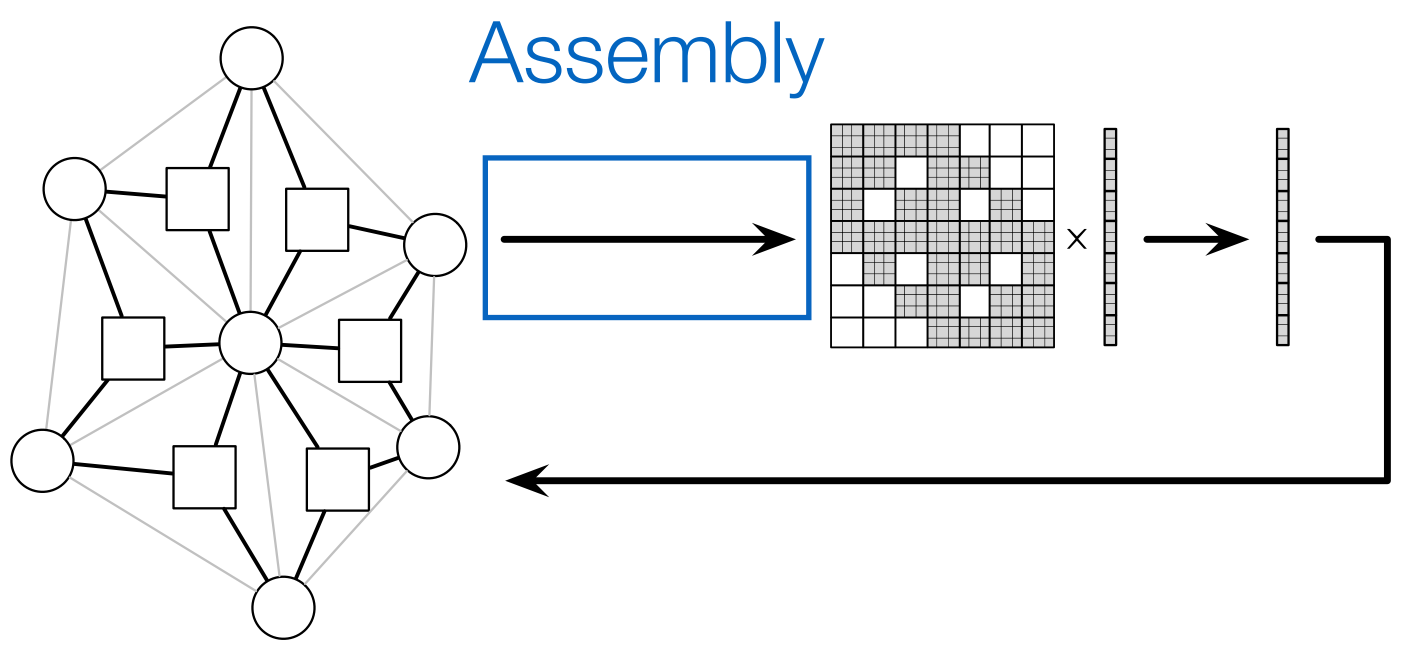

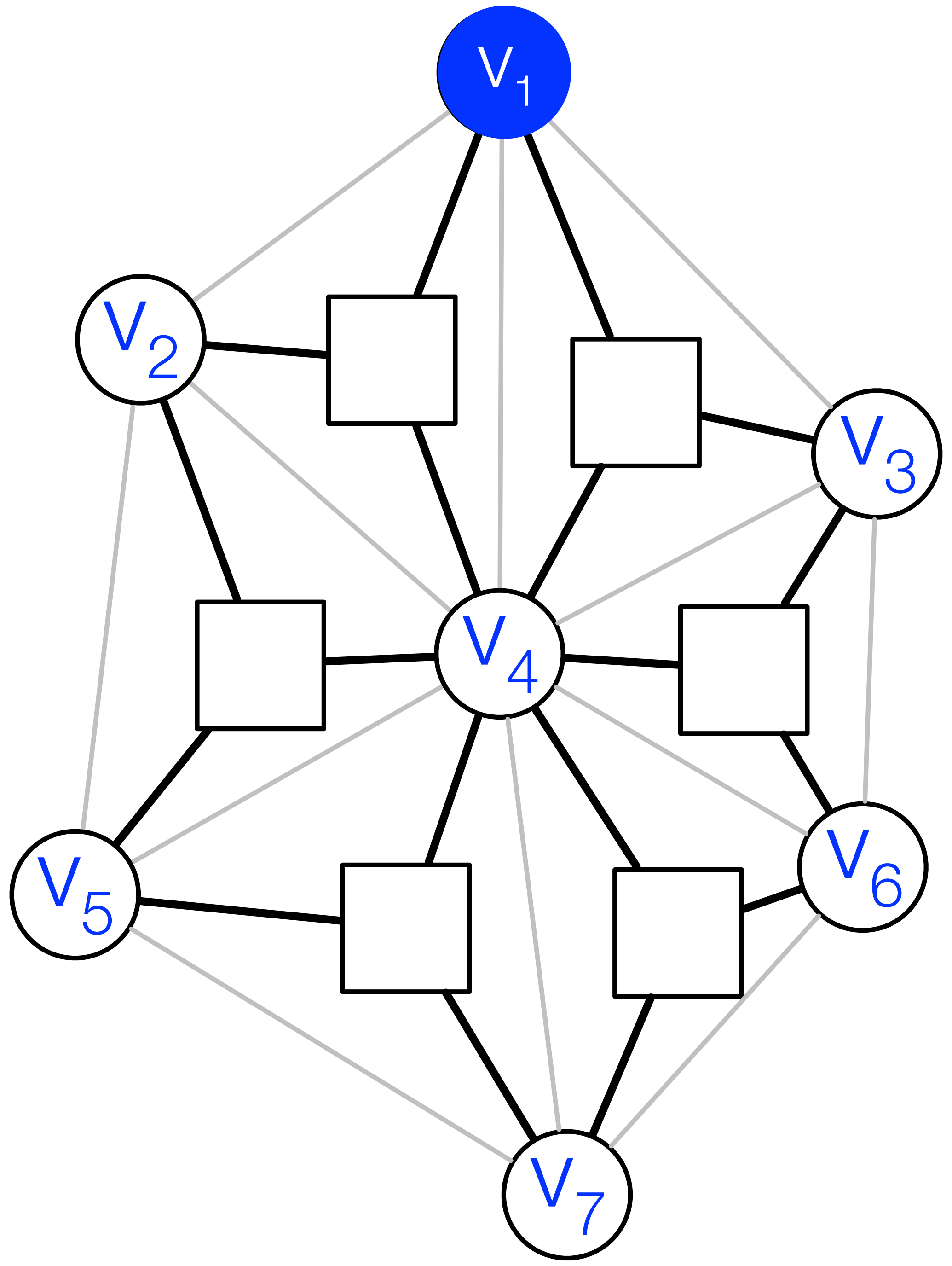

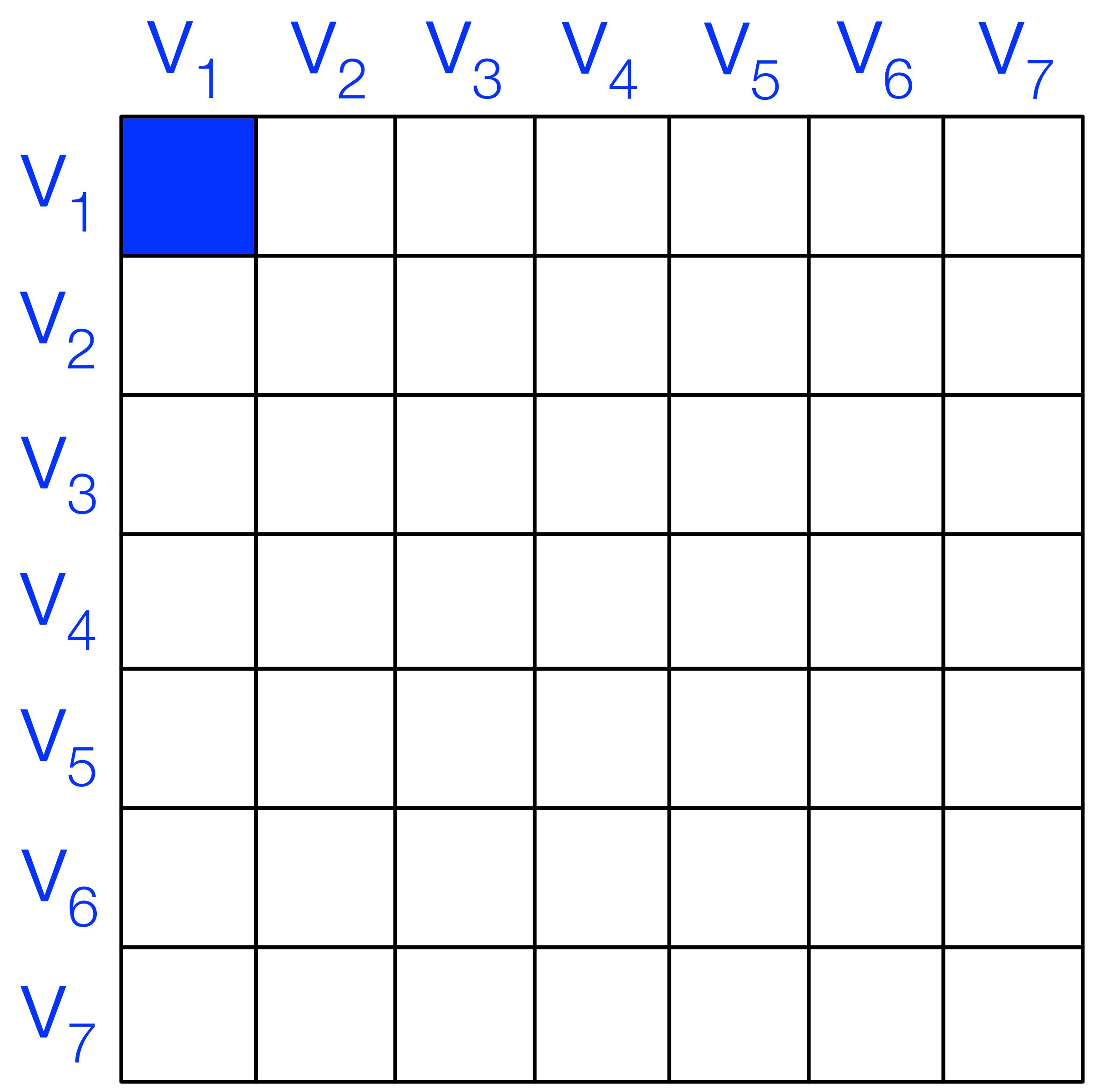


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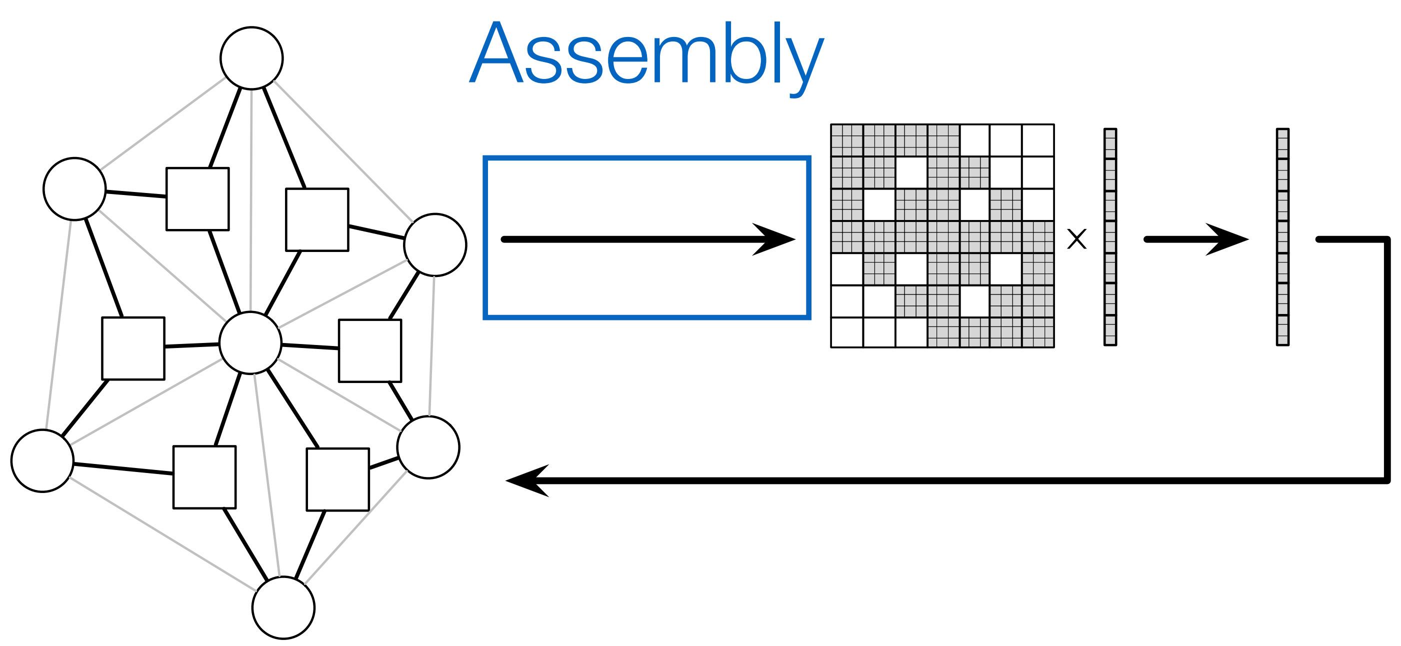

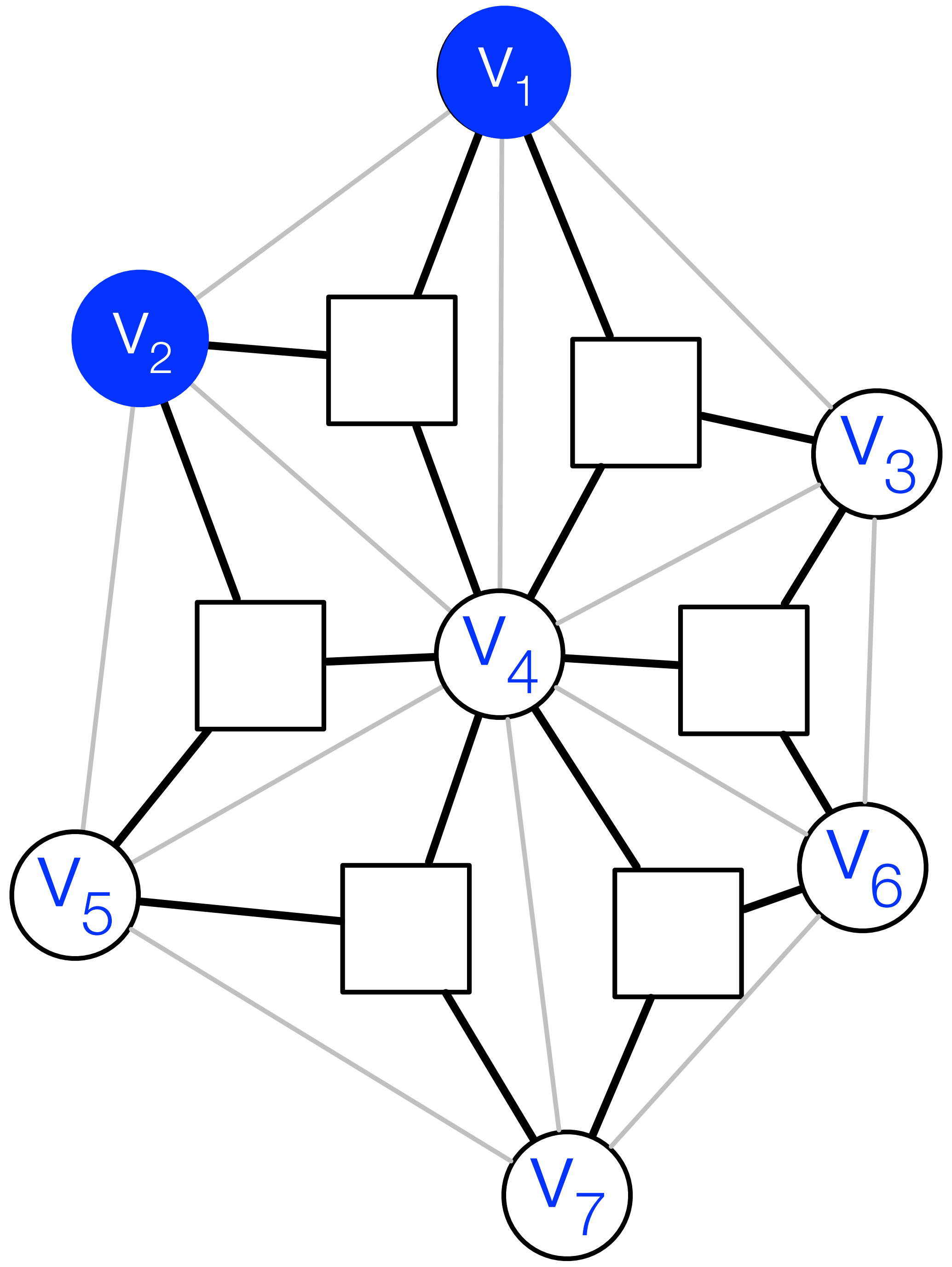

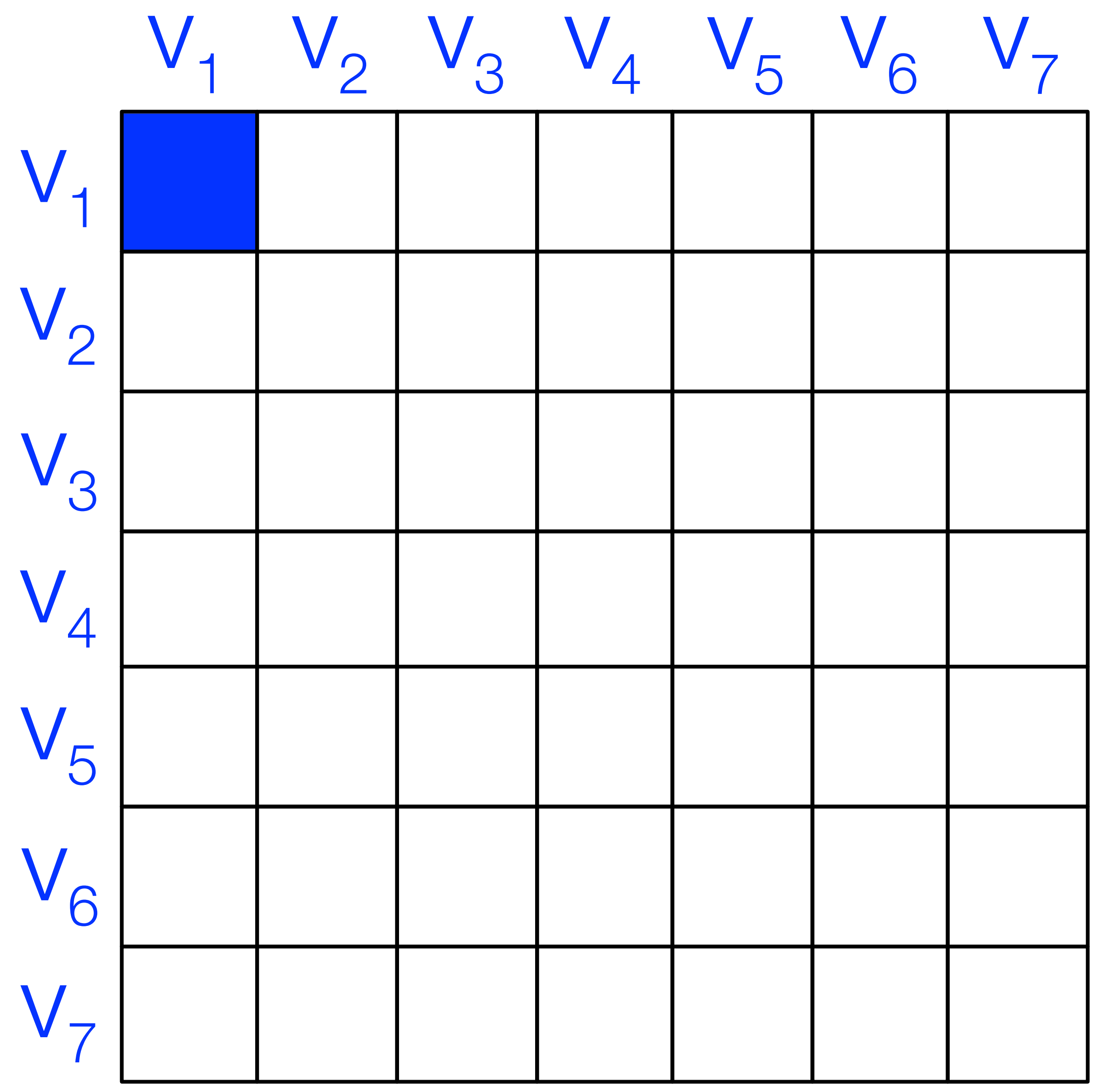


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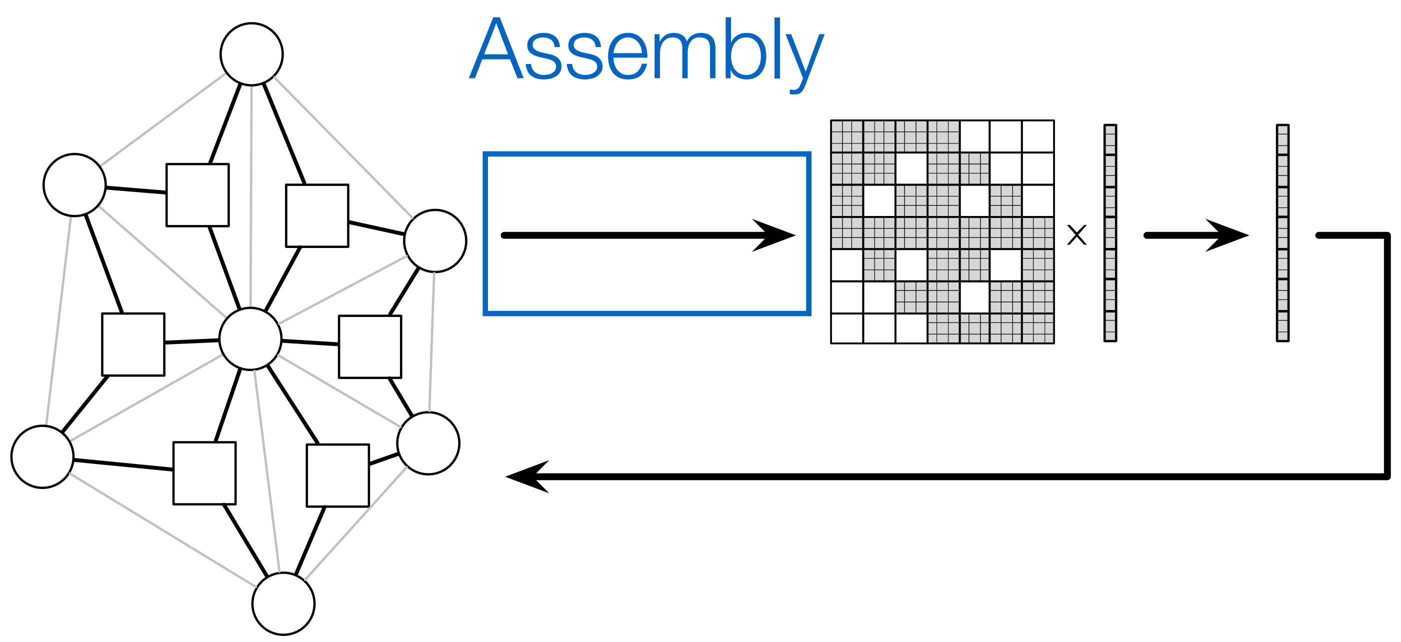
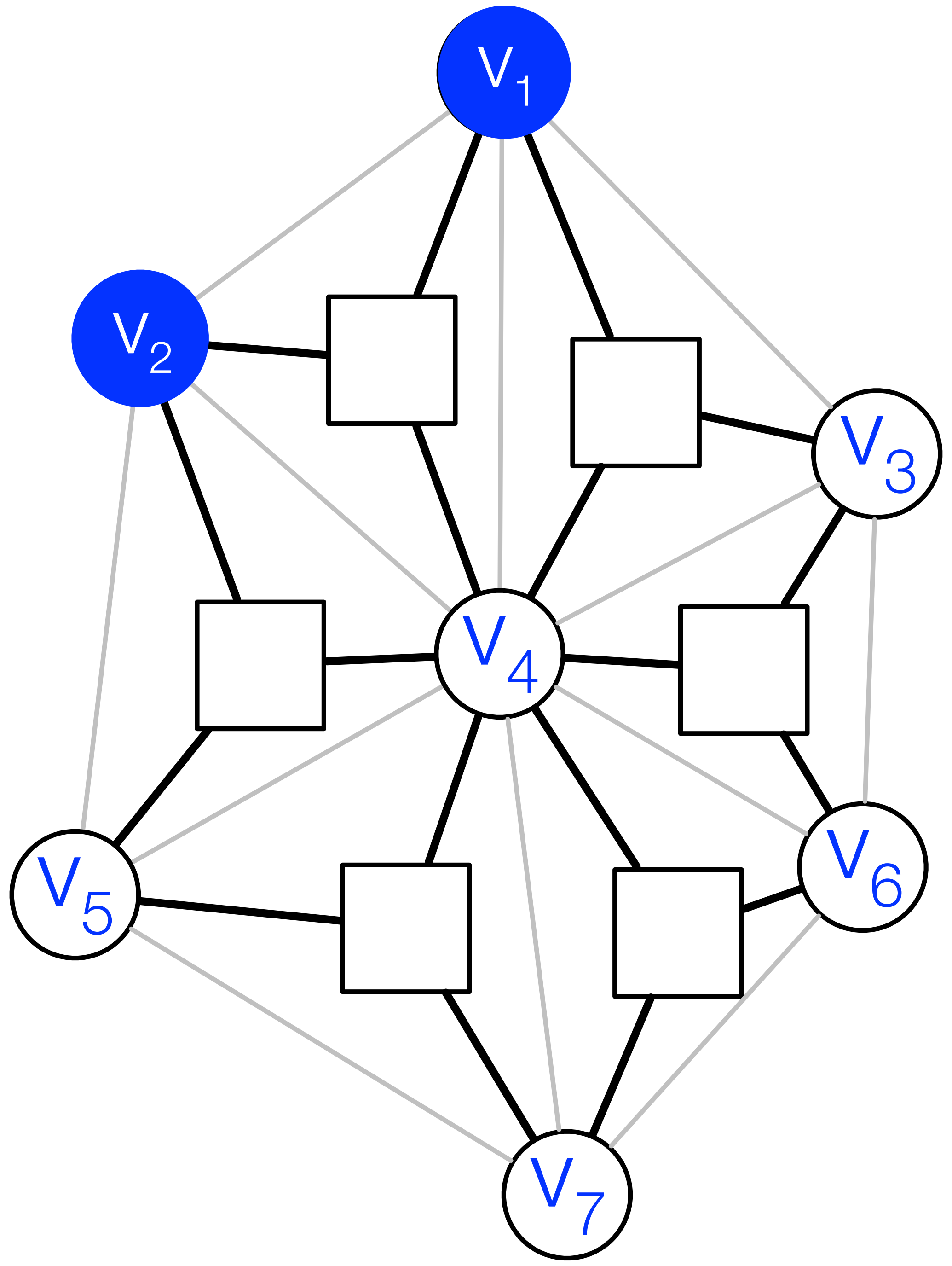
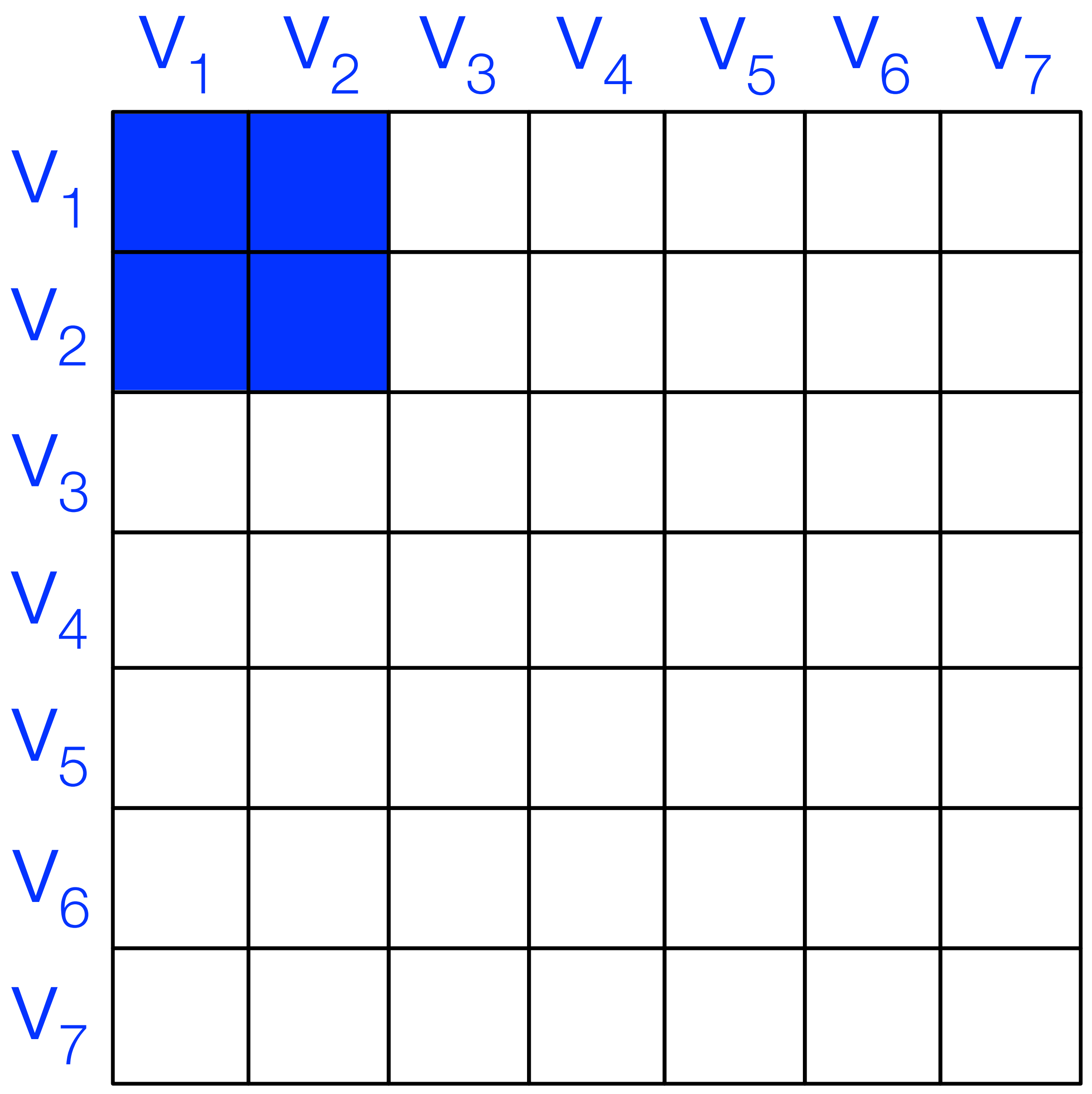
Statics Triangular Neo-Hookean FEM Simulation





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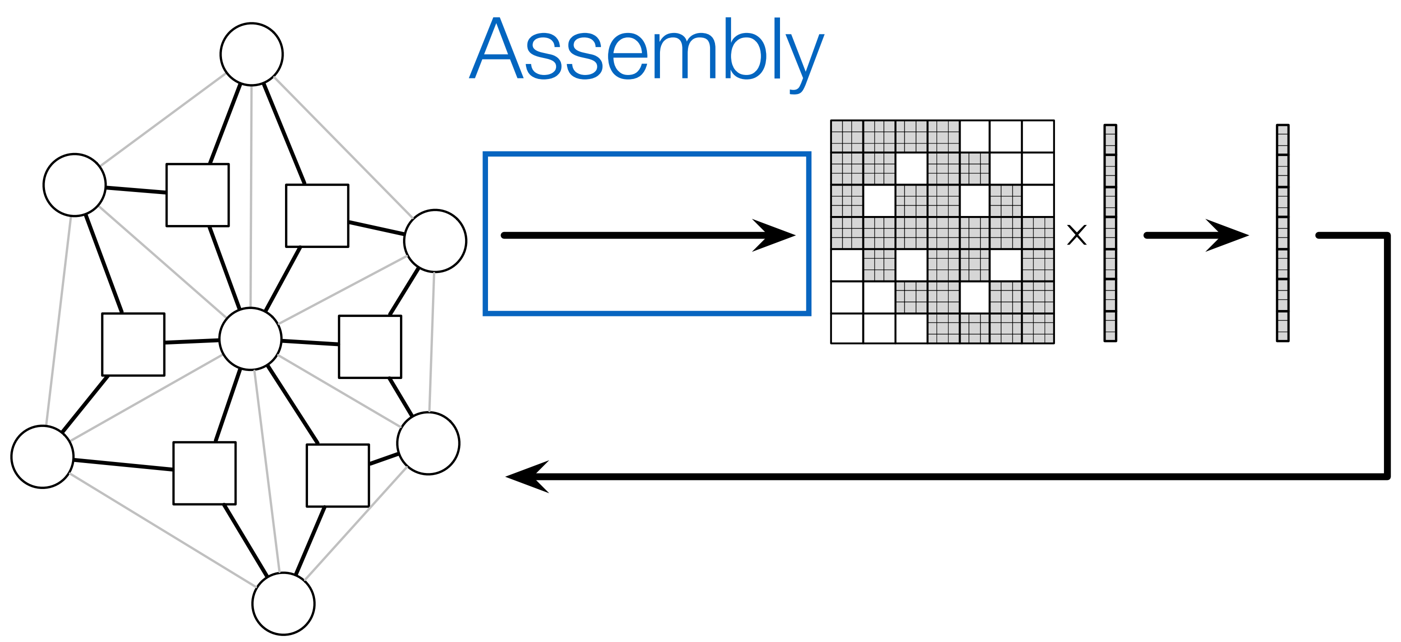
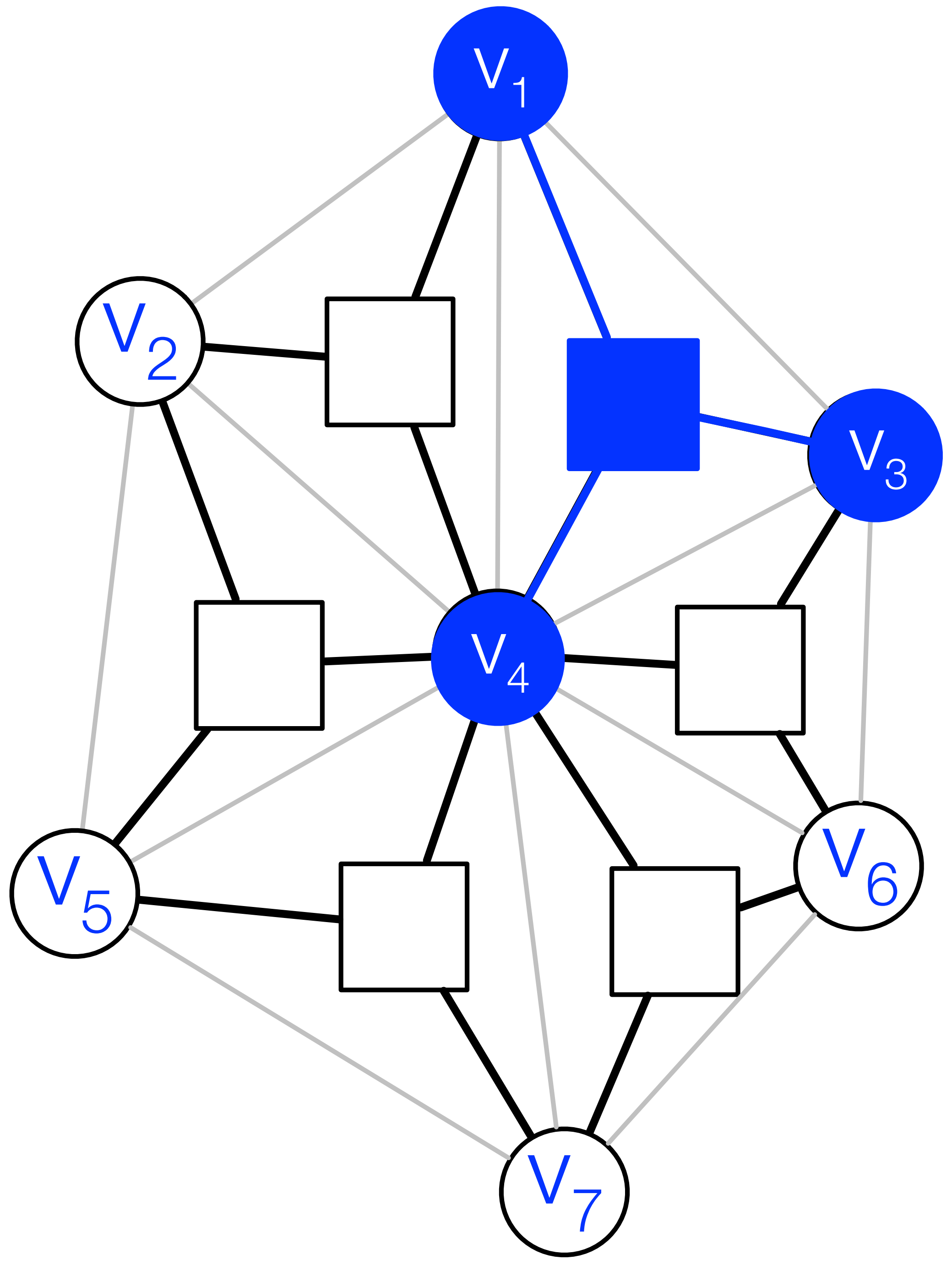
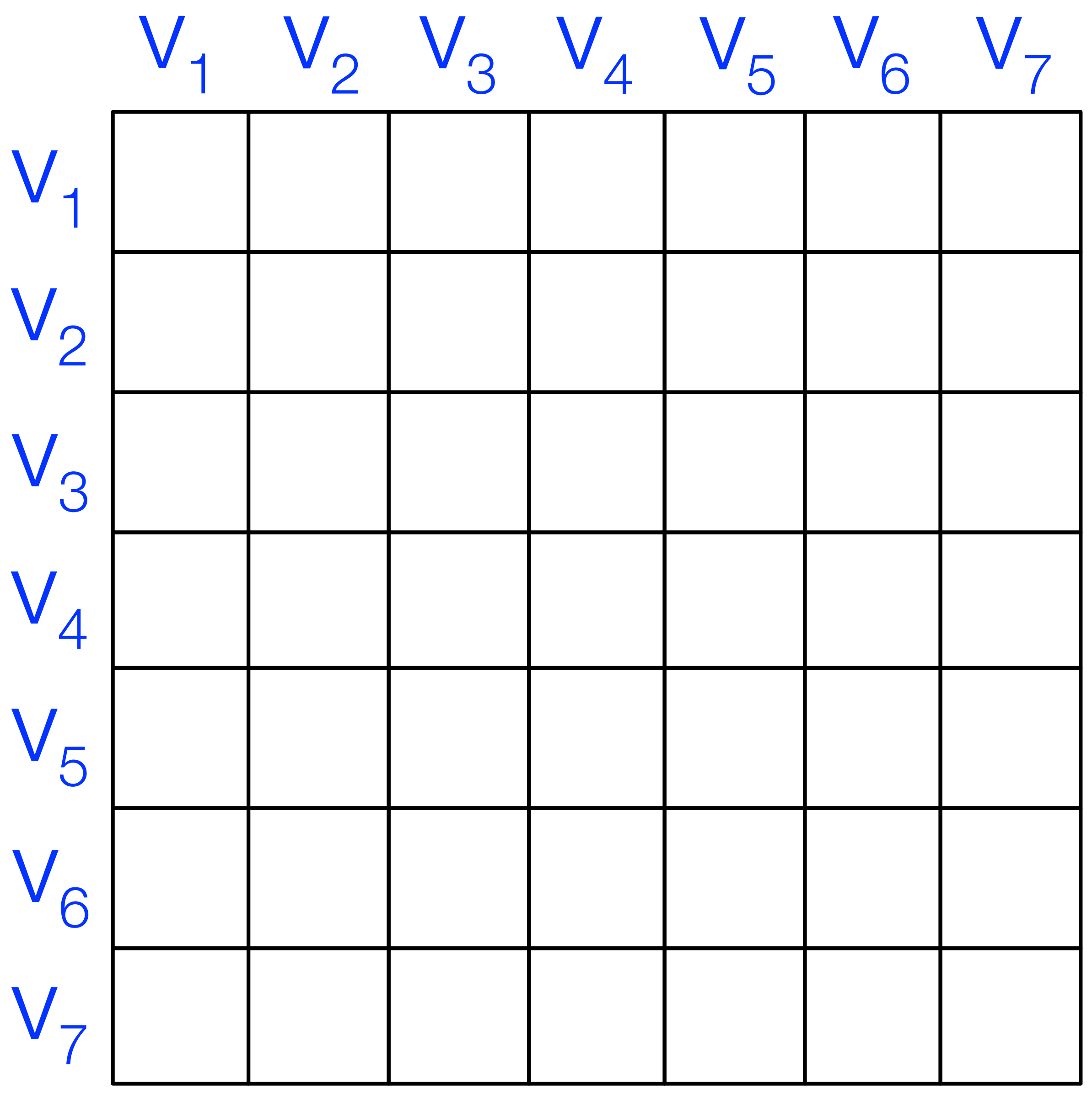
Statics Triangular Neo-Hookean FEM Simulation








```
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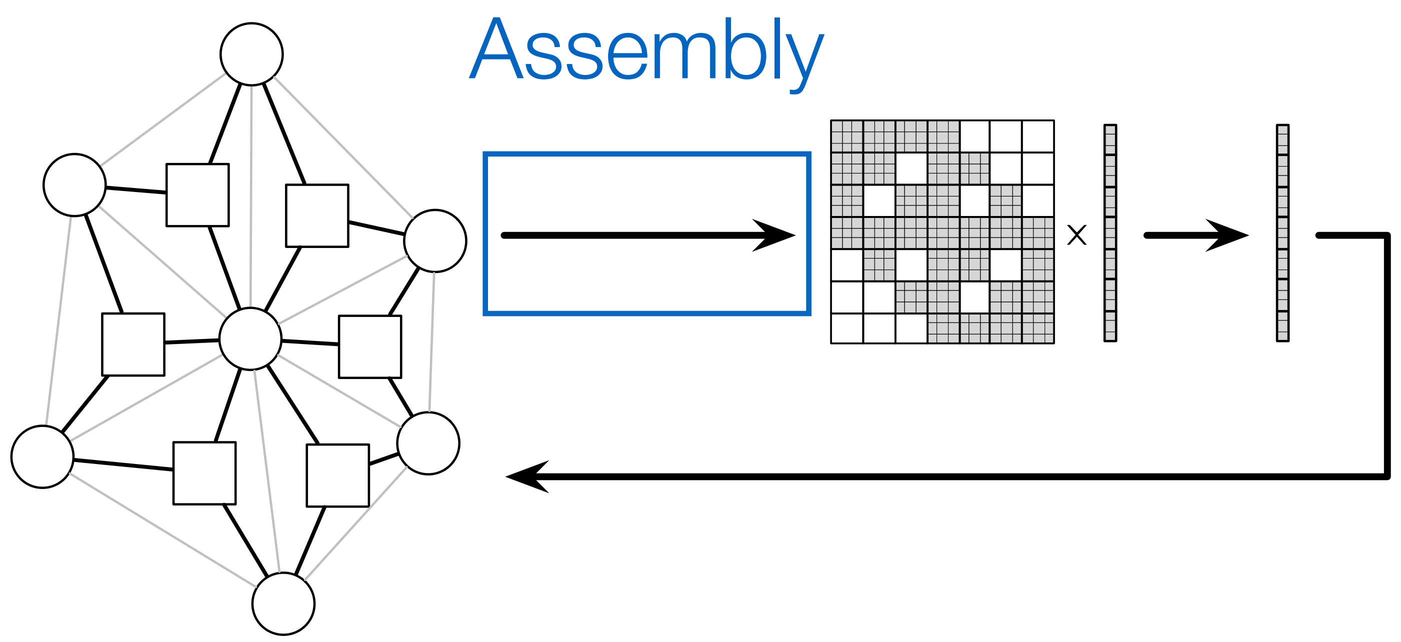
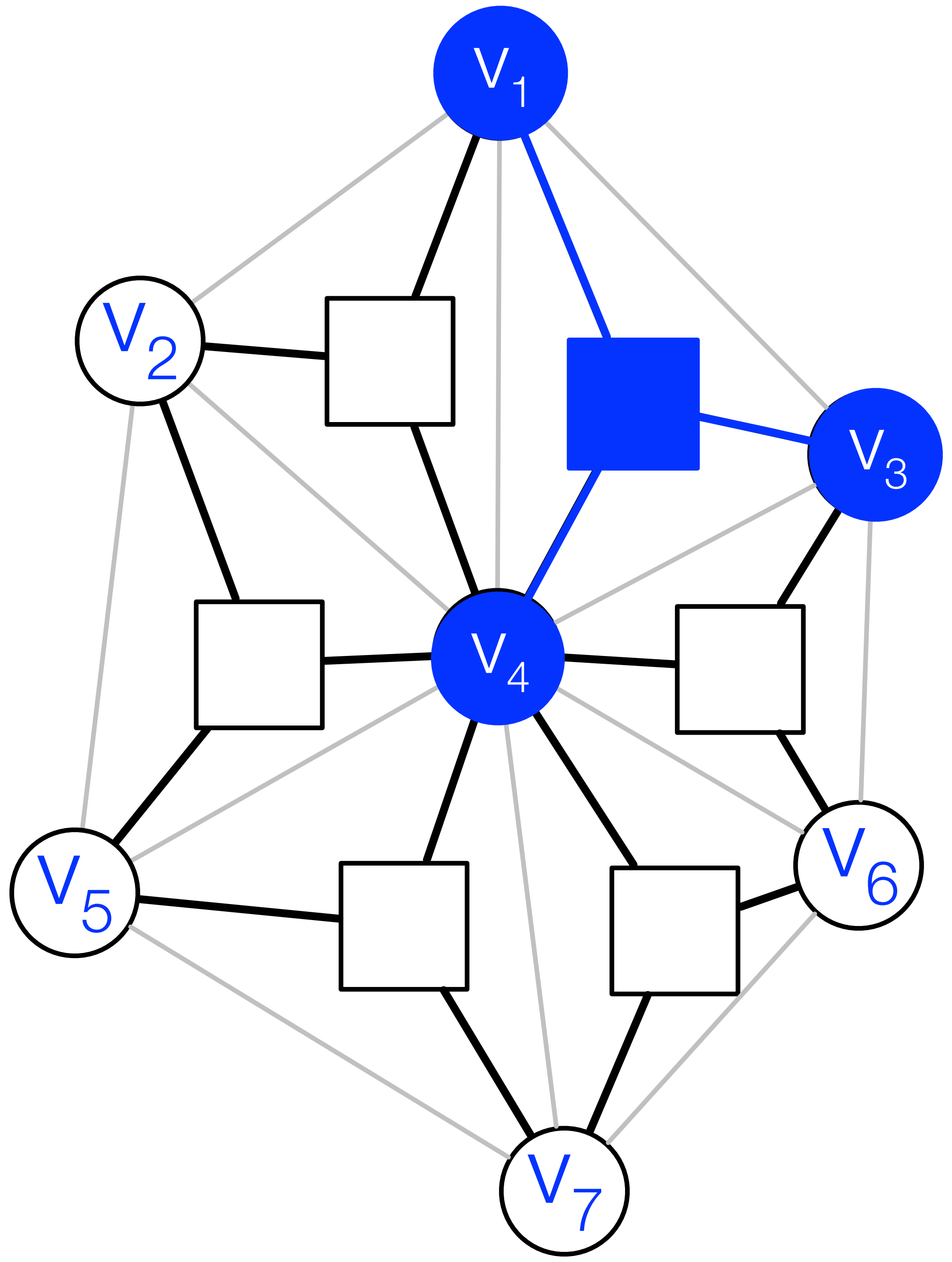
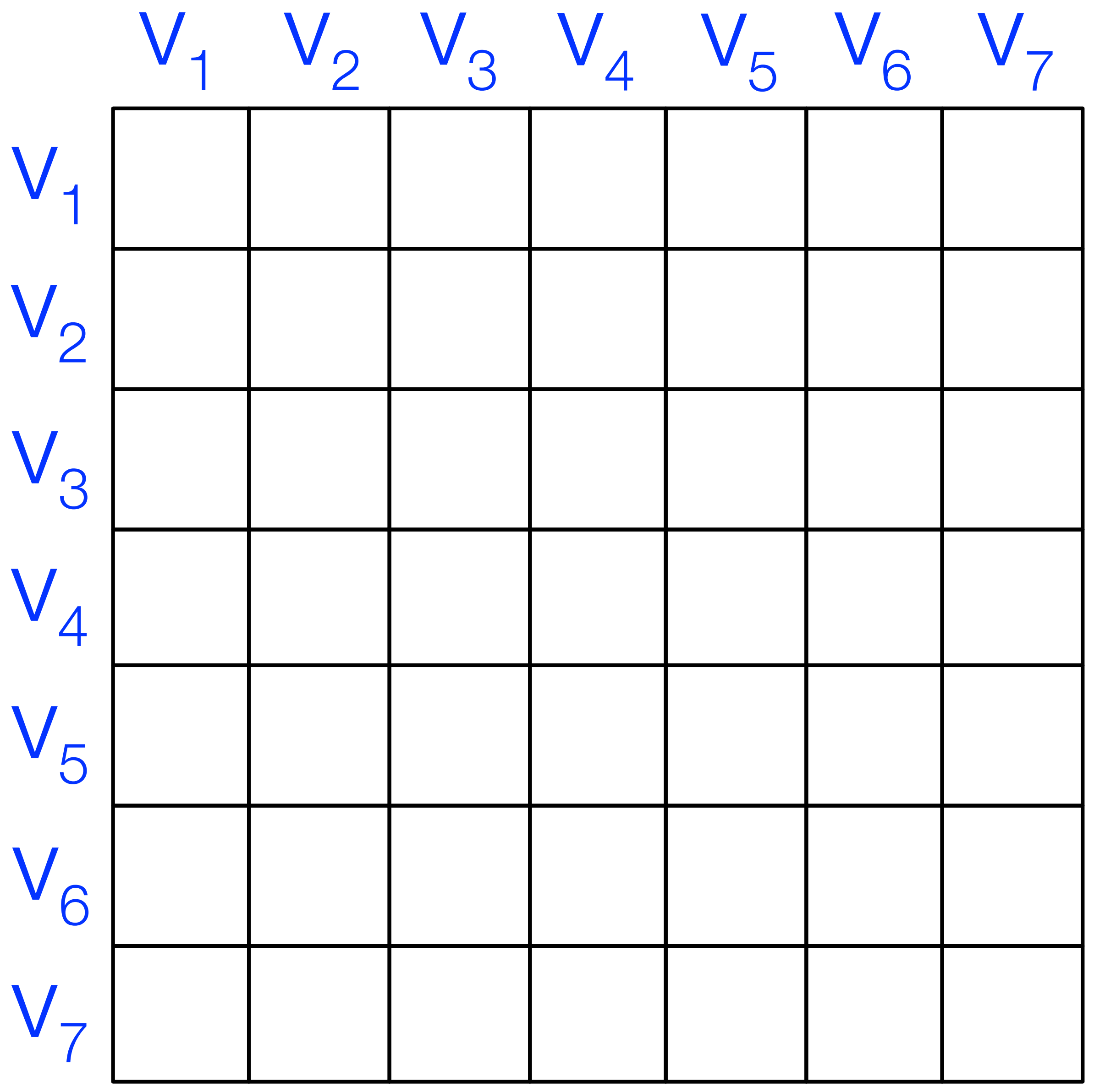
```

Statics Triangular Neo-Hookean FEM Simulation








```
element Vertex
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end

element Triangle
  u  : float;              % shear modulus
  l
  W
  B
end

% gr
exte
exte

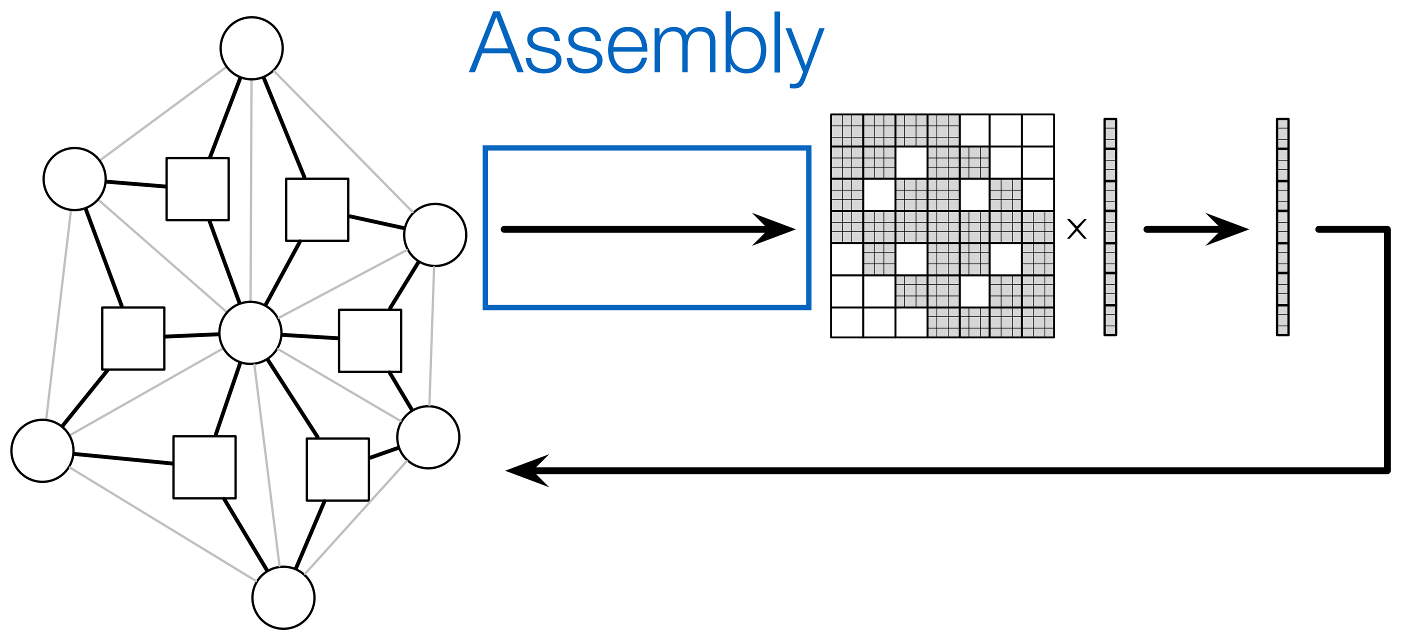
% co
func
  t.
  t.
end

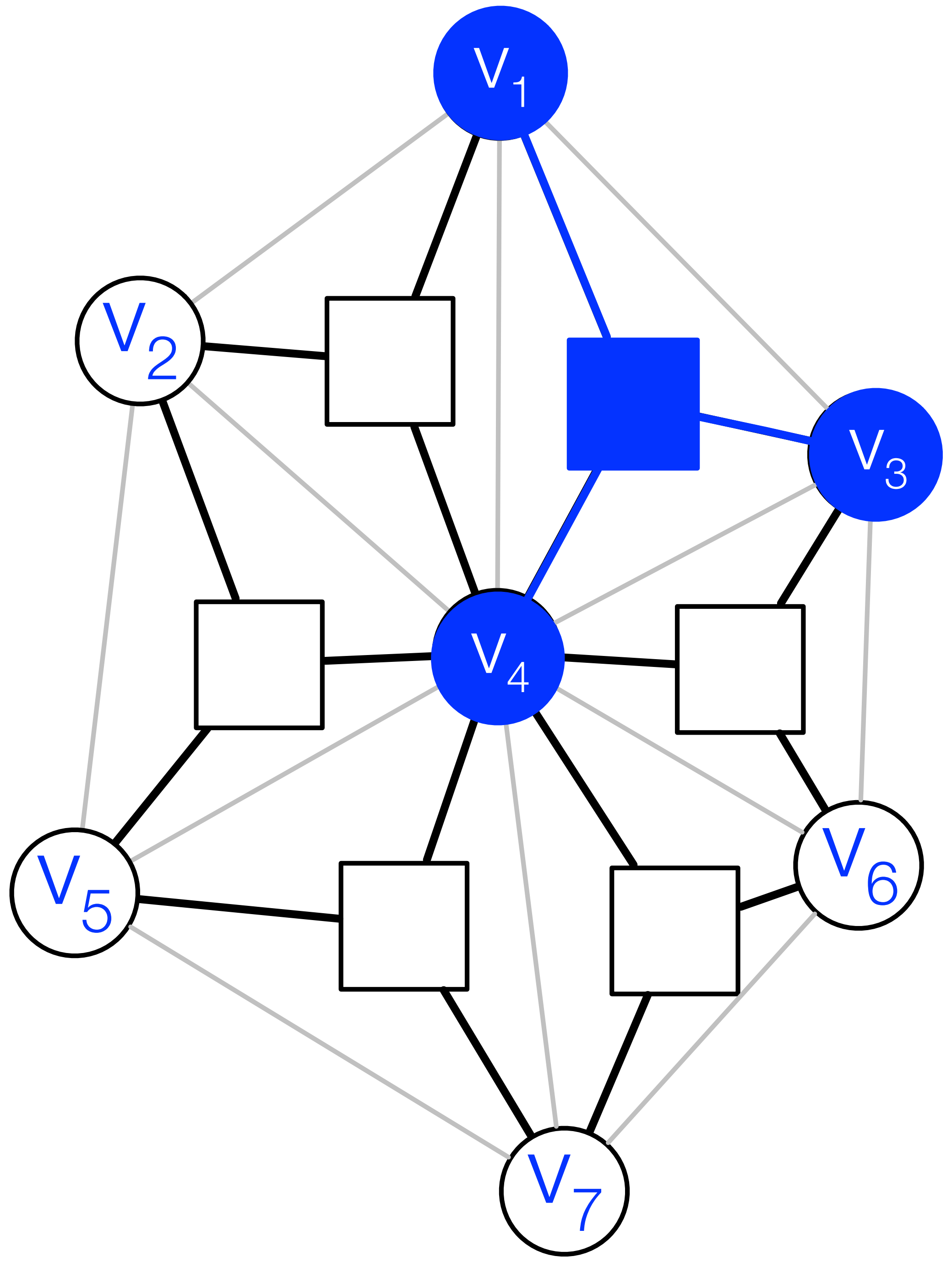
expo
  ap
end
```

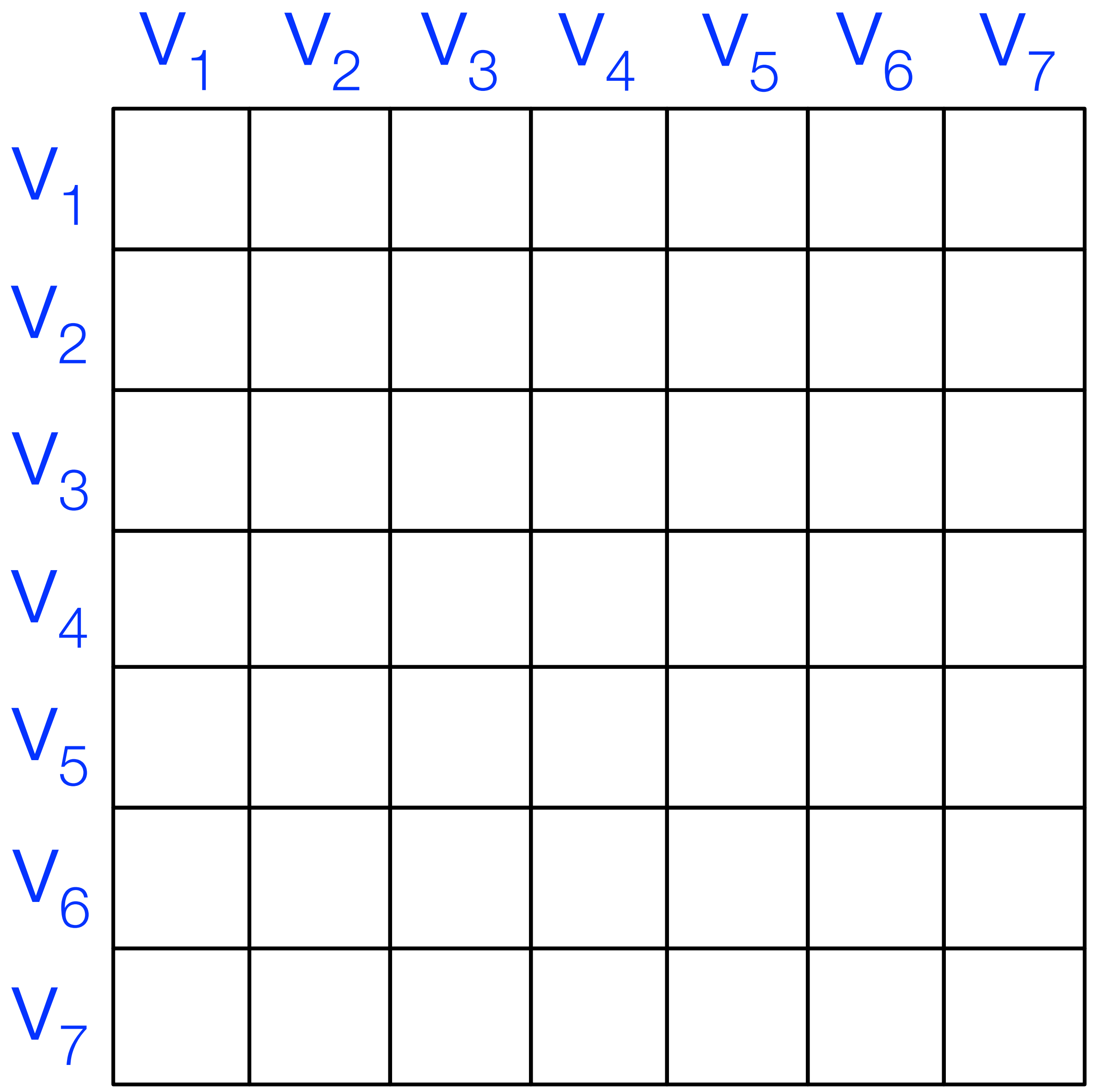
```
ce +;
```

```
% computes the stiffness of a triangle
func triangle_stiffness(t : Triangle, v : (Vertex*3))
    -> K : matrix[verts,verts](matrix[3,3](float))
  for i in 0:3
    for j in 0:3
      K(v(i),v(j)) += compute_stiffness(t,v,i,j);
    end
  end
end
```

Statics Triangular Neo-Hookean FEM Simulation





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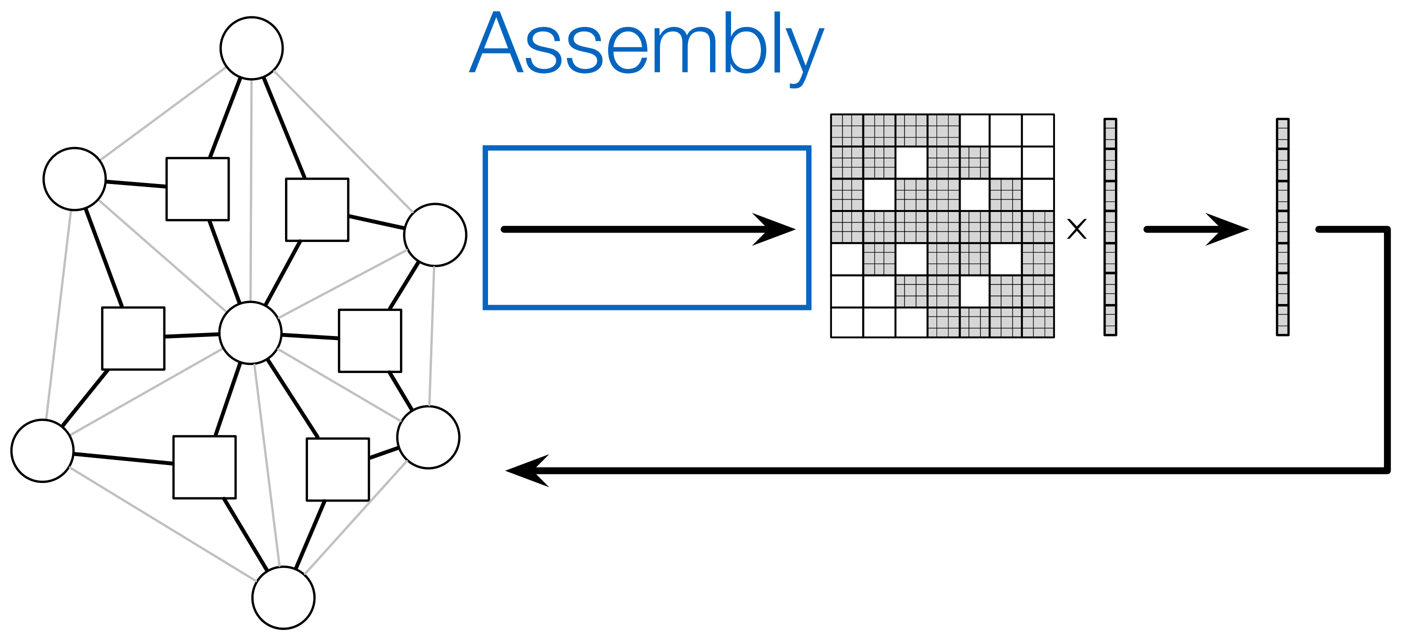

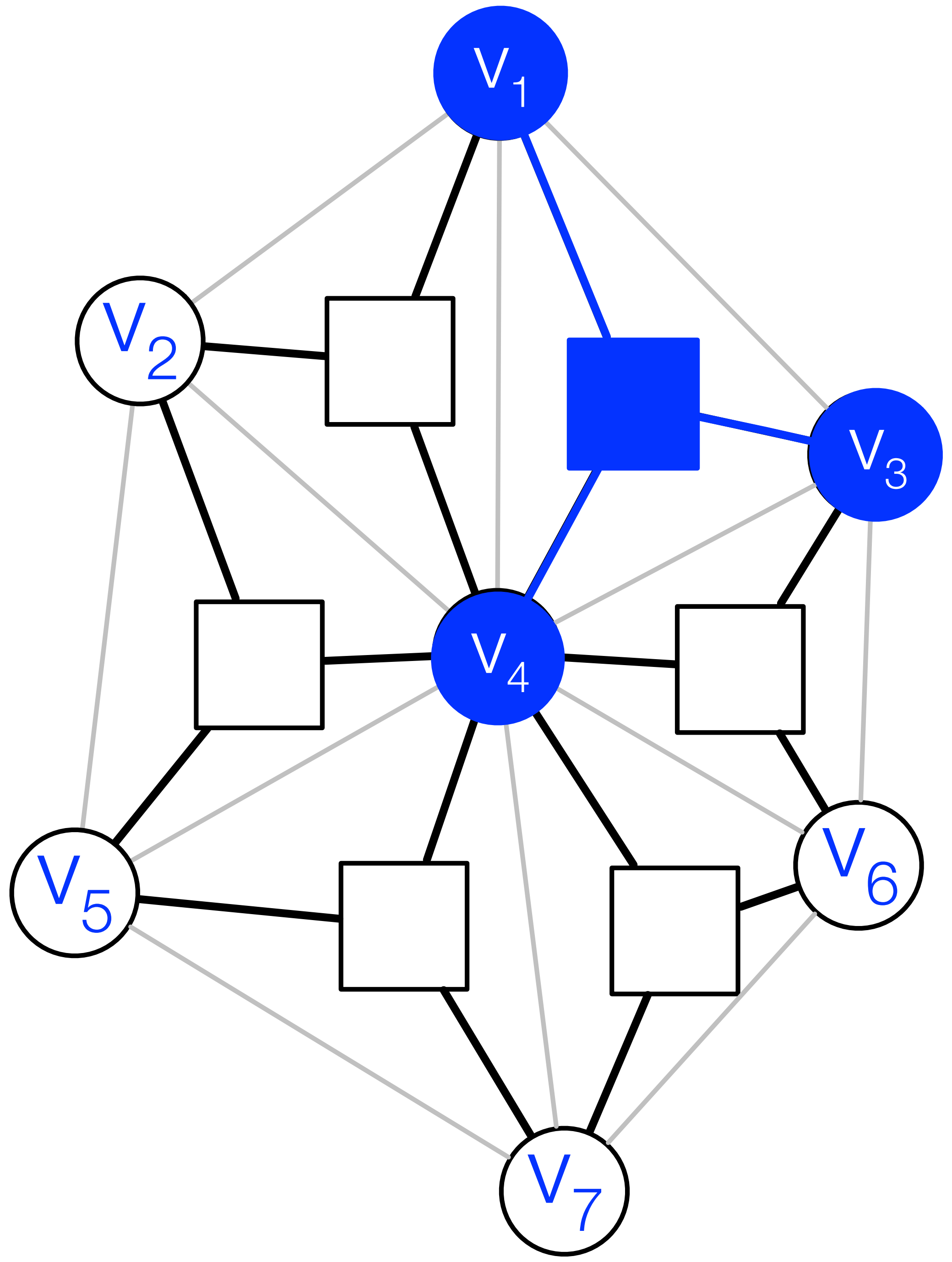

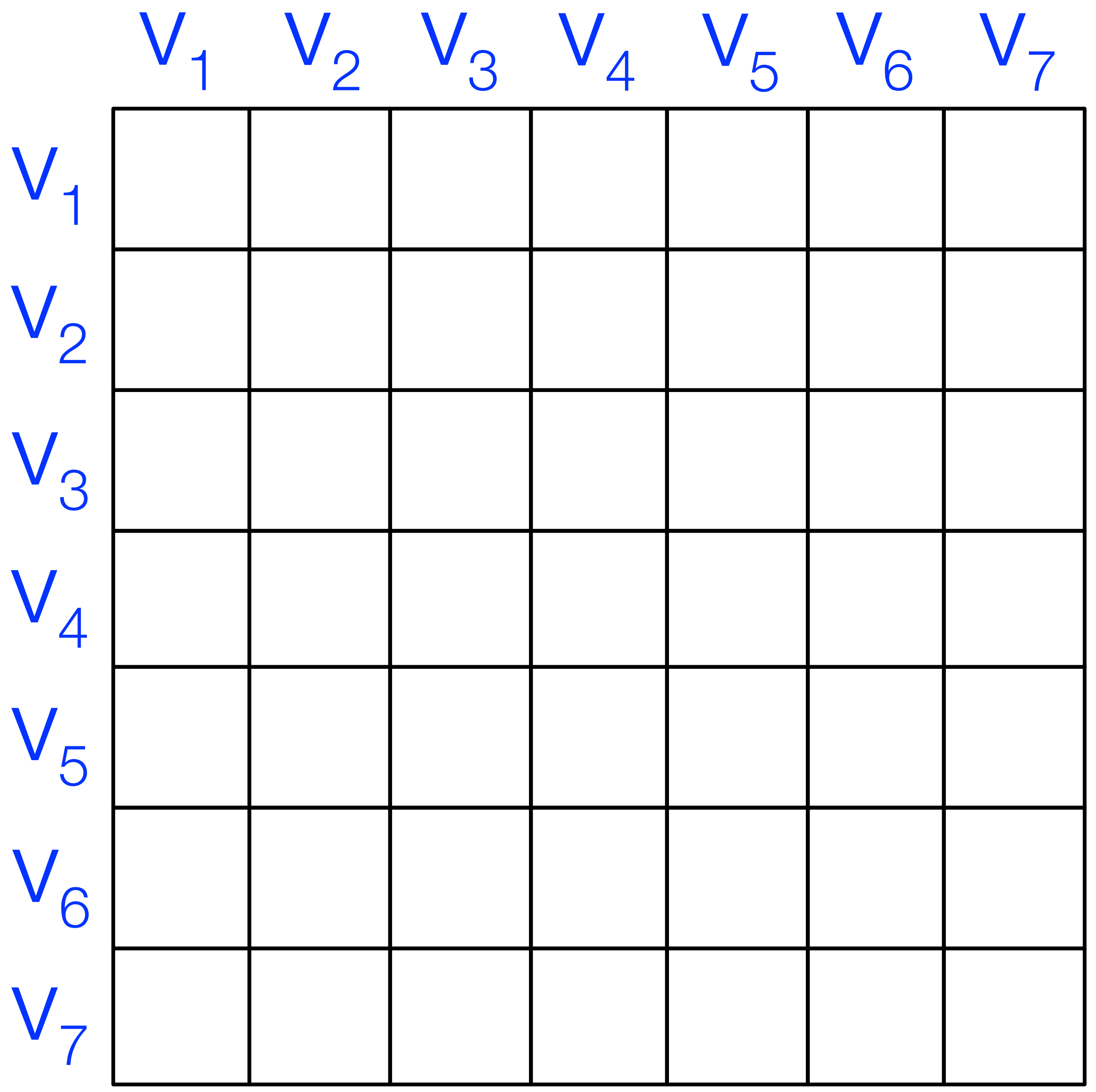


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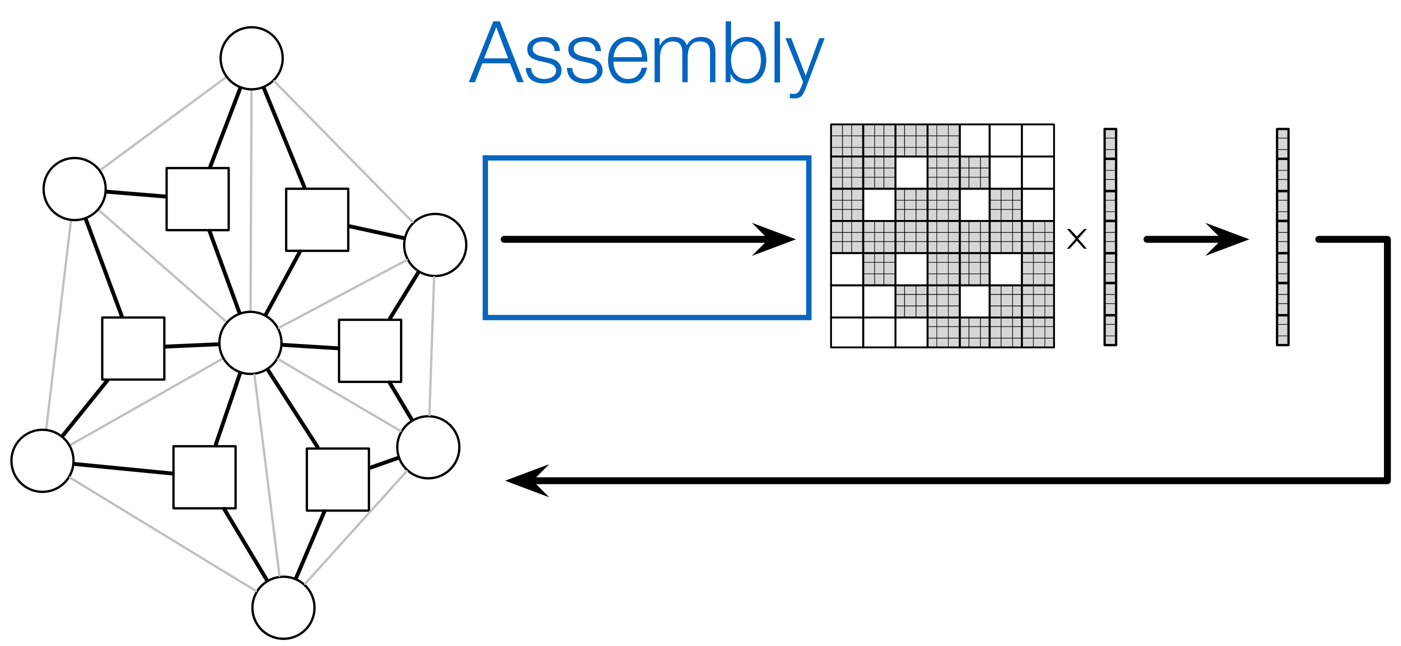

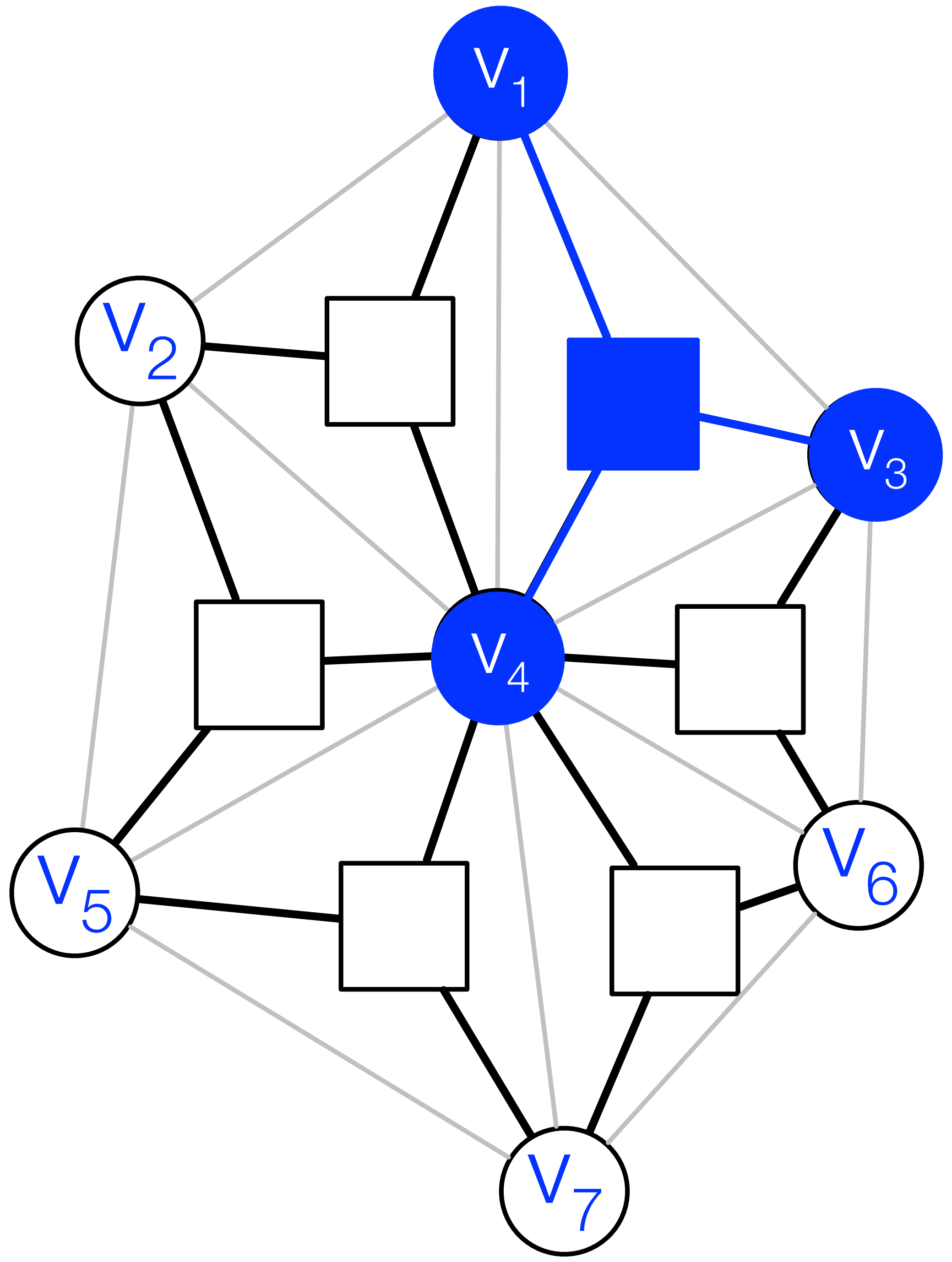

	V1	V2	V3	V4	V5	V6	V7
V1							
V2							
V3							
V4							
V5							
V6							
V7							

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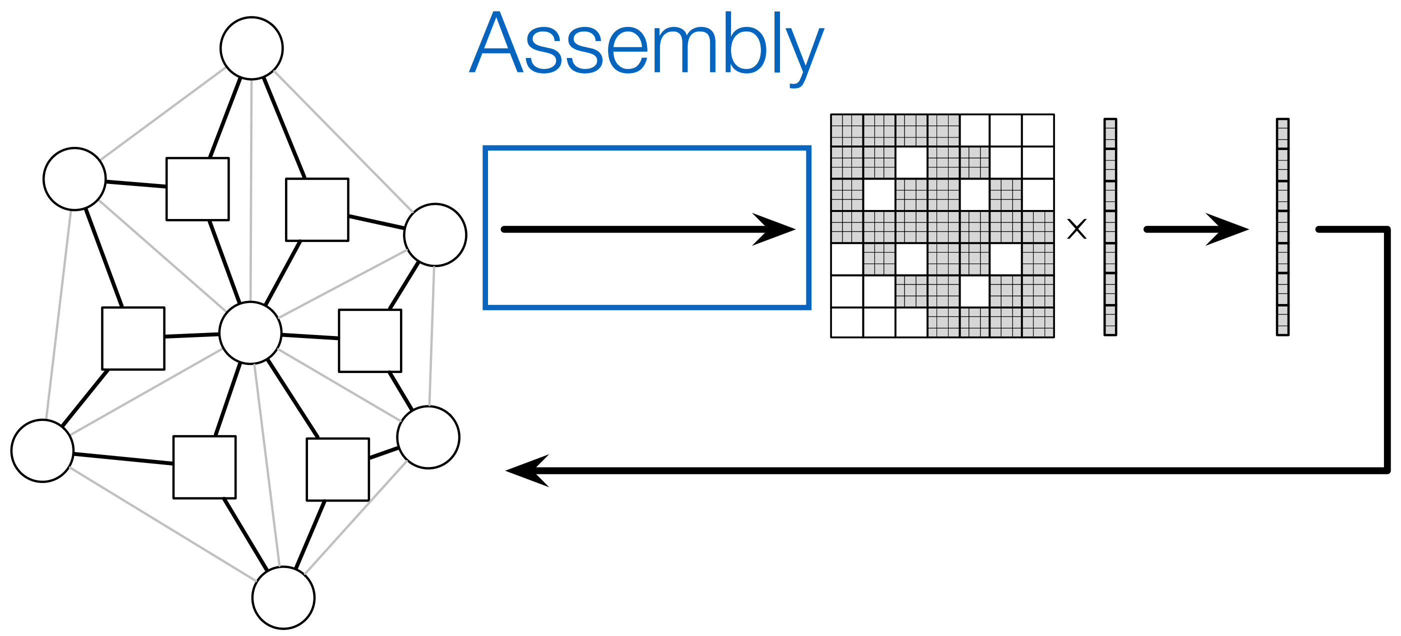

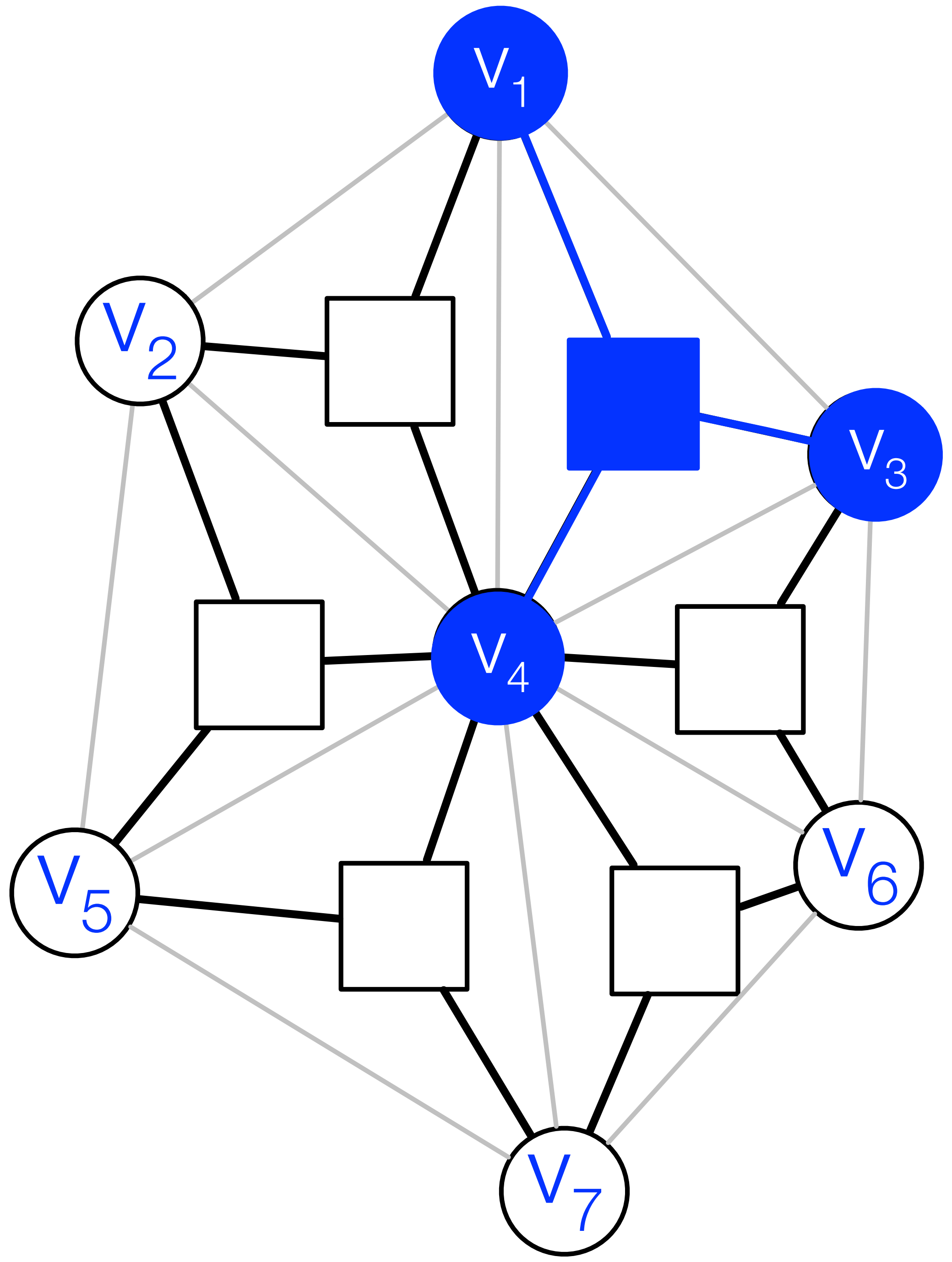

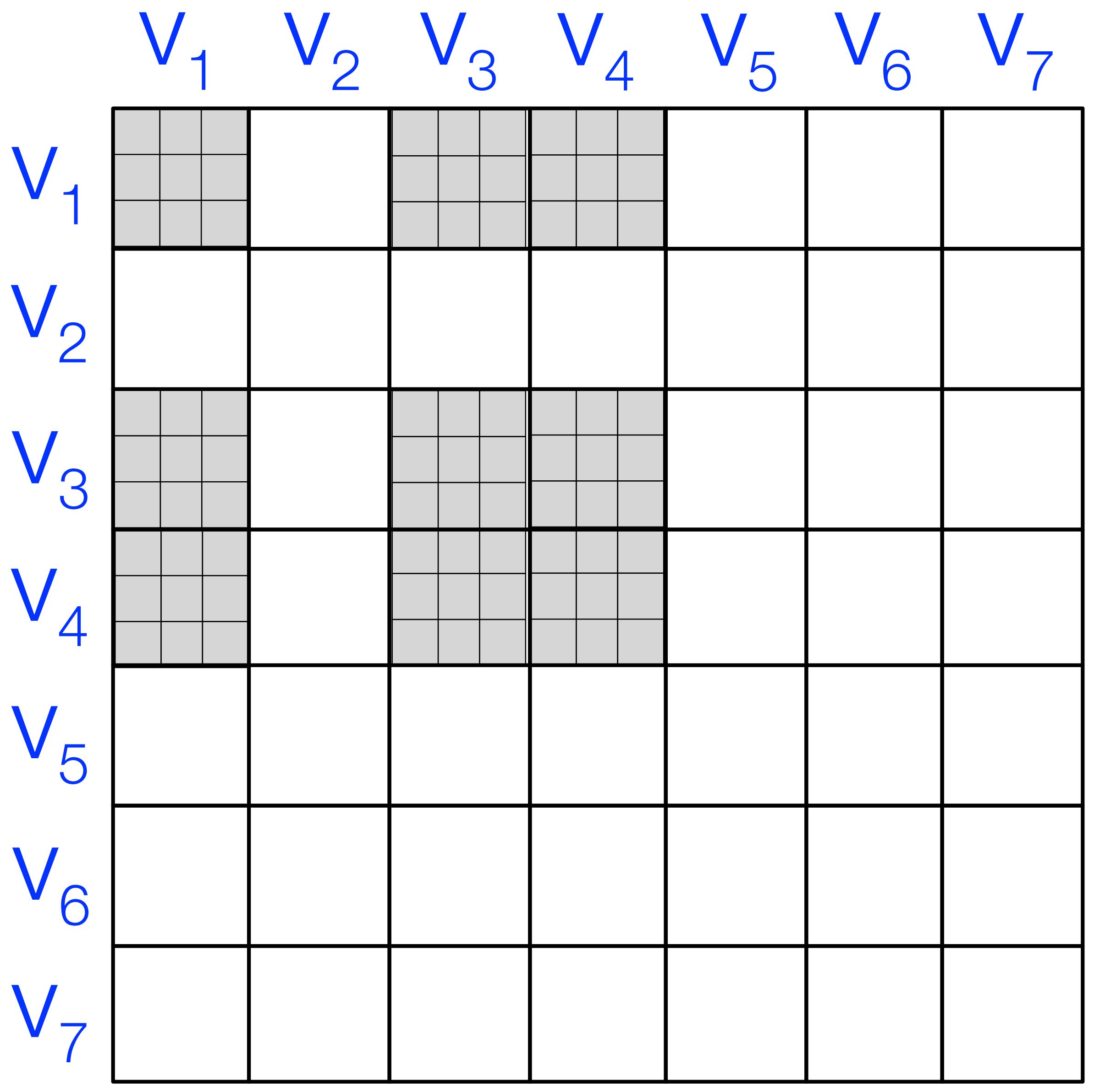


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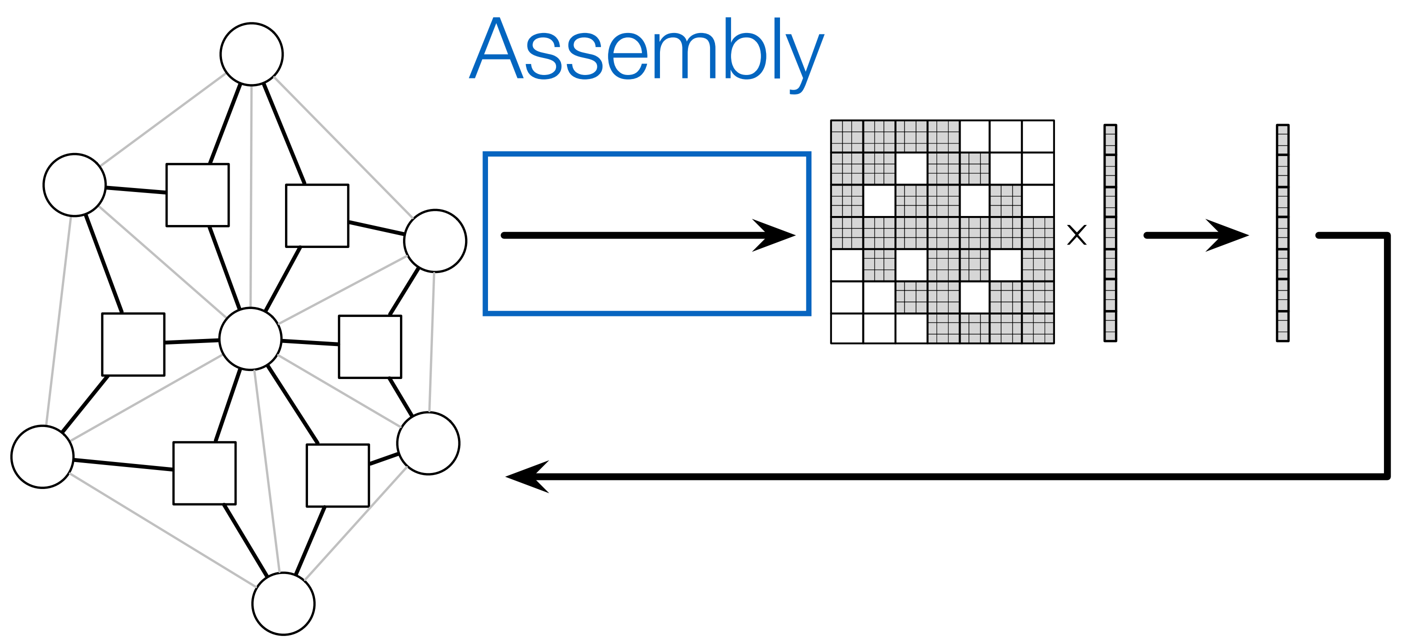

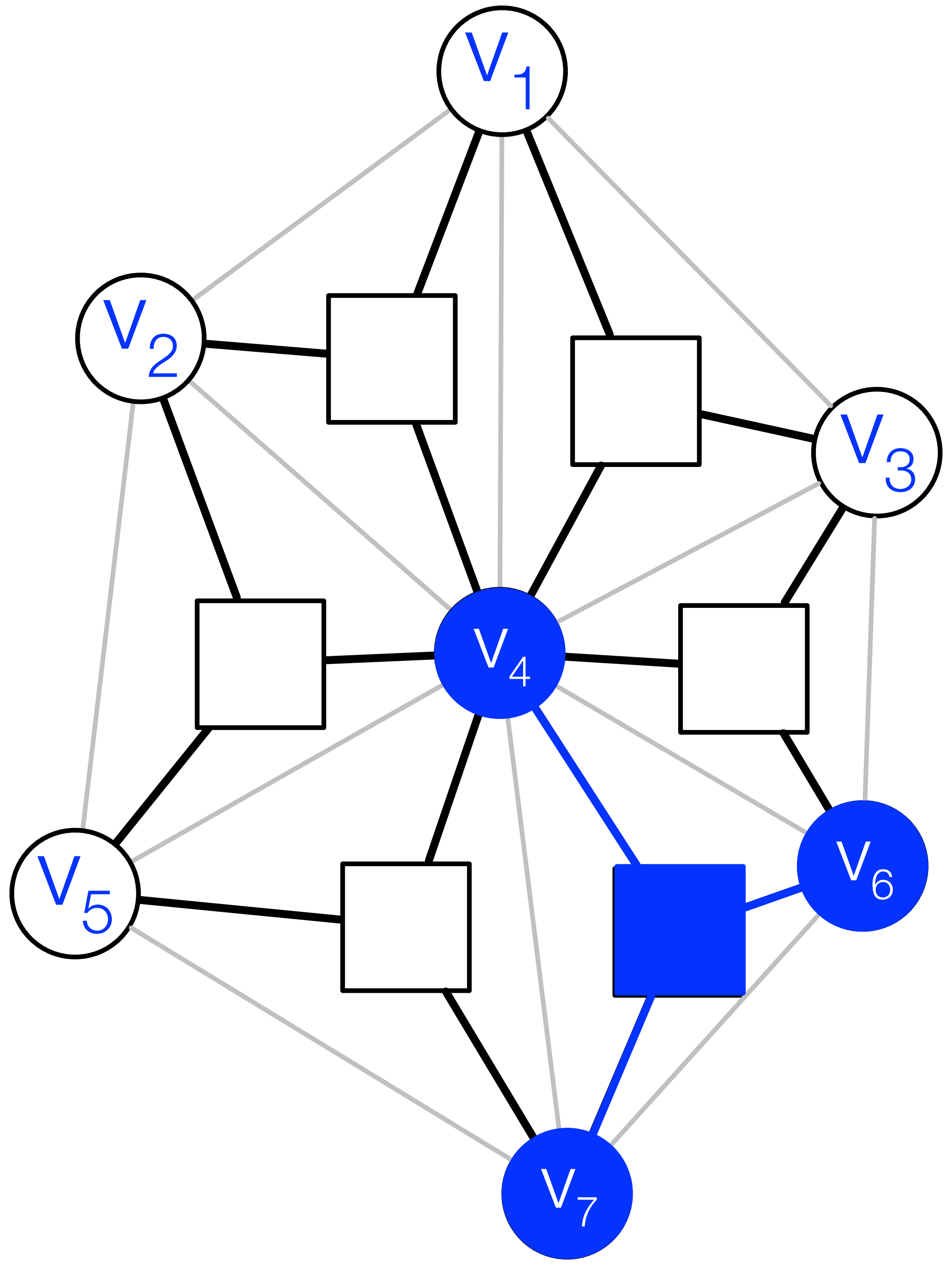

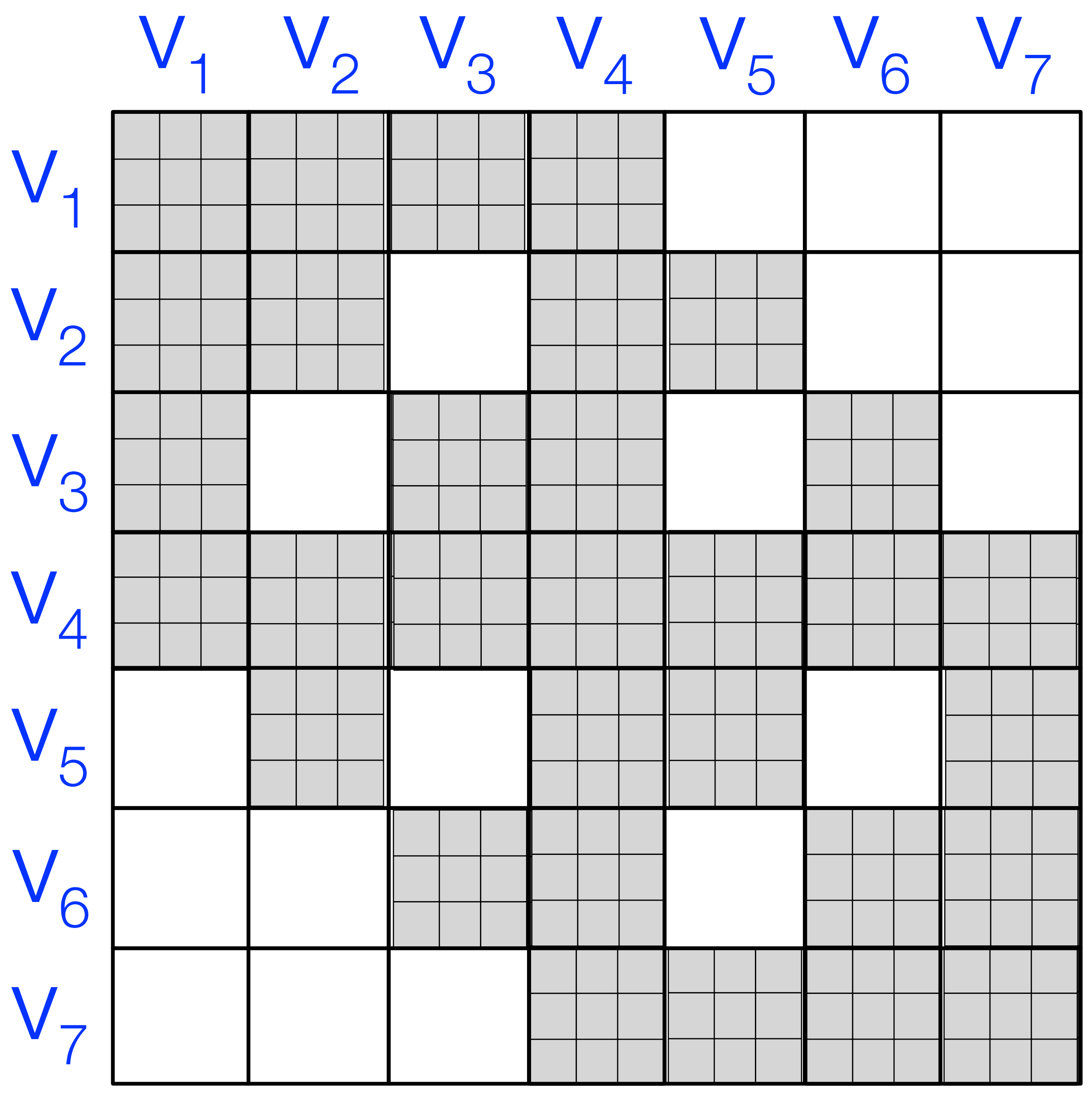


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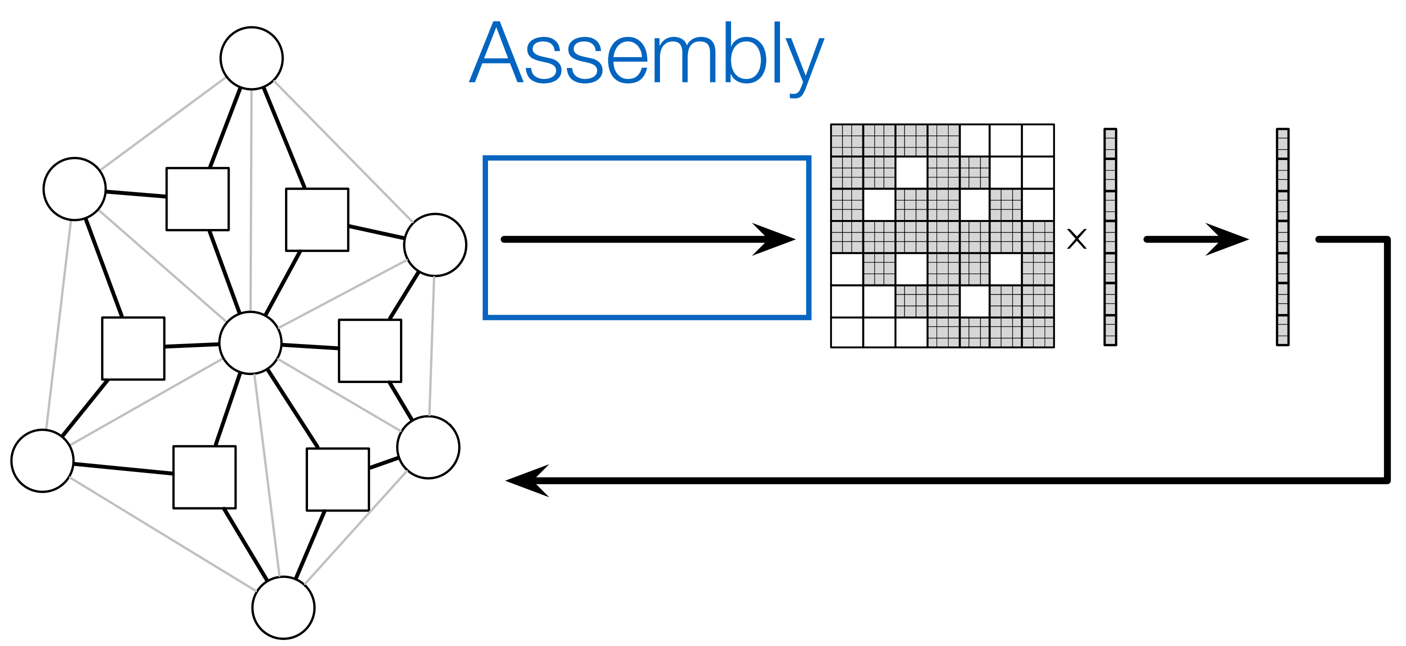
Statics Triangular Neo-Hookean FEM Simulation




```
element Vertex
  x  : vector[3](float);   % position
  v  : vector[3](float);   % velocity
  fe : vector[3](float);   % external force
end

element Triangle
  u  : float;              % shear modulus
  l  : float;              % lame's first parameter
  W  : float;              % volume
  B  : matrix[3,3](float); % strain-displacement
end

% graph vertices and triangle hyperedges
extern verts     : set{Vertex};
extern triangles : set{Triangle}(verts, verts, verts);

% compute triangle area
func compute_area(inout t : Triangle, v : (Vertex*3))
  t.B = compute_B(v);
  t.W = det(B) / 2.0;
end

export func init()
  apply compute_area to triangles;
end
```

```
% computes the stiffness of a triangle
func triangle_stiffness(t : Triangle, v : (Vertex*3))
    -> K : matrix[verts,verts](matrix[3,3](float))
  for i in 0:3
    for j in 0:3
      K(v(i),v(j)) += compute_stiffness(t,v,i,j);
    end
  end
end
```

```
% newton's method
export func newton_method()
  while abs(f - verts.fe) > 1e-6
    K = map triangle_stiffness to triangles reduce +;
    // assemble force vector
    // compute new position
  end
end
```

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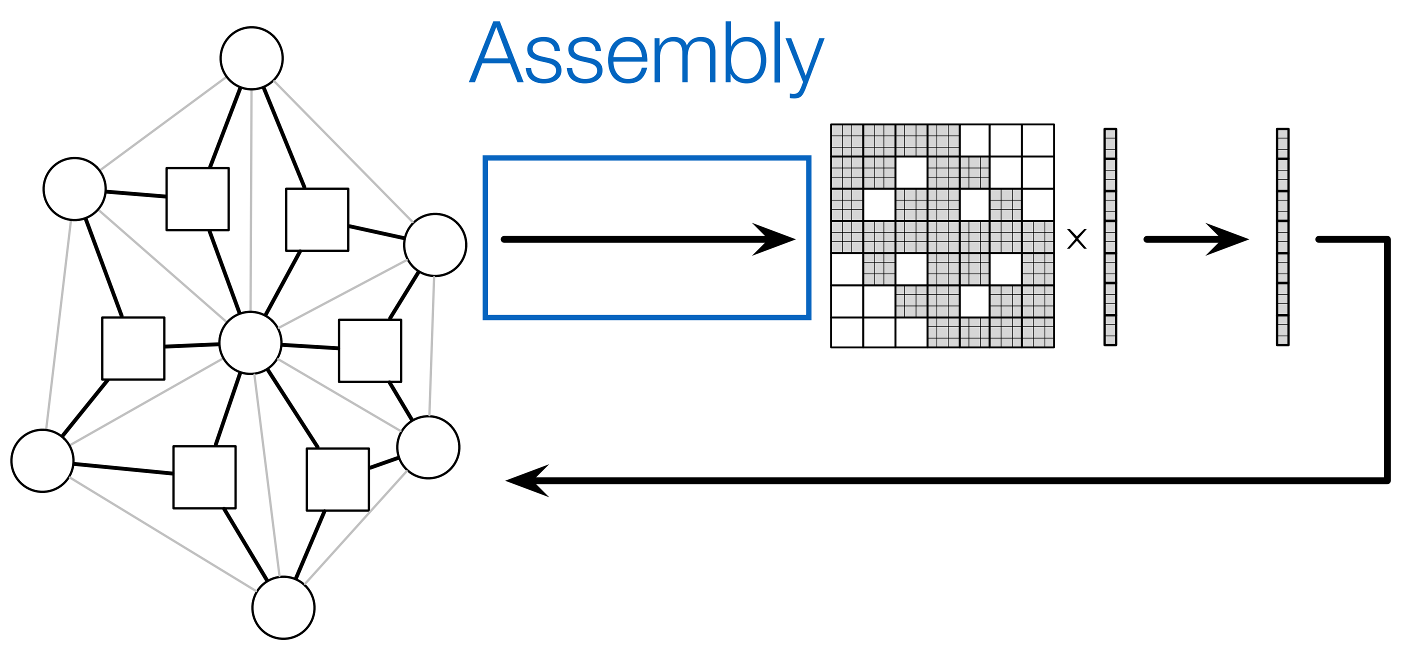


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% computes the force of a triangle on its vertices
func triangle_force(t : Triangle, v : (Vertex*3))
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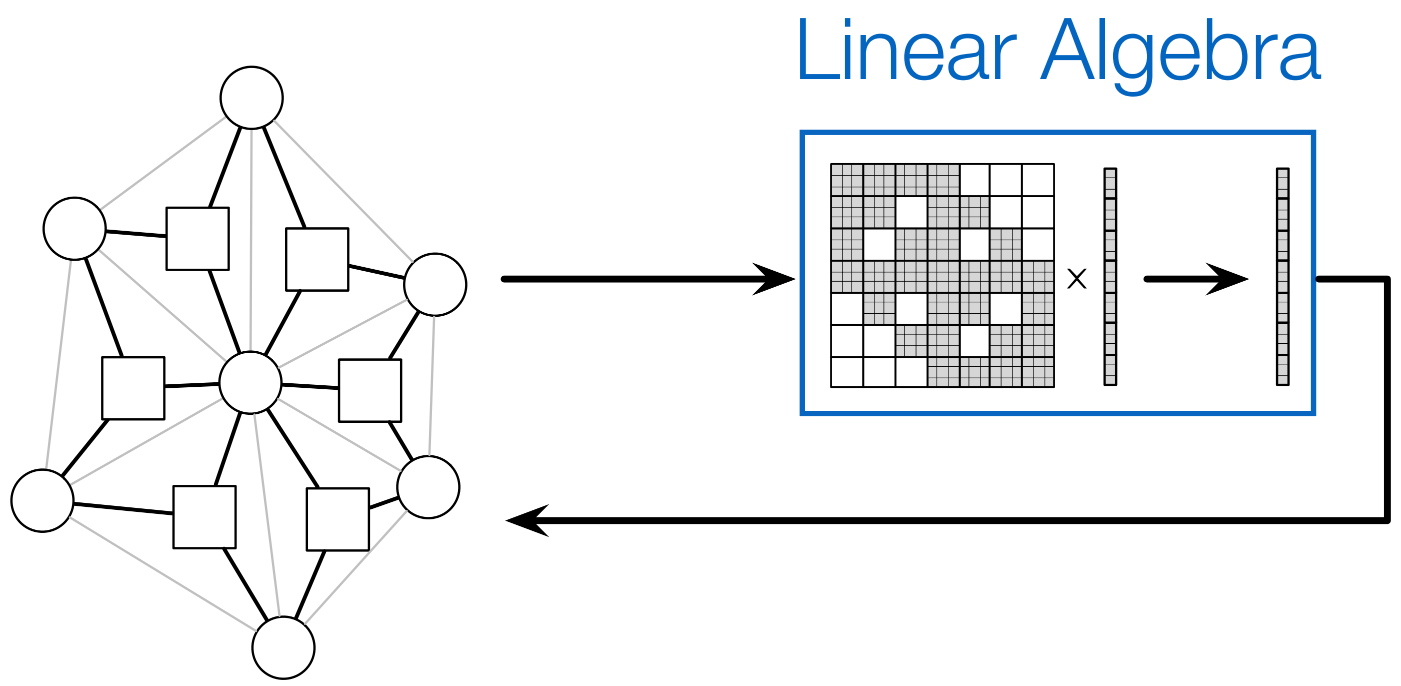

$$x_{t+1} = x_t + K^{-1}(f_{external} - f)$$

```
verts.x = verts.x + K \ (verts.fe - f);
```

```
element Vert
  x  : vecto
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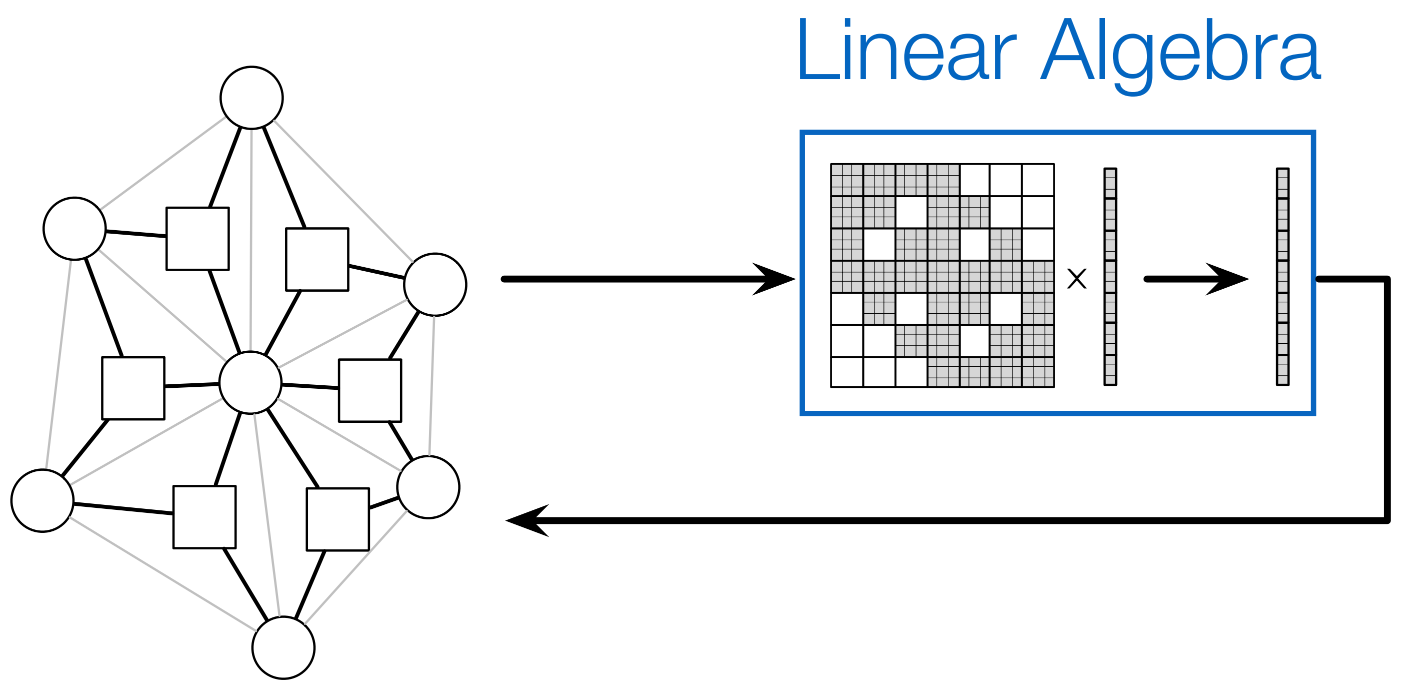

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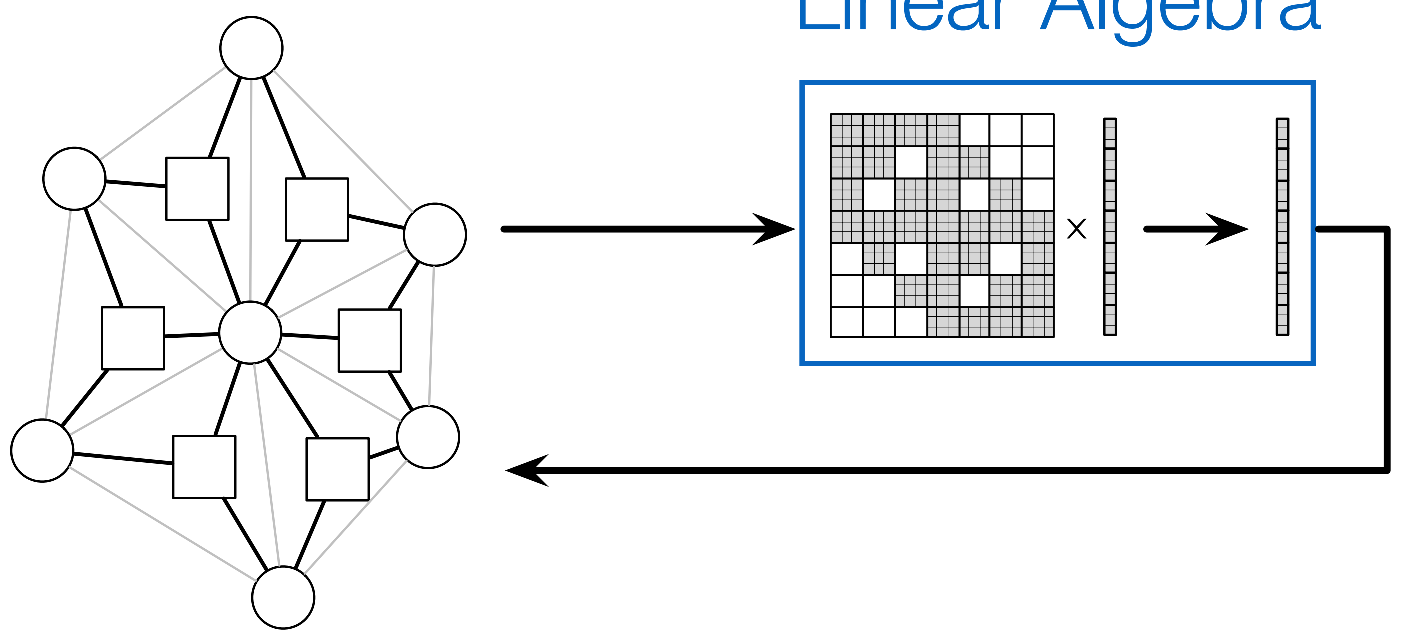

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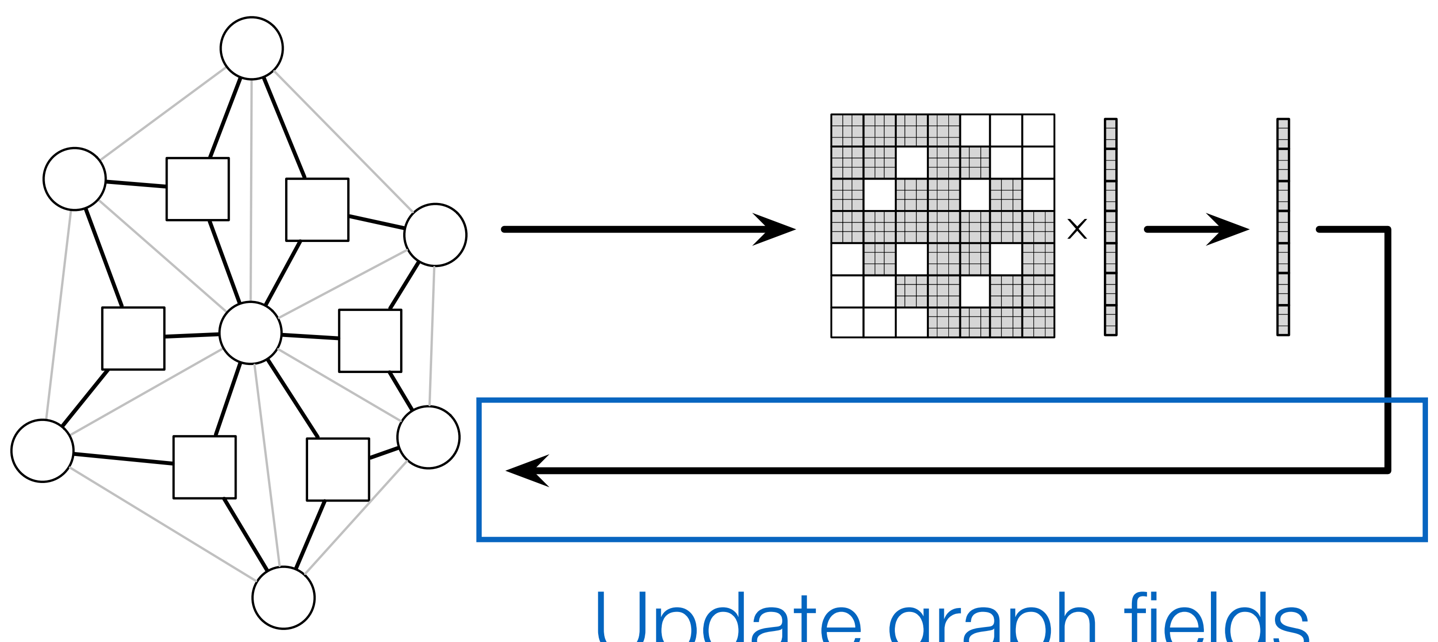

Update graph fields

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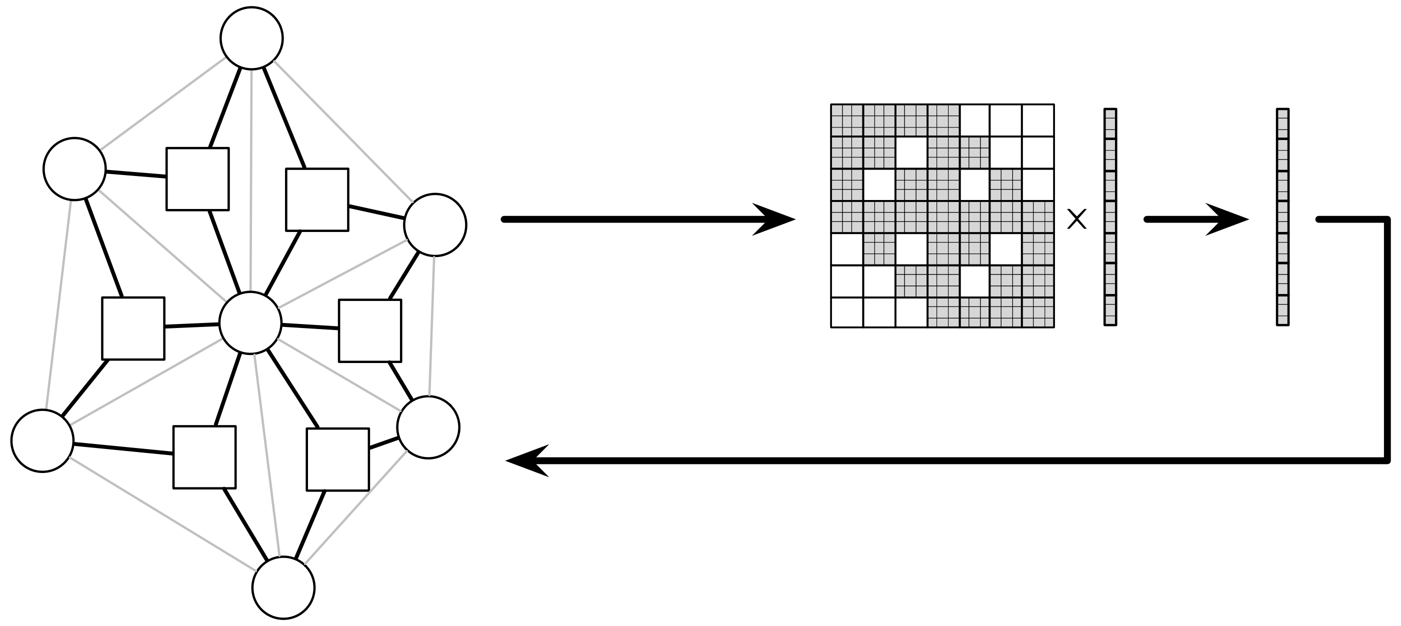

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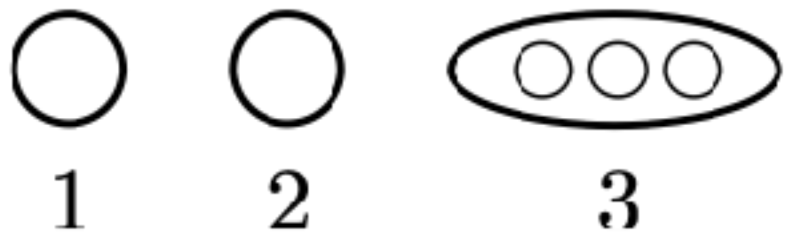
Collection-Oriented Languages

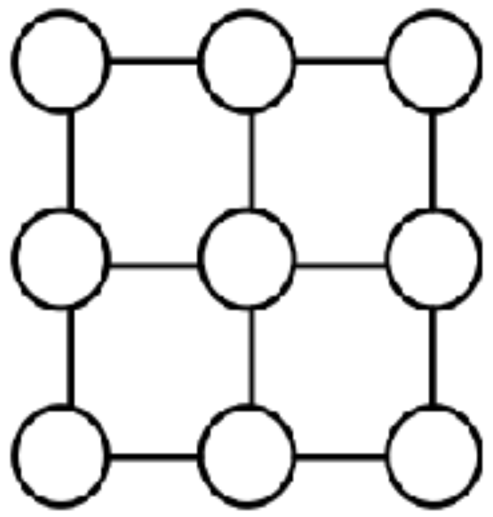
Lists
Lisp M58

Sets
SETL S70

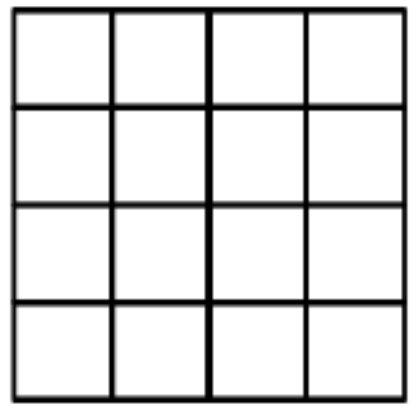
Nested Sequences
NESL B94



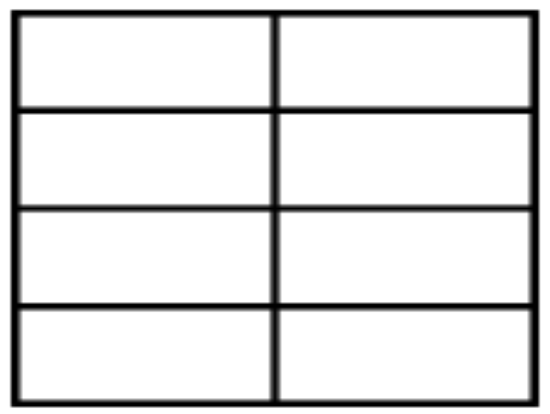
Grids
Sejits S09, Halide



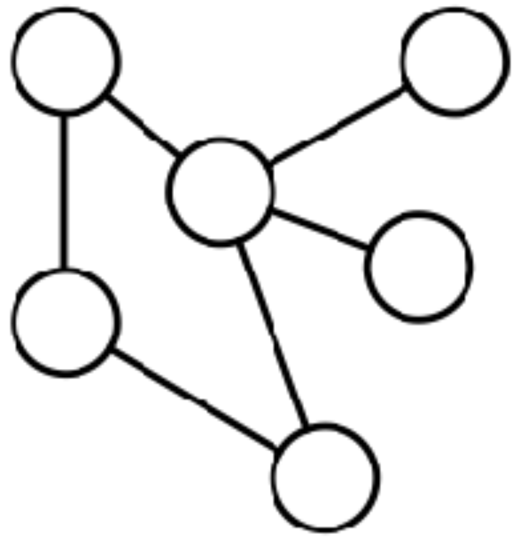
Arrays
APL I62
NumPy



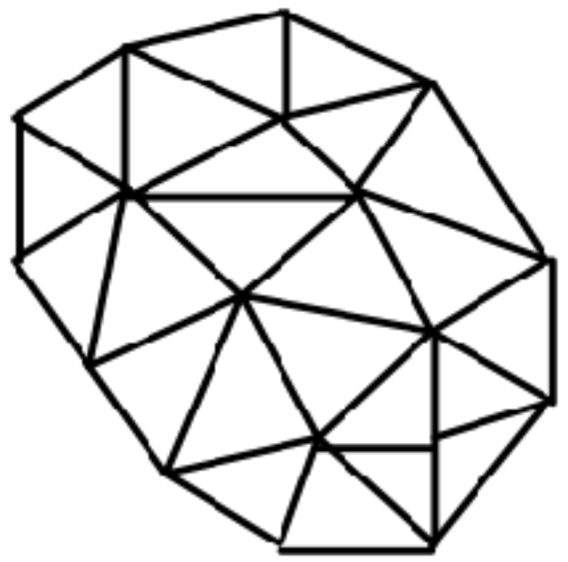
Relations
Relational Algebra C70,



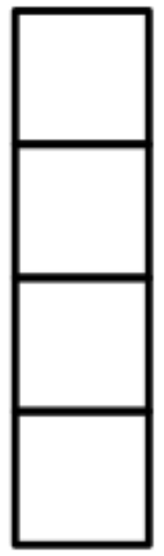
Graphs
GraphLab L10



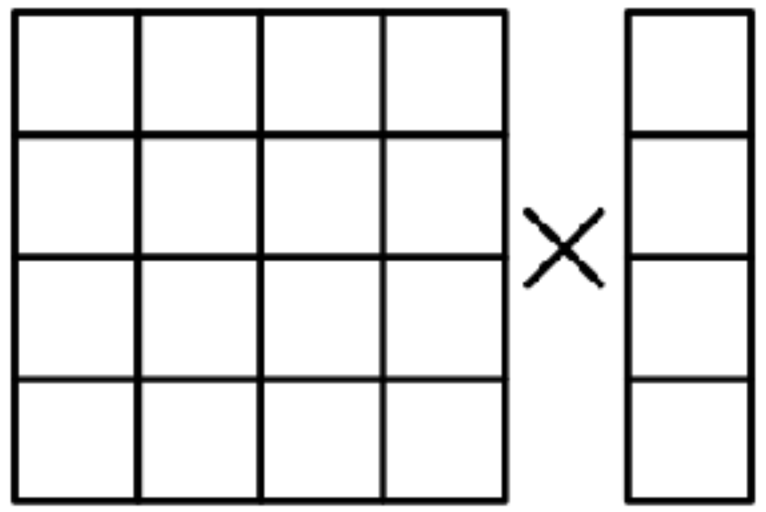
Meshes
Liszt D11



Vectors
Vector Model B90



Matrices and Tensors
Matlab M79, taco K17



A collection-oriented programming model provides collective operations on some collection/abstract data structure